# Shammas Polynomial Approximations

By

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# INTRODUCTION

In the HHC 2008 conference held in Corvallis, Oregon, I introduced the Shammas polynomials as polynomials with non-integer powers. I explained that the powers of such polynomial change using various math expressons that involves the polynomial term sequence number. Moreover, the study explained that such expressons must never produce the same powers of two or more terms. In the 2008 presentation I showed that the Shammas polynomials can have multiple terms with a smaller power range compared to regular polynomials. This feature allows better

 

curve fitting with Shammas polynomials that avoids the instability of raising large or small numbers to high powers. If you are not familiar with the Shammas polynomials, then you can download my 2008 presentation using the following link:

http://www.namirshammas.com/NEW/ShammasPolynomials.pdf.

A Shammas polynomial has the following general form:

$$y = = a_0 + \sum_{i=1}^{n} a_i \, x^{gx(i,A,B,C)} \quad (1)$$

Where A, B, and C are fixed constants. Using C is optional. The function gx(i, A, B, C) supplies the math expressions used to calculate the powers of the Shammas polynomial. The power can be integers and non-integers.

This study looks at Shammas polynomials used to approximate common functions that include:

- Trigonometric functions and their inverses.
- Hyperbolic functions and their inverses.
- Logarithmic functions.
- Exponential functions.
- Inverse student-t functions.
- The common logarithm of the gamma function.
- The digamma function.
- The trigamma function.

The digamma function is defined as:

$$\varphi(x) = \Gamma'(x)) / \Gamma(x)) \quad (2)$$

The following Matlab function implements the code for the digamma function:

```
function y = digamma(x)
%DIGAMMA Summary of this function goes here
%   Detailed explanation goes here
  h = 0.001;
  fp = gamma(x+h);
  fm = gamma(x-h);
  f0 = gamma(x);
  y = (fp -fm)/2/h./f0;
end
```

The trigamma function is defined as:

 

$$\varphi_1(x) = \frac{d^2}{dx^2} \ln(\Gamma(x)) \quad (3)$$

The following Matlab function implements the code for the trigamma function:

```
function y = trigamma(x)
%DIGAMMA Summary of this function goes here
%   Detailed explanation goes here
  h = 0.001;
  fp = log(gamma(x+h));
  fm = log(gamma(x-h));
  f0 = log(gamma(x));
  y = (fp -2*f0 + fm)/h/h;
end
```

The study works with Shammas Polynomials that use the following nine equations to calculate the powers of Shammas polynomial terms:

Power = A + B*i (4)

Power = A + B/I (5)

Power = A+B*sqrt(i) (6)

Power = A+B*sqrt(i+C) (7)

Power = A+B*i+C*(i–1) (8)

Power = A+B*i–C*(i–1) (9)

Power = A+B*i+C*sqrt(i–1) (10)

Power = A+B*sqrt(i)+C*(i-1) (11)

Power = A+B*sqrt(i)+C*sqrt(i–1) (12)

Where A, B, and C are constants, and i is a variable that represents the term number in a Shammas polynomial.

There are several choices that we can use to determine the best Shammas polynomial order and the best values for parameters A, B, and C:

1. Iterate over ranges of finite values for A, B, and C, and for a range of Shammas polynomial orders, say from 3 to 7.
2. Iterate over random values that fall within ranges of values for A, B, and C, and for a range of Shammas polynomial orders. The random values can be randomly selected values from a list of finite-values (which make up the

 

ranges) in a range or from pure random values that fall with in the given ranges.

3. Use an optimization function to determine the best Shammas polynomial order and the best values for parameters A, B, and C. The function would use ranges for the polynomial orders as well as the values for A, B, C.

The study uses the Matlab particle swarm optimization function to select the best Shammas polynomial order along with the best values for A, B, and C. I have carried out multiple studies using all of the other approaches. I decided in the end that focusing on approach number 3 yields the results needed without overwhelming you with a multi-document study.

The approximations that I obtain are typically for a defined and suitable interval. It is your responsibility to implement expanded versions of the approximation functions that take wider ranges of arguments and map them onto the interval used. For example, given that my approximation for $\log_{10}(x)$ uses the range (1, 10), to calculate $\log_{10}(235)$ use:

$\log_{10}(235) =$

$\log_{10}(2.35 * 100) =$

$\log_{10}(2.35) + \log_{10}(100) =$

$\log_{10}(2.35) + 2$

The argument of the $\log_{10}(x)$ function in the last line falls in the interval (1, 10).

## MATLAB CODE

The algorithm in this study uses particle swarm optimization to obtain the best values for parameters A, B, and C in a prespecified range of Shammas polynomial orders.

This study uses the function ShamPoly2 () to perform various Shammas polynomial curve fitting that involve Shammas polynomial parameters A and B:

```
function ShamPoly2(fx,gx,xRange,Lb,Ub,runNum,sFxName,diaryFilename)
%SHAMPOLY2 Summary of this function goes here
%   Detailed explanation goes here
clc
global bDeleteIfExists
global bUseDiary
global xdata
global ydata
```

 

```
global order
global ggx

warning('off','all')
if isempty(sFxName)
  sFxName = getFuncName(fx);
end
xdata = xRange';
ydata=fx(xdata);
ggx = gx;

fprintf('Fitting %s in range (%f, %f)\n', sFxName, min(xdata),max(xdata));

options = optimoptions('particleswarm', 'Display', 'iter');
[x,psAICc] = particleswarm(@optimFunc,3,Lb,Ub,options);
A = x(1);
B = x(2);
order = round(x(3));
if bUseDiary
  diaryFilename = strrep(diaryFilename, ".txt", strcat("_",
num2str(order),"_run", num2str(runNum),".txt"));
  if exist(diaryFilename, 'file')==2
    if bDeleteIfExists
      delete(diaryFilename);
    else
      return;
    end
  end
end
X = [];
for i=1:order
  xs = xdata.^gx(i,A,B);
  X = [X;xs'];
end
X = X';
lm = fitlm(X,ydata);
if bUseDiary
  diary(diaryFilename)
end
fprintf('Fitting %s in range (%f, %f)\n', sFxName, min(xdata),max(xdata));
sp = getFuncName(gx);
fprintf('Shammas polynomial power is %s\n', sp);
format long
disp(lm);
anva = anova(lm,'summary');
disp(anva);
format short
fprintf('A = %f, B = %f\n', A, B);
lstPwr = [];
for i=1:order
  lstPwr = [lstPwr,gx(i,A,B)];
end
fprintf('List of powers: [');
for i=1:order-1
 fprintf('%f, ', lstPwr(i));
end
fprintf('%f]\n', lstPwr(order));
```

 

```
fprintf('Fitting %s in range (%f, %f)\n', sFxName, min(xdata),max(xdata));
n = length(xdata);
sumsqr = 0;
for i=1:n
  yc = lm.Coefficients{1,1};
  for j=2:length(lm.Coefficients{:,1})
    yc = yc + lm.Coefficients{j,1} * xdata(i)^lstPwr(j-1);
  end
  sumsqr = sumsqr + (ydata(i) - yc)^2;
end
k = order + 1;
fprintf('MSS of errors squared = %e\n', sqrt(sumsqr)/n);
fprintf('Corrected MSS of errors squared = %e\n', sqrt(sumsqr)/n*sqrt(2));
AIC = lm.ModelCriterion.AIC;
AICc = AIC + 2*k*(1 + (k+1)/(n-k-1));
fprintf('Particle swarm AICc =  %e\n', psAICc);
fprintf('AIC =  %e\n', AIC);
fprintf('AICc =  %e\n', AICc);

if bUseDiary
  diary off
end
end

function AICc = optimFunc(x)
  global xdata
  global ydata
  global order
  global ggx

  A = x(1);
  B = x(2);
  order = round(x(3));
  X = [];
  for i=1:order
    xs = xdata.^ggx(i,A,B);
    X = [X;xs'];
  end
  X = X';
  lm = fitlm(X,ydata);

  n = length(xdata);
  k = order + 1;
  AIC = lm.ModelCriterion.AIC;
  AICc = AIC + 2*k*(1 + (k+1)/(n-k-1));
  if isinf(AICc), AICc = -1e+99; end
end

function sFx = getFuncName(fx)
  sFx = func2str(fx);
  if sFx(1:2)=="@("
    i = strfind(sFx,")");
    sFx = sFx(i(1)+1:end);
  elseif sFx(1)=="@"
    sFx = strcat(sFx(2:end),".m");
  else
    % return sFx as is
```

 

```
  end
end
```

The parameters of function ShamPoly2 () are:

- The parameter fx is the handle (or inline function) for the function being approximated. An example is @(x)cos(x) which also shows the ***recommended format*** for the argument of parameter fx.
- The parameter gx is the handle (or inline function) for the function that calculates the powers of the Shammas polynomial. An example is @(i,A,B)A+B*sqrt(i) which also shows the ***recommended format*** for the argument of parameter gx. You can assign the inline expression to a variable (function handle) and use that variable as an argument. The result is the same.
- The parameter xRange is the array that specifies the minimum value, increment value, and maximum value for the range of approximation.
- The parameter Lb is is the array of lower limits for the parameters A and B, and the Shammas polynomial order. An example is [0.1 0 3].
- The parameter Ub is is the array of upper limits for the parameters A and B, and the Shammas polynomial order. An example is [1 5 7].
- The parameter runNum specifies the run number. The arguments for this parameter have nothing to do with the calculations and serve in fine tuning the name of the diary files, when used.
- The optional parameter sFxName is the name of the approximated function. An example is “cos(x)”.
- The parameter diaryFilename is the name of the diary file. An example is “cos_1.txt”.

The above listing performs the following tasks:

1. Initialize the data for the curve fitting. The function uses the global variables xdata and ydata to store the data for the polynomial fitting.
2. Store the handle of function gx in the global handle ggx.
3. Set the optimization options and then call the Matlab function particleswarm(). The function call returns the optimized values of A and B and the optimum value for *corrected* Akaike information criterion (AICc). The arguments for this function call are:
    a. The handle to the local function optimFunc() that calculates AICc.
    b. The number of optimized variables which is 3.

 

   c. The lower and upper bounds arrays, Lb and Ub, respectively,
   d. The optimization parameters for function particleswarm().
4. Retrieve the optimum values and perform a Shammas polynomial fit for the best values of A and B.
5. Display the results of the regression and its associated ANOVA table.
6. Display the list of Shammas polynomial powers.
7. Display the range of the approximated function.
8. Calculate and display the value of the mean square root of the sum of squared errors. This statistic serves as a check that the Shammas polynomial performs well in checking the training data.
9. Calculate and display the value of the *corrected* mean square root of the sum of squared errors. This value equals the mean square root of the sum of squared errors multiplied by the square root of the number of Shammas polynomial parameters (i.e. A, B, and C, if used). In the case of ShamPoly2(), the function uses square root of 2.
10. Calculate and display the *corrected* Akaike information criterion. This statistic is calculated using:

$$AIC = n * \ln(SSE/n) + 2*k \quad (8)$$

$$AICc = AIC + 2*k*(k+1)/(n-k-1) \quad (9)$$

Where n is the number of observations, k is the total number of regression coefficients (including the intercept), and SSE is the sum of squared errors. The program obtains the value of AIC using lm.ModelCriterion.AIC. The program uses equation (9) to calculate the value for AICc.

11. Close the diary file, if one is used.

The function optimFunc() obtains the array x containing the current values of A and B, and the best Shammas polynomial order. The function calculates the transformed variables needed to perform a curve fit for a Shammas polynomial. This task calls the Matlab function fitlm(). The optimFunc() function returns the AICc as its result. I am using this statistic since the optimization is dealing with different Shammas polynomial orders and thus a varying number of polynomial coefficients.

The function getFuncName() returns a string-type function name given a handle of a function. The best way to take advantage of this function is to supply arguments like @(x)cos(x) and @(x,A,B)A+B*sqrt(x). Such arguments allow the function to

 

discard the part that declares the variable(s) and return the part that comes after the first closed parenthesis (e.g. cos(x) and A+B*sqrt(x) for the above examples). If you supply an argument like @fx1 which refers to the file fx1.m that defines the function fx1() then the function getFuncName() returns *fx1.m.* This string value indicates that you a rereferencing a separate Matlab file that implements the code for fx1.

Under the current calculation scheme, note that, the function ShamPoly2 () does not explicitly iterate over different values of A and B. It delegates that task to the Matlab function particleswarm().

The study also uses the function ShamPoly3() to work with Shammas polynomial powers that use parameters A, B, and C:

```
function ShamPoly3(fx,gx,xRange,Lb,Ub,runNum,sFxName,diaryFilename)
%SHAMPOLY3 Summary of this function goes here
%   Detailed explanation goes here
clc
global bDeleteIfExists
global bUseDiary
global xdata
global ydata
global order
global ggx

warning('off','all')
if isempty(sFxName)
  sFxName = getFuncName(fx);
end
xdata = xRange';
ydata=fx(xdata);
ggx = gx;

fprintf('Fitting %s in range (%f, %f)\n', sFxName, min(xdata),max(xdata));

options = optimoptions('particleswarm', 'Display', 'iter');
[x,psAICc] = particleswarm(@optimFunc,4,Lb,Ub,options);
A = x(1);
B = x(2);
C = x(3);
order = round(x(4));
if bUseDiary
  diaryFilename = strrep(diaryFilename, ".txt", strcat("_",
num2str(order),"_run", num2str(runNum),".txt"));
  if exist(diaryFilename, 'file')==2
    if bDeleteIfExists
      delete(diaryFilename);
    else
      return;
    end
  end
end
```

 

```
X = [];
for i=1:order
  xs = xdata.^gx(i,A,B,C);
  X = [X;xs'];
end
X = X';
lm = fitlm(X,ydata);

if bUseDiary
  diary(diaryFilename)
end
fprintf('Fitting %s in range (%f, %f)\n', sFxName, min(xdata),max(xdata));
sp = getFuncName(gx);
fprintf('Shammas polynomial power is %s\n', sp);
format long
disp(lm);
anva = anova(lm,'summary');
disp(anva);
format short
fprintf('A = %f, B = %f, C = %f\n', A, B, C);
lstPwr = [];
for i=1:order
  lstPwr = [lstPwr,gx(i,A,B,C)];
end
fprintf('List of powers: [');
for i=1:order-1
 fprintf('%f, ', lstPwr(i));
end
fprintf('%f]\n', lstPwr(order));
fprintf('Fitting %s in range (%f, %f)\n', sFxName, min(xdata),max(xdata));
n = length(xdata);
sumsqr = 0;
for i=1:n
  yc = lm.Coefficients{1,1};
  for j=2:length(lm.Coefficients{:,1})
    yc = yc + lm.Coefficients{j,1} * xdata(i)^lstPwr(j-1);
  end
  sumsqr = sumsqr + (ydata(i) - yc)^2;
end
k = order + 1;
fprintf('MSS of errors squared = %e\n', sqrt(sumsqr)/n);
fprintf('Corrected MSS of errors squared = %e\n', sqrt(sumsqr)/n*sqrt(3));
AIC = lm.ModelCriterion.AIC;
AICc = AIC + 2*k*(1 + (k+1)/(n-k-1));
fprintf('Particle swarm AICc =  %e\n', psAICc);
fprintf('AIC =  %e\n', AIC);
fprintf('AICc =  %e\n', AICc);

if bUseDiary
  diary off
end
end

function AICc = optimFunc(x)
  global xdata
  global ydata
  global order
```

 

```
  global ggx

  A = x(1);
  B = x(2);
  C = x(3);
  order = round(x(4));
  X = [];
  for i=1:order
    xs = xdata.^ggx(i,A,B,C);
    X = [X;xs'];
  end
  X = X';
  lm = fitlm(X,ydata);

  n = length(xdata);
  k = order + 1;
  AIC = lm.ModelCriterion.AIC;
  AICc = AIC + 2*k*(1 + (k+1)/(n-k-1));
  if isinf(AICc), AICc = -1e+99; end
end

function sFx = getFuncName(fx)
  sFx = func2str(fx);
  if sFx(1:2)=="@("
    i = strfind(sFx,")");
    sFx = sFx(i(1)+1:end);
  elseif sFx(1)=="@"
    sFx = strcat(sFx(2:end),".m");
  else
    % return sFx as is
  end
end
```

The parameters of function ShamPoly3rand3() are:

- The parameter fx is the handle (or inline function) for the function being approximated. An example is @(x)cos(x) which also shows the ***recommended format*** for the argument of parameter fx.
- The parameter gx is the handle (or inline function) for the function that calculates the powers of the Shammas polynomial. An example is @(i,A,B,c)A+B*sqrt(i+C) which also shows the ***recommended format*** for the argument of parameter gx. You can assign the inline expression to a variable (function handle) and use that variable as an argument. The result is the same.
- The parameter xRange is the array that specifies the minimum value, increment value, and maximum value for the range of approximation.
- The parameter Lb is is the array of lower limits for the parameters A, B, C and the Shammas polynomial order. An example is [0.1 0 0 3].

 

- The parameter Ub is is the array of upper limits for the parameters A, B and C, and the Shammas polynomial order. An example is [1 5 3 7].
- The parameter runNum specifies the run number. The arguments for this parameter have nothing to do with the calculations and serve in fine tuning the name of the diary files, when used.
- The optional parameter sFxName is the name of the approximated function. An example is “cos(x)”.
- The parameter diaryFilename is the name of the diary file. An example is “cos_1.txt”.

The above listing performs basically the same tasks as function ShamPoly2 () with the exception that it optimizes the parameter C and manages values related to that parameter. The function optimFunc() is the function to be optimized to get the best values for the Shammas polynomial power parameters A, B, and C, and the best Shammas polynomial order. The function getFuncName() returns a string-type function name given a handle of a function.

Using the calculation scheme, note that function ShamPoly3 () does not explicitly iterate over different values of A, B and C. It delegates that task to the Matlab function particleswarm().

The following Matlab script goAll () performs the various Shammas polynomial fittings for the various tested functions:

```
% Version 1.0.0 8/12/2020
global bUseDiary
global bDeleteIfExists

bUseDiary = true;
bDeleteIfExists = false; % or false
selIdx = 0; % note a zero value will execute all the models
runNum = 1;
bShutdown = false;

tic;

if selIdx==0 || selIdx==1
  lstA = [0 0.5];
  lstB = [0.1 0.5];
  lstOrder = [3 7];
  Lb = [lstA(1) lstB(1) lstOrder(1)];
  Ub = [lstA(2) lstB(2) lstOrder(2)];
  gx = @(i,A,B)A+B*i;
  ShamPoly2(@(x)acos(x),gx,[0:.01:1],Lb,Ub,runNum,"arccos(x)","arccos_1.txt")
  ShamPoly2(@(x)asin(x),gx,[0:.01:1],Lb,Ub,runNum,"arcsin(x)","arcsin_1.txt")
  ShamPoly2(@(x)atan(x),gx,[0:.01:1],Lb,Ub,runNum,"arctan(x)","arctan_1.txt")
  ShamPoly2(@(x)sin(x),gx,[0:.01:1],Lb,Ub,runNum,"sin(x)","sin_1.txt")
```

 

```
  ShamPoly2(@(x)cos(x),gx,[0:.01:1],Lb,Ub,runNum,"cos(x)","cos_1.txt")
  ShamPoly2(@(x)tan(x),gx,[0:.01:1],Lb,Ub,runNum,"tan(x)","tan_1.txt")
  ShamPoly2(@(x)sinh(x),gx,[0:.01:5],Lb,Ub,runNum,"sinh(x)","sinh_1.txt")
  ShamPoly2(@(x)cosh(x),gx,[0:.01:5],Lb,Ub,runNum,"cosh(x)","cosh_1.txt")
  ShamPoly2(@(x)tanh(x),gx,[0:.01:3],Lb,Ub,runNum,"tanh(x)","tanh_1.txt")
  ShamPoly2(@(x)erf(x),gx,[0:.01:2.1],Lb,Ub,runNum,"erf(x)","erf_1.txt")
  ShamPoly2(@(x)exp(x),gx,[0:.01:2],Lb,Ub,runNum,"exp(x)","exp_1.txt")
  ShamPoly2(@(x)log(x),gx,[1:.01:10],Lb,Ub,runNum,"ln(x)","ln_1.txt")
  ShamPoly2(@(x)log10(x),gx,[1:.01:10],Lb,Ub,runNum,"log(x)","log_1.txt")
  ShamPoly2(@(x)10.^x,gx,[0:.01:1],Lb,Ub,runNum,"10^x","pwr10_1.txt")

ShamPoly2(@(x)tinv(0.95,x),gx,[2:100],Lb,Ub,runNum,"tinv(0.95,x)","tinv1_1.tx
t")

ShamPoly2(@(x)tinv(0.975,x),gx,[2:100],Lb,Ub,runNum,"tinv(0.975,x)","tinv2_1.
txt")

ShamPoly2(@(x)log10(gamma(x)),gx,[2:100],Lb,Ub,runNum,"log10Gamma(x)","log10G
amma_1.txt")

ShamPoly2(@(x)digamma(x),gx,[2:100],Lb,Ub,runNum,"digamma(x)","digamma_1.txt"
)

ShamPoly2(@(x)trigamma(x),gx,[1:100],Lb,Ub,runNum,"tigamma(x)","trigamma_1.tx
t")
  ShamPoly2(@(x)asinh(x),gx,[0:100],Lb,Ub,runNum,"asinh(x)","asinh_1.txt")
  ShamPoly2(@(x)acosh(x),gx,[1:100],Lb,Ub,runNum,"acosh(x)","acosh_1.txt")

ShamPoly2(@(x)atanh(x),gx,[0:0.01:0.99],Lb,Ub,runNum,"atanh(x)","atanh_1.txt"
)
end

if selIdx==0 || selIdx==2
  lstA = [0 2];
  lstB = [0.1 2];
  lstOrder = [3 7];
  Lb = [lstA(1) lstB(1) lstOrder(1)];
  Ub = [lstA(2) lstB(2)  lstOrder(2)];
  gx = @(i,A,B)A+B/i;
  ShamPoly2(@(x)acos(x),gx,[0:.01:1],Lb,Ub,runNum,"arccos(x)","arccos_2.txt")
  ShamPoly2(@(x)asin(x),gx,[0:.01:1],Lb,Ub,runNum,"arcsin(x)","arcsin_2.txt")
  ShamPoly2(@(x)atan(x),gx,[0:.01:1],Lb,Ub,runNum,"arctan(x)","arctan_2.txt")
  ShamPoly2(@(x)sin(x),gx,[0:.01:1],Lb,Ub,runNum,"sin(x)","sin_2.txt")
  ShamPoly2(@(x)cos(x),gx,[0:.01:1],Lb,Ub,runNum,"cos(x)","cos_2.txt")
  ShamPoly2(@(x)tan(x),gx,[0:.01:1],Lb,Ub,runNum,"tan(x)","tan_2.txt")
  ShamPoly2(@(x)sinh(x),gx,[0:.01:5],Lb,Ub,runNum,"sinh(x)","sinh_2.txt")
  ShamPoly2(@(x)cosh(x),gx,[0:.01:5],Lb,Ub,runNum,"cosh(x)","cosh_2.txt")
  ShamPoly2(@(x)tanh(x),gx,[0:.01:3],Lb,Ub,runNum,"tanh(x)","tanh_2.txt")
  ShamPoly2(@(x)erf(x),gx,[0:.01:2.1],Lb,Ub,runNum,"erf(x)","erf_2.txt")
  ShamPoly2(@(x)exp(x),gx,[0:.01:2],Lb,Ub,runNum,"exp(x)","exp_2.txt")
  ShamPoly2(@(x)log(x),gx,[1:.01:10],Lb,Ub,runNum,"ln(x)","ln_2.txt")
  ShamPoly2(@(x)log10(x),gx,[1:.01:10],Lb,Ub,runNum,"log(x)","log_2.txt")
  ShamPoly2(@(x)10.^x,gx,[0:.01:1],Lb,Ub,runNum,"10^x","pwr10_2.txt")

ShamPoly2(@(x)tinv(0.95,x),gx,[2:100],Lb,Ub,runNum,"tinv(0.95,x)","tinv1_2.tx
t")
```

 

```
ShamPoly2(@(x)tinv(0.975,x),gx,[2:100],Lb,Ub,runNum,"tinv(0.975,x)","tinv2_2.
txt")

ShamPoly2(@(x)log10(gamma(x)),gx,[2:100],Lb,Ub,runNum,"log10Gamma(x)","log10G
amma_2.txt")

ShamPoly2(@(x)digamma(x),gx,[2:100],Lb,Ub,runNum,"digamma(x)","diamma_2.txt")

ShamPoly2(@(x)trigamma(x),gx,[1:100],Lb,Ub,runNum,"trigamma(x)","trigamma_2.t
xt")
  ShamPoly2(@(x)asinh(x),gx,[0:100],Lb,Ub,runNum,"asinh(x)","asinh_2.txt")
  ShamPoly2(@(x)acosh(x),gx,[1:100],Lb,Ub,runNum,"acosh(x)","acosh_2.txt")

ShamPoly2(@(x)atanh(x),gx,[0:0.01:0.99],Lb,Ub,runNum,"atanh(x)","atanh_2.txt"
)
end

if selIdx==0 || selIdx==3
  lstA = [0 1.4];
  lstB = [0.1 1];
  lstOrder = [3 7];
  Lb = [lstA(1) lstB(1) lstOrder(1)];
  Ub = [lstA(2) lstB(2) lstOrder(2)];
  gx = @(i,A,B)A+B*sqrt(i);
  ShamPoly2(@(x)acos(x),gx,[0:.01:1],Lb,Ub,runNum,"arccos(x)","arccos_3.txt")
  ShamPoly2(@(x)asin(x),gx,[0:.01:1],Lb,Ub,runNum,"arcsin(x)","arcsin_3.txt")
  ShamPoly2(@(x)atan(x),gx,[0:.01:1],Lb,Ub,runNum,"arctan(x)","arctan_3.txt")
  ShamPoly2(@(x)sin(x),gx,[0:.01:1],Lb,Ub,runNum,"sin(x)","sin_3.txt")
  ShamPoly2(@(x)cos(x),gx,[0:.01:1],Lb,Ub,runNum,"cos(x)","cos_3.txt")
  ShamPoly2(@(x)tan(x),gx,[0:.01:1],Lb,Ub,runNum,"tan(x)","tan_3.txt")
  ShamPoly2(@(x)sinh(x),gx,[0:.01:5],Lb,Ub,runNum,"sinh(x)","sinh_3.txt")
  ShamPoly2(@(x)cosh(x),gx,[0:.01:5],Lb,Ub,runNum,"cosh(x)","cosh_3.txt")
  ShamPoly2(@(x)tanh(x),gx,[0:.01:3],Lb,Ub,runNum,"tanh(x)","tanh_3.txt")
  ShamPoly2(@(x)erf(x),gx,[0:.01:2.1],Lb,Ub,runNum,"erf(x)","erf_3.txt")
  ShamPoly2(@(x)exp(x),gx,[0:.01:2],Lb,Ub,runNum,"exp(x)","exp_3.txt")
  ShamPoly2(@(x)log(x),gx,[1:.01:10],Lb,Ub,runNum,"ln(x)","ln_3.txt")
  ShamPoly2(@(x)log10(x),gx,[1:.01:10],Lb,Ub,runNum,"log(x)","log_3.txt")
  ShamPoly2(@(x)10.^x,gx,[0:.01:1],Lb,Ub,runNum,"10^x","pwr10_3.txt")

ShamPoly2(@(x)tinv(0.95,x),gx,[2:100],Lb,Ub,runNum,"tinv(0.95,x)","tinv1_3.tx
t")

ShamPoly2(@(x)tinv(0.975,x),gx,[2:100],Lb,Ub,runNum,"tinv(0.975,x)","tinv2_3.
txt")

ShamPoly2(@(x)log10(gamma(x)),gx,[2:100],Lb,Ub,runNum,"log10Gamma(x)","log10G
amma_3.txt")

ShamPoly2(@(x)digamma(x),gx,[2:100],Lb,Ub,runNum,"digamma(x)","digamma_3.txt"
)

ShamPoly2(@(x)trigamma(x),gx,[1:100],Lb,Ub,runNum,"trigamma(x)","trigamma_3.t
xt")
  ShamPoly2(@(x)asinh(x),gx,[0:100],Lb,Ub,runNum,"asinh(x)","asinh_3.txt")
  ShamPoly2(@(x)acosh(x),gx,[1:100],Lb,Ub,runNum,"acosh(x)","acosh_3.txt")
```

 

```
ShamPoly2(@(x)atanh(x),gx,[0:0.01:0.99],Lb,Ub,runNum,"atanh(x)","atanh_3.txt"
)
end

if selIdx==0 || selIdx==4
  lstA = [0 1];
  lstB = [0.1 1];
  lstC = [0 2];
  lstOrder = [3 7];
  Lb = [lstA(1) lstB(1) lstC(1) lstOrder(1)];
  Ub = [lstA(2) lstB(2) lstC(2) lstOrder(2)];
  gx = @(i,A,B,C)A+B*sqrt(i+C);
  ShamPoly3(@(x)acos(x),gx,[0:.01:1],Lb,Ub,runNum,"arccos(x)","arccos_4.txt")
  ShamPoly3(@(x)asin(x),gx,[0:.01:1],Lb,Ub,runNum,"arcsin(x)","arcsin_4.txt")
  ShamPoly3(@(x)atan(x),gx,[0:.01:1],Lb,Ub,runNum,"arctan(x)","arctan_4.txt")
  ShamPoly3(@(x)sin(x),gx,[0:.01:1],Lb,Ub,runNum,"sin(x)","sin_4.txt")
  ShamPoly3(@(x)cos(x),gx,[0:.01:1],Lb,Ub,runNum,"cos(x)","cos_4.txt")
  ShamPoly3(@(x)tan(x),gx,[0:.01:1],Lb,Ub,runNum,"tan(x)","tan_4.txt")
  ShamPoly3(@(x)sinh(x),gx,[0:.01:5],Lb,Ub,runNum,"sinh(x)","sinh_4.txt")
  ShamPoly3(@(x)cosh(x),gx,[0:.01:5],Lb,Ub,runNum,"cosh(x)","cosh_4.txt")
  ShamPoly3(@(x)tanh(x),gx,[0:.01:3],Lb,Ub,runNum,"tanh(x)","tanh_4.txt")
  ShamPoly3(@(x)erf(x),gx,[0:.01:2.1],Lb,Ub,runNum,"erf(x)","erf_4.txt")
  ShamPoly3(@(x)exp(x),gx,[0:.01:2],Lb,Ub,runNum,"exp(x)","exp_4.txt")
  ShamPoly3(@(x)log(x),gx,[1:.01:10],Lb,Ub,runNum,"ln(x)","ln_4.txt")
  ShamPoly3(@(x)log10(x),gx,[1:.01:10],Lb,Ub,runNum,"log(x)","log_4.txt")
  ShamPoly3(@(x)10.^x,gx,[0:.01:1],Lb,Ub,runNum,"10^x","pwr10_4.txt")

ShamPoly3(@(x)tinv(0.95,x),gx,[2:100],Lb,Ub,runNum,"tinv(0.95,x)","tinv1_4.tx
t")

ShamPoly3(@(x)tinv(0.975,x),gx,[2:100],Lb,Ub,runNum,"tinv(0.975,x)","tinv2_4.
txt")

ShamPoly3(@(x)log10(gamma(x)),gx,[2:100],Lb,Ub,runNum,"log10Gamma(x)","log10G
amma_4.txt")

ShamPoly3(@(x)digamma(x),gx,[2:100],Lb,Ub,runNum,"digamma(x)","digamma_4.txt"
)

ShamPoly3(@(x)trigamma(x),gx,[1:100],Lb,Ub,runNum,"trigamma(x)","trigamma_4.t
xt")
  ShamPoly3(@(x)asinh(x),gx,[0:100],Lb,Ub,runNum,"asinh(x)","asinh_4.txt")
  ShamPoly3(@(x)acosh(x),gx,[1:100],Lb,Ub,runNum,"acosh(x)","acosh_4.txt")

ShamPoly3(@(x)atanh(x),gx,[0:0.01:0.99],Lb,Ub,runNum,"atanh(x)","atanh_4.txt"
)
end

if selIdx==0 || selIdx==5
  lstA = [0 0.5];
  lstB = [0.1 0.27];
  lstC = [0 0.27];
  lstOrder = [3 7];
  Lb = [lstA(1) lstB(1) lstC(1) lstOrder(1)];
  Ub = [lstA(2) lstB(2) lstC(2) lstOrder(2)];
  gx = @(i,A,B,C)A+B*i+C*(i-1);
```

 

```
  ShamPoly3(@(x)acos(x),gx,[0:.01:1],Lb,Ub,runNum,"arccos(x)","arccos_5.txt")
  ShamPoly3(@(x)asin(x),gx,[0:.01:1],Lb,Ub,runNum,"arcsin(x)","arcsin_5.txt")
  ShamPoly3(@(x)atan(x),gx,[0:.01:1],Lb,Ub,runNum,"arctan(x)","arctan_5.txt")
  ShamPoly3(@(x)sin(x),gx,[0:.01:1],Lb,Ub,runNum,"sin(x)","sin_5.txt")
  ShamPoly3(@(x)cos(x),gx,[0:.01:1],Lb,Ub,runNum,"cos(x)","cos_5.txt")
  ShamPoly3(@(x)tan(x),gx,[0:.01:1],Lb,Ub,runNum,"tan(x)","tan_5.txt")
  ShamPoly3(@(x)sinh(x),gx,[0:.01:5],Lb,Ub,runNum,"sinh(x)","sinh_5.txt")
  ShamPoly3(@(x)cosh(x),gx,[0:.01:5],Lb,Ub,runNum,"cosh(x)","cosh_5.txt")
  ShamPoly3(@(x)tanh(x),gx,[0:.01:3],Lb,Ub,runNum,"tanh(x)","tanh_5.txt")
  ShamPoly3(@(x)erf(x),gx,[0:.01:2.1],Lb,Ub,runNum,"erf(x)","erf_5.txt")
  ShamPoly3(@(x)exp(x),gx,[0:.01:2],Lb,Ub,runNum,"exp(x)","exp_5.txt")
  ShamPoly3(@(x)log(x),gx,[1:.01:10],Lb,Ub,runNum,"ln(x)","ln_5.txt")
  ShamPoly3(@(x)log10(x),gx,[1:.01:10],Lb,Ub,runNum,"log(x)","log_5.txt")
  ShamPoly3(@(x)10.^x,gx,[0:.01:1],Lb,Ub,runNum,"10^x","pwr10_5.txt")

ShamPoly3(@(x)tinv(0.95,x),gx,[2:100],Lb,Ub,runNum,"tinv(0.95,x)","tinv1_5.tx
t")

ShamPoly3(@(x)tinv(0.975,x),gx,[2:100],Lb,Ub,runNum,"tinv(0.975,x)","tinv2_5.
txt")

ShamPoly3(@(x)log10(gamma(x)),gx,[2:100],Lb,Ub,runNum,"log10Gamma(x)","log10G
amma_5.txt")

ShamPoly3(@(x)digamma(x),gx,[2:100],Lb,Ub,runNum,"digamma(x)","digamma_5.txt"
)

ShamPoly3(@(x)trigamma(x),gx,[1:100],Lb,Ub,runNum,"trigamma(x)","trigamma_5.t
xt")
  ShamPoly3(@(x)asinh(x),gx,[0:100],Lb,Ub,runNum,"asinh(x)","asinh_5.txt")
  ShamPoly3(@(x)acosh(x),gx,[1:100],Lb,Ub,runNum,"acosh(x)","acosh_5.txt")

ShamPoly3(@(x)atanh(x),gx,[0:0.01:0.99],Lb,Ub,runNum,"atanh(x)","atanh_5.txt"
)
end

if selIdx==0 || selIdx==6
  lstA = [0 0.75];
  lstB = [0.1 0.25];
  lstC = [0 0.25];
  lstOrder = [3 7];
  Lb = [lstA(1) lstB(1) lstC(1) lstOrder(1)];
  Ub = [lstA(2) lstB(2) lstC(2) lstOrder(2)];
  gx = @(i,A,B,C)A+B*i-C*(i-1);
  ShamPoly3(@(x)acos(x),gx,[0:.01:1],Lb,Ub,runNum,"arccos(x)","arccos_6.txt")
  ShamPoly3(@(x)asin(x),gx,[0:.01:1],Lb,Ub,runNum,"arcsin(x)","arcsin_6.txt")
  ShamPoly3(@(x)atan(x),gx,[0:.01:1],Lb,Ub,runNum,"arctan(x)","arctan_6.txt")
  ShamPoly3(@(x)sin(x),gx,[0:.01:1],Lb,Ub,runNum,"sin(x)","sin_6.txt")
  ShamPoly3(@(x)cos(x),gx,[0:.01:1],Lb,Ub,runNum,"cos(x)","cos_6.txt")
  ShamPoly3(@(x)tan(x),gx,[0:.01:1],Lb,Ub,runNum,"tan(x)","tan_6.txt")
  ShamPoly3(@(x)sinh(x),gx,[0:.01:5],Lb,Ub,runNum,"sinh(x)","sinh_6.txt")
  ShamPoly3(@(x)cosh(x),gx,[0:.01:5],Lb,Ub,runNum,"cosh(x)","cosh_6.txt")
  ShamPoly3(@(x)tanh(x),gx,[0:.01:3],Lb,Ub,runNum,"tanh(x)","tanh_6.txt")
  ShamPoly3(@(x)erf(x),gx,[0:.01:2.1],Lb,Ub,runNum,"erf(x)","erf_6.txt")
  ShamPoly3(@(x)exp(x),gx,[0:.01:2],Lb,Ub,runNum,"exp(x)","exp_6.txt")
  ShamPoly3(@(x)log(x),gx,[1:.01:10],Lb,Ub,runNum,"ln(x)","ln_6.txt")
  ShamPoly3(@(x)log10(x),gx,[1:.01:10],Lb,Ub,runNum,"log(x)","log_6.txt")
```

 

```
  ShamPoly3(@(x)10.^x,gx,[0:.01:1],Lb,Ub,runNum,"10^x","pwr10_6.txt")

ShamPoly3(@(x)tinv(0.95,x),gx,[2:100],Lb,Ub,runNum,"tinv(0.95,x)","tinv1_6.tx
t")

ShamPoly3(@(x)tinv(0.975,x),gx,[2:100],Lb,Ub,runNum,"tinv(0.975,x)","tinv2_6.
txt")

ShamPoly3(@(x)log10(gamma(x)),gx,[2:100],Lb,Ub,runNum,"log10Gamma(x)","log10G
amma_6.txt")

ShamPoly3(@(x)digamma(x),gx,[2:100],Lb,Ub,runNum,"digamma(x)","digamma_6.txt"
)

ShamPoly3(@(x)trigamma(x),gx,[1:100],Lb,Ub,runNum,"trigamma(x)","trigamma_6.t
xt")
  ShamPoly3(@(x)asinh(x),gx,[0:100],Lb,Ub,runNum,"asinh(x)","asinh_6.txt")
  ShamPoly3(@(x)acosh(x),gx,[1:100],Lb,Ub,runNum,"acosh(x)","acosh_6.txt")

ShamPoly3(@(x)atanh(x),gx,[0:0.01:0.99],Lb,Ub,runNum,"atanh(x)","atanh_6.txt"
)
end

if selIdx==0 || selIdx==7
  lstA = [0 0.34];
  lstB = [0.1 0.39];
  lstC = [0 0.36];
  lstOrder = [3 7];
  Lb = [lstA(1) lstB(1) lstC(1) lstOrder(1)];
  Ub = [lstA(2) lstB(2) lstC(2) lstOrder(2)];
  gx = @(i,A,B,C)A+B*i+C*sqrt(i-1);
  ShamPoly3(@(x)acos(x),gx,[0:.01:1],Lb,Ub,runNum,"arccos(x)","arccos_7.txt")
  ShamPoly3(@(x)asin(x),gx,[0:.01:1],Lb,Ub,runNum,"arcsin(x)","arcsin_7.txt")
  ShamPoly3(@(x)atan(x),gx,[0:.01:1],Lb,Ub,runNum,"arctan(x)","arctan_7.txt")
  ShamPoly3(@(x)sin(x),gx,[0:.01:1],Lb,Ub,runNum,"sin(x)","sin_7.txt")
  ShamPoly3(@(x)cos(x),gx,[0:.01:1],Lb,Ub,runNum,"cos(x)","cos_7.txt")
  ShamPoly3(@(x)tan(x),gx,[0:.01:1],Lb,Ub,runNum,"tan(x)","tan_7.txt")
  ShamPoly3(@(x)sinh(x),gx,[0:.01:5],Lb,Ub,runNum,"sinh(x)","sinh_7.txt")
  ShamPoly3(@(x)cosh(x),gx,[0:.01:5],Lb,Ub,runNum,"cosh(x)","cosh_7.txt")
  ShamPoly3(@(x)tanh(x),gx,[0:.01:3],Lb,Ub,runNum,"tanh(x)","tanh_7.txt")
  ShamPoly3(@(x)erf(x),gx,[0:.01:2.1],Lb,Ub,runNum,"erf(x)","erf_7.txt")
  ShamPoly3(@(x)exp(x),gx,[0:.01:2],Lb,Ub,runNum,"exp(x)","exp_7.txt")
  ShamPoly3(@(x)log(x),gx,[1:.01:10],Lb,Ub,runNum,"ln(x)","ln_7.txt")
  ShamPoly3(@(x)log10(x),gx,[1:.01:10],Lb,Ub,runNum,"log(x)","log_7.txt")
  ShamPoly3(@(x)10.^x,gx,[0:.01:1],Lb,Ub,runNum,"10^x","pwr10_7.txt")

ShamPoly3(@(x)tinv(0.95,x),gx,[2:100],Lb,Ub,runNum,"tinv(0.95,x)","tinv1_7.tx
t")

ShamPoly3(@(x)tinv(0.975,x),gx,[2:100],Lb,Ub,runNum,"tinv(0.975,x)","tinv2_7.
txt")

ShamPoly3(@(x)log10(gamma(x)),gx,[2:100],Lb,Ub,runNum,"log10Gamma(x)","log10G
amma_7.txt")

ShamPoly3(@(x)digamma(x),gx,[2:100],Lb,Ub,runNum,"digamma(x)","digamma_7.txt"
)
```

 

```
ShamPoly3(@(x)trigamma(x),gx,[1:100],Lb,Ub,runNum,"trigamma(x)","trigamma_7.t
xt")
  ShamPoly3(@(x)asinh(x),gx,[0:100],Lb,Ub,runNum,"asinh(x)","asinh_7.txt")
  ShamPoly3(@(x)acosh(x),gx,[1:100],Lb,Ub,runNum,"acosh(x)","acosh_7.txt")

ShamPoly3(@(x)atanh(x),gx,[0:0.01:0.99],Lb,Ub,runNum,"atanh(x)","atanh_7.txt"
)
end

if selIdx==0 || selIdx==8
  lstA = [0 0.55];
  lstB = [0.1 0.4];
  lstC = [0 0.4];
  lstOrder = [3 7];
  Lb = [lstA(1) lstB(1) lstC(1) lstOrder(1)];
  Ub = [lstA(2) lstB(2) lstC(2) lstOrder(2)];
  gx = @(i,A,B,C)A+B*sqrt(i)+C*(i-1);
  ShamPoly3(@(x)acos(x),gx,[0:.01:1],Lb,Ub,runNum,"arccos(x)","arccos_8.txt")
  ShamPoly3(@(x)asin(x),gx,[0:.01:1],Lb,Ub,runNum,"arcsin(x)","arcsin_8.txt")
  ShamPoly3(@(x)atan(x),gx,[0:.01:1],Lb,Ub,runNum,"arctan(x)","arctan_8.txt")
  ShamPoly3(@(x)sin(x),gx,[0:.01:1],Lb,Ub,runNum,"sin(x)","sin_8.txt")
  ShamPoly3(@(x)cos(x),gx,[0:.01:1],Lb,Ub,runNum,"cos(x)","cos_8.txt")
  ShamPoly3(@(x)tan(x),gx,[0:.01:1],Lb,Ub,runNum,"tan(x)","tan_8.txt")
  ShamPoly3(@(x)sinh(x),gx,[0:.01:5],Lb,Ub,runNum,"sinh(x)","sinh_8.txt")
  ShamPoly3(@(x)cosh(x),gx,[0:.01:5],Lb,Ub,runNum,"cosh(x)","cosh_8.txt")
  ShamPoly3(@(x)tanh(x),gx,[0:.01:3],Lb,Ub,runNum,"tanh(x)","tanh_8.txt")
  ShamPoly3(@(x)erf(x),gx,[0:.01:2.1],Lb,Ub,runNum,"erf(x)","erf_8.txt")
  ShamPoly3(@(x)exp(x),gx,[0:.01:2],Lb,Ub,runNum,"exp(x)","exp_8.txt")
  ShamPoly3(@(x)log(x),gx,[1:.01:10],Lb,Ub,runNum,"ln(x)","ln_8.txt")
  ShamPoly3(@(x)log10(x),gx,[1:.01:10],Lb,Ub,runNum,"log(x)","log_8.txt")
  ShamPoly3(@(x)10.^x,gx,[0:.01:1],Lb,Ub,runNum,"10^x","pwr10_8.txt")

ShamPoly3(@(x)tinv(0.95,x),gx,[2:100],Lb,Ub,runNum,"tinv(0.95,x)","tinv1_8.tx
t")

ShamPoly3(@(x)tinv(0.975,x),gx,[2:100],Lb,Ub,runNum,"tinv(0.975,x)","tinv2_8.
txt")

ShamPoly3(@(x)log10(gamma(x)),gx,[2:100],Lb,Ub,runNum,"log10Gamma(x)","log10G
amma_8.txt")

ShamPoly3(@(x)digamma(x),gx,[2:100],Lb,Ub,runNum,"digamma(x)","digamma_8.txt"
)
  ShamPoly3(@(x)trigamma(x),
gx,[1:100],Lb,Ub,runNum,"trigamma(x)","trigamma_8.txt")
  ShamPoly3(@(x)asinh(x),gx,[0:100],Lb,Ub,runNum,"asinh(x)","asinh_8.txt")
  ShamPoly3(@(x)acosh(x),gx,[1:100],Lb,Ub,runNum,"acosh(x)","acosh_8.txt")

ShamPoly3(@(x)atanh(x),gx,[0:0.01:0.99],Lb,Ub,runNum,"atanh(x)","atanh_8.txt"
)
end

if selIdx==0 || selIdx==9
  lstA = [0 0.5];
  lstB = [0.1 0.7];
  lstC = [0 0.7];
```

 

```
  lstOrder = [3 7];
  Lb = [lstA(1) lstB(1) lstC(1) lstOrder(1)];
  Ub = [lstA(2) lstB(2) lstC(2) lstOrder(2)];
  gx = @(i,A,B,C)A+B*sqrt(i)+C*sqrt(i-1);
  ShamPoly3(@(x)acos(x),gx,[0:.01:1],Lb,Ub,runNum,"arccos(x)","arccos_9.txt")
  ShamPoly3(@(x)asin(x),gx,[0:.01:1],Lb,Ub,runNum,"arcsin(x)","arcsin_9.txt")
  ShamPoly3(@(x)atan(x),gx,[0:.01:1],Lb,Ub,runNum,"arctan(x)","arctan_9.txt")
  ShamPoly3(@(x)sin(x),gx,[0:.01:1],Lb,Ub,runNum,"sin(x)","sin_9.txt")
  ShamPoly3(@(x)cos(x),gx,[0:.01:1],Lb,Ub,runNum,"cos(x)","cos_9.txt")
  ShamPoly3(@(x)tan(x),gx,[0:.01:1],Lb,Ub,runNum,"tan(x)","tan_9.txt")
  ShamPoly3(@(x)sinh(x),gx,[0:.01:5],Lb,Ub,runNum,"sinh(x)","sinh_9.txt")
  ShamPoly3(@(x)cosh(x),gx,[0:.01:5],Lb,Ub,runNum,"cosh(x)","cosh_9.txt")
  ShamPoly3(@(x)tanh(x),gx,[0:.01:3],Lb,Ub,runNum,"tanh(x)","tanh_9.txt")
  ShamPoly3(@(x)erf(x),gx,[0:.01:2.1],Lb,Ub,runNum,"erf(x)","erf_9.txt")
  ShamPoly3(@(x)exp(x),gx,[0:.01:2],Lb,Ub,runNum,"exp(x)","exp_9.txt")
  ShamPoly3(@(x)log(x),gx,[1:.01:10],Lb,Ub,runNum,"ln(x)","ln_9.txt")
  ShamPoly3(@(x)log10(x),gx,[1:.01:10],Lb,Ub,runNum,"log(x)","log_9.txt")
  ShamPoly3(@(x)10.^x,gx,[0:.01:1],Lb,Ub,runNum,"10^x","pwr10_9.txt")

ShamPoly3(@(x)tinv(0.95,x),gx,[2:100],Lb,Ub,runNum,"tinv(0.95,x)","tinv1_9.tx
t")

ShamPoly3(@(x)tinv(0.975,x),gx,[2:100],Lb,Ub,runNum,"tinv(0.975,x)","tinv2_9.
txt")

ShamPoly3(@(x)log10(gamma(x)),gx,[2:100],Lb,Ub,runNum,"log10Gamma(x)","log10G
amma_9.txt")

ShamPoly3(@(x)digamma(x),gx,[2:100],Lb,Ub,runNum,"digamma(x)","digamma_9.txt"
)

ShamPoly3(@(x)trigamma(x),gx,[1:100],Lb,Ub,runNum,"trigamma(x)","trigamma_9.t
xt")
  ShamPoly3(@(x)asinh(x),gx,[0:100],Lb,Ub,runNum,"asinh(x)","asinh_9.txt")
  ShamPoly3(@(x)acosh(x),gx,[1:100],Lb,Ub,runNum,"acosh(x)","acosh_9.txt")

ShamPoly3(@(x)atanh(x),gx,[0:0.01:0.99],Lb,Ub,runNum,"atanh(x)","atanh_9.txt"
)
end

toc;

for i=1:7
  beep;
  pause(3)
end

if bShutdown
  system('shutdown -s');
else
  fprintf("\n\nDone!\n\n");
end
```

The above listing has the following global and operational variables:

 

- The global variable bUseDiary is a Boolean flag used to tell the functions ShamPoly2() and ShamPoly3 () whether you want to copy the screen output to diary text files.
- The global variable bDeleteIfExists is a Boolean flag used to tell the functions ShamPoly2 () and ShamPoly3 () whether you want to delete diary files if they exist.
- The variable selIdx allows you to select calculations for one of the nine groups (when set to the targeted group number) or all of the groups (when set to 0). I am using this scheme to reduce calculation time which can be done by working with a specific set of approximations.
- The Boolean variable bShutdown tells the Matlab script whether to shut down the computer when done

The ranges in script goAll.m have the values of A, B, and C calculated to give a maximum power of 4 (for i=7 and for i=3 in the case of A+B/i). Aiming at a high power of 4 follows the basic purpose of Shammas polynomials which is having several terms while keeping the range of powers in check. In an earlier iteration I chose consistent values for all the fitted functions. The prevailing Shammas polynomials were those whose maximum powers were in the range of (3, 4). Those Shammas polynomials with smaller maximum powers did not fair as well. So, I decided to calculate A, B, and C for the highest powers for each fitted function to be at 4 or slightly above it. This way each Shammas polynomial can be optimized to use powers up to 4. This puts the proverbial race between the various Shammas polynomial models on a more even footing.

You can edit the script in goAll.m to perform Shammas polynomial fitting for other functions you are interested in or for other Shammas polynomial orders.

## General Comments on Results

The next sections show the results of fitting various common functions each with a variety of Shammas polynomials. There is a summary table for each fitted function showing the Shammas polynomial parameters, the F statistic, the AICc statistic, the root mean sum of errors squared, and the range of powers of the fitted Shammas polynomial. I use the root mean sum of errors squared to select the best Shammas polynomials. The coefficients of the fitted Shammas polynomials are inversely proportional with the range of powers in the same polynomials. Thus, the Shammas polynomials with the lowest power range will have high valued coefficients. I have deliberately not included the coefficient of determination and

 

its adjusted value in the table. Why? These values are very close to 1 and we are better served to look at other statistics.

 

# RESULTS FOR THE ARC COSINE

## Using Power A+B* i

```
Fitting arccos(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat               pValue
                ________________    ________________    ________________    ____________________

    (Intercept)     1.58015780077143    0.0104052126698892     151.862133999828    3.13160692502676e-113
    x1             -8.39503512549006     2.53182216781442    -3.31580757614466      0.00130421184391248
    x2              89.6959076272839     25.1871013115279     3.56118421559812      0.00058420995359831
    x3             -435.327951035025      105.65873457837    -4.12013216675552     8.20801709349963e-05
    x4              1075.19339372594     231.893044109279     4.63659182989231     1.15383074776098e-05
    x5             -1430.48695111677     278.761006063302    -5.13158913909188      1.5661270478351e-06
    x6              976.806713526825     173.907707141032     5.61681094866413     2.01064779496624e-07
    x7             -268.984822589819     44.0514793890985    -6.10614731491594     2.34057400014351e-08


Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.0112
R-squared: 0.999,  Adjusted R-Squared: 0.999
F-statistic vs. constant model: 1.71e+04, p-value = 1.24e-141
                    SumSq           DF            MeanSq                F                   pValue
                ________________    ___    ____________________    ________________    ____________________

    Total         14.8796738812062    100       0.148796738812062
    Model         14.8681117894905      7        2.12401596992722    17084.5803735872    1.23548709304724e-141
    Residual    0.0115620917156747     93    0.000124323566835212

A = 0.500000, B = 0.500000
List of powers: [1.000000, 1.500000, 2.000000, 2.500000, 3.000000, 3.500000, 4.000000]
Fitting arccos(x) in range (0.000000, 1.000000)
MSS of errors squared = 1.064625e-03
Corrected MSS of errors squared = 1.505608e-03
Particle swarm AICc =  -5.963987e+02
AIC =  -6.139640e+02
AICc =  -5.963987e+02
```

## Using Power A+B/i

```
Fitting arccos(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B/i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat               pValue
                ________________    ________________    ________________    ____________________

    (Intercept)     1.58002850493741    0.00802566894891247     196.871876350134    1.10666677393361e-123
    x1             -449.524818312817     71.3318679137188    -6.30187925061138      9.71112509597125e-09
    x2              34304.7929190518     6196.09732128227     5.53651615529378      2.84060510705204e-07
    x3             -471342.719845201     88787.8743769611    -5.30863840533056      7.48051033986365e-07
    x4              2249062.47202831     432176.998943627     5.20403093530129      1.15922576666714e-06
    x5             -4666165.32624902     906825.926792279     -5.1456020261294      1.47782532561169e-06
```

 

| | | | | |
|---|---|---|---|---|
| x6 | 4350327.34390859 | 851512.546221202 | 5.10894098180261 | 1.71984932409242e-06 |
| x7 | -1495738.54253386 | 294200.522149456 | -5.08407847683566 | 1.90557626583421e-06 |

Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.0104
R-squared: 0.999,  Adjusted R-Squared: 0.999
F-statistic vs. constant model: 1.96e+04, p-value = 2.23e-144

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 14.879673822563 | 100 | 0.14879673822563 | | |
| Model | 14.8695801725269 | 7 | 2.12422573893241 | 19572.007451582 | 2.23353873416829e-144 |
| Residual | 0.0100936500361257 | 93 | 0.000108533871356191 | | |

A = 2.000000, B = 2.000000
List of powers: [4.000000, 3.000000, 2.666667, 2.500000, 2.400000, 2.333333, 2.285714]
Fitting arccos(x) in range (0.000000, 1.000000)
MSS of errors squared = 9.947243e-04
Corrected MSS of errors squared = 1.406753e-03
Particle swarm AICc =  -6.101171e+02
AIC =  -6.276823e+02
AICc =  -6.101171e+02

## Using Power A+B*sqrt(i)

Fitting arccos(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*sqrt(i)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 1.57215118073349 | 0.00540239197780893 | 291.010201997804 | 1.93015750652625e-139 |
| x1 | -2354.79962383759 | 342.439897032105 | -6.87653408451066 | 6.95473453861354e-10 |
| x2 | 34959.981887762 | 5179.07195269014 | 6.75024062363198 | 1.24914478742729e-09 |
| x3 | -173848.247812636 | 25708.5606586602 | -6.7622707517884 | 1.18153575414229e-09 |
| x4 | 407596.845097221 | 59807.5407043054 | 6.81514137343352 | 9.24886543484131e-10 |
| x5 | -495707.161434963 | 72006.8319903394 | -6.88416845642463 | 6.71215790415617e-10 |
| x6 | 303385.051381152 | 43587.7866354116 | 6.96032248480072 | 4.70755792659717e-10 |
| x7 | -74033.1776175608 | 10516.5764855115 | -7.03966521039952 | 3.24921952313195e-10 |

Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.00909
R-squared: 0.999,  Adjusted R-Squared: 0.999
F-statistic vs. constant model: 2.57e+04, p-value = 6.82e-150

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 14.879673880622 | 100 | 0.14879673880622 | | |
| Model | 14.8719924970262 | 7 | 2.12457035671803 | 25722.5850929956 | 6.82114244292921e-150 |
| Residual | 0.00768138359579496 | 93 | 8.25955225354296e-05 | | |

A = 1.400000, B = 1.000000
List of powers: [2.400000, 2.814214, 3.132051, 3.400000, 3.636068, 3.849490, 4.045751]
Fitting arccos(x) in range (0.000000, 1.000000)
MSS of errors squared = 8.677575e-04
Corrected MSS of errors squared = 1.227194e-03

 

```
Particle swarm AICc =  -6.377009e+02
AIC =  -6.552661e+02
AICc =  -6.377009e+02
```

## Using Power A+B*sqrt(i+C)

```
Fitting arccos(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*sqrt(i+C)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7
```

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 1.5701892064967 | 0.00503281795808476 | 311.990065918108 | 2.99788190235824e-142 |
| x1 | -25462.7293959234 | 3462.49404954263 | -7.35386950319433 | 7.40536712714337e-11 |
| x2 | 248019.695286829 | 34068.3345702895 | 7.28006515185288 | 1.04959589134168e-10 |
| x3 | -947687.053825414 | 130442.446324561 | -7.2651738795777 | 1.12600613861157e-10 |
| x4 | 1845626.74662571 | 253508.538303783 | 7.28033366834403 | 1.04826642989768e-10 |
| x5 | -1951332.97320753 | 266848.288555557 | -7.31251822438155 | 9.00450934409078e-11 |
| x6 | 1069254.45322208 | 145377.506709162 | 7.35501988874527 | 7.36516911150089e-11 |
| x7 | -238419.646509964 | 32201.1599694214 | -7.40407012469021 | 5.8386585763778e-11 |

```
Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.00891
R-squared: 1,  Adjusted R-Squared: 0.999
F-statistic vs. constant model: 2.68e+04, p-value = 1.03e-150
```

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 14.8796738809584 | 100 | 0.148796738809584 | | |
| Model | 14.8722982728549 | 7 | 2.12461403897927 | 26789.5342123248 | 1.03167665516019e-150 |
| Residual | 0.00737560810348725 | 93 | 7.9307614015992e-05 | | |

```
A = 1.000000, B = 1.000000, C = 2.000000
List of powers: [2.732051, 3.000000, 3.236068, 3.449490, 3.645751, 3.828427, 4.000000]
Fitting arccos(x) in range (0.000000, 1.000000)
MSS of errors squared = 8.503105e-04
Corrected MSS of errors squared = 1.472781e-03
Particle swarm AICc =  -6.418036e+02
AIC =  -6.593689e+02
AICc =  -6.418036e+02
```

## Using Power A+B*i+C*(i–1)

```
Fitting arccos(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*i+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7
```

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 1.57812244151 | 0.0111110739582466 | 142.03149465482 | 1.53924062313324e-110 |
| x1 | -2.72243765690148 | 0.941970406099978 | -2.89015200400312 | 0.00479195349914641 |
| x2 | 32.9629698896198 | 10.1442075993565 | 3.24943762898847 | 0.00161013391425577 |
| x3 | -186.7487238267 | 46.6057882687457 | -4.00698562912057 | 0.000123862429021658 |
| x4 | 510.729443960495 | 110.748982837208 | 4.61159489574028 | 1.27258835354955e-05 |

 

| | | | | |
|---|---|---|---|---|
| x5 | -736.810106223047 | 142.402653814676 | -5.17413184716305 | 1.31281687445223e-06 |
| x6 | 538.069863915511 | 94.061696860899 | 5.72039291095528 | 1.28334771834388e-07 |
| x7 | -156.97505479642 | 25.0202860773967 | -6.27391126987278 | 1.10187037075947e-08 |

Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.0115
R-squared: 0.999,  Adjusted R-Squared: 0.999
F-statistic vs. constant model: 1.62e+04, p-value = 1.69e-140

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 14.8796738812142 | 100 | 0.148796738812142 | | |
| Model | 14.8674432814164 | 7 | 2.12392046877377 | 16150.0340834642 | 1.68623459045603e-140 |
| Residual | 0.0122305997978174 | 93 | 0.000131511825782983 | | |

A = 0.500000, B = 0.270000, C = 0.270000
List of powers: [0.770000, 1.310000, 1.850000, 2.390000, 2.930000, 3.470000, 4.010000]
Fitting arccos(x) in range (0.000000, 1.000000)
MSS of errors squared = 1.094971e-03
Corrected MSS of errors squared = 1.896545e-03
Particle swarm AICc =  -5.907216e+02
AIC =  -6.082868e+02
AICc =  -5.907216e+02

## Using Power A+B*i–C*(i–1)

Fitting arccos(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*i-C*(i-1)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 1.57518054508707 | 0.0136019257330074 | 115.805701046038 | 2.43227943232584e-102 |
| x1 | -207.119071729484 | 53.2090661466068 | -3.89255227969626 | 0.000186453771415449 |
| x2 | 1930.00437712015 | 454.790660875679 | 4.24372033806501 | 5.19587574796997e-05 |
| x3 | -7358.14825122478 | 1604.31266682001 | -4.58648018145349 | 1.40380357175191e-05 |
| x4 | 14647.2006548285 | 2980.3793613397 | 4.91454237162766 | 3.80861693409093e-06 |
| x5 | -16087.4473957901 | 3073.09712174407 | -5.23492970071182 | 1.0189811897616e-06 |
| x6 | 9260.95347740212 | 1667.92142429164 | 5.55239194276507 | 2.65347215347144e-07 |
| x7 | -2186.91720095095 | 372.489836156878 | -5.87107885550442 | 6.63734147144916e-08 |

Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.0137
R-squared: 0.999,  Adjusted R-Squared: 0.999
F-statistic vs. constant model: 1.14e+04, p-value = 2.05e-133

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 14.8796738812062 | 100 | 0.148796738812062 | | |
| Model | 14.86230381967 | 7 | 2.12318625995286 | 11367.6236416115 | 2.04641154815774e-133 |
| Residual | 0.0173700615362407 | 93 | 0.000186774855228394 | | |

A = 0.750000, B = 0.250000, C = 0.000000
List of powers: [1.000000, 1.250000, 1.500000, 1.750000, 2.000000, 2.250000, 2.500000]
Fitting arccos(x) in range (0.000000, 1.000000)
MSS of errors squared = 1.304906e-03

 

```
Corrected MSS of errors squared = 2.260164e-03
Particle swarm AICc =  -5.552901e+02
AIC =  -5.728553e+02
AICc =  -5.552901e+02
```

## Using Power A+B*i+C*sqrt(i–1)

```
Fitting arccos(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*i+C*sqrt(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat               pValue
                ________________    ________________    ________________    ____________________

    (Intercept)     1.5787924731432    0.0108303478322724     145.774863152474    1.38434203095036e-111
    x1             -1.56620313347052    0.534011902950905    -2.9328992945959      0.00422789992967286
    x2               41.896516315444     12.7464211959111     3.2869238880074      0.00142996253141571
    x3             -299.503717337532     74.2570300917078    -4.03333821683473     0.000112612923272695
    x4              894.657854671759     194.542902664745     4.59876892149348     1.33803985393723e-05
    x5             -1332.09732996285     260.936044683851    -5.10507213204987     1.74754249444973e-06
    x6              977.314306916945     175.026158232179     5.58381853768681     2.31801577471363e-07
    x7             -282.197672393626     46.6039509583262    -6.05523065299701      2.9373980627602e-08


Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.0113
R-squared: 0.999,  Adjusted R-Squared: 0.999
F-statistic vs. constant model: 1.67e+04, p-value = 3.98e-141
                    SumSq           DF            MeanSq                F                  pValue
                ________________    ___    ____________________    ________________    ____________________

    Total         14.8796738812046    100       0.148796738812046
    Model         14.8678169518662      7         2.1239738502666    16659.4201953645    3.9841239472172e-141
    Residual    0.0118569293383786     93    0.000127493863853533

A = 0.340000, B = 0.390000, C = 0.360000
List of powers: [0.730000, 1.480000, 2.019117, 2.523538, 3.010000, 3.484984, 3.951816]
Fitting arccos(x) in range (0.000000, 1.000000)
MSS of errors squared = 1.078114e-03
Corrected MSS of errors squared = 1.867348e-03
Particle swarm AICc =  -5.938555e+02
AIC =  -6.114207e+02
AICc =  -5.938555e+02
```

## Using Power A+B*sqrt(i)+C*(i–1)

```
Fitting arccos(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*sqrt(i)+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat               pValue
                ________________    ________________    ________________    ____________________

    (Intercept)     1.58009946895005    0.010356384526793     152.572499105472    2.03246944711955e-113
    x1             -5.55040791283454    1.69511280167034    -3.27435903225158      0.00148813674379653
    x2               72.4897314907438     20.6379454303308     3.51244903401133     0.000687270037545397
    x3             -405.063825764153     98.3600127776308    -4.11817581479893     8.26710621615534e-05
```

| | | | | |
|---|---|---|---|---|
| x4 | 1088.32494137559 | 234.356851257027 | 4.64387934697922 | 1.12128152469142e-05 |
| x5 | -1527.40936917224 | 297.575064954062 | -5.13285402259101 | 1.55794936592645e-06 |
| x6 | 1079.75357974665 | 192.717901844348 | 5.60276740984203 | 2.13625472323844e-07 |
| x7 | -304.043774977862 | 50.0976367450524 | -6.06902430398355 | 2.76230635787344e-08 |

Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.0111
R-squared: 0.999,  Adjusted R-Squared: 0.999
F-statistic vs. constant model: 1.72e+04, p-value = 8.11e-142

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 14.8796738812072 | 100 | 0.148796738812072 | | |
| Model | 14.8682159533356 | 7 | 2.12403085047651 | 17240.0168082206 | 8.11120723122689e-142 |
| Residual | 0.011457927871632 | 93 | 0.00012320352550142 | | |

A = 0.550000, B = 0.400000, C = 0.400000
List of powers: [0.950000, 1.515685, 2.042820, 2.550000, 3.044427, 3.529796, 4.008301]
Fitting arccos(x) in range (0.000000, 1.000000)
MSS of errors squared = 1.059819e-03
Corrected MSS of errors squared = 1.835660e-03
Particle swarm AICc =  -5.973128e+02
AIC =  -6.148780e+02
AICc =  -5.973128e+02

## Using Power A+B*sqrt(i)+C*sqrt(i–1)

Fitting arccos(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*sqrt(i)+C*sqrt(i-1)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 1.58024016839286 | 0.00792321197778722 | 199.44438856654 | 3.3181477145156e-124 |
| x1 | -6.91367632843073 | 1.47770181814439 | -4.67866808008161 | 9.77782480426327e-06 |
| x2 | 408.084081255225 | 92.4859510408971 | 4.41238995395929 | 2.74871058664573e-05 |
| x3 | -3455.64342574311 | 721.573533903669 | -4.78903848793939 | 6.30921329474645e-06 |
| x4 | 11256.2703046656 | 2205.21464438242 | 5.10438760840807 | 1.75248742982296e-06 |
| x5 | -17367.7276238955 | 3228.23904150207 | -5.37993853634038 | 5.53652566011216e-07 |
| x6 | 12804.6453552449 | 2275.10565761439 | 5.62815415292469 | 1.91451236554433e-07 |
| x7 | -3640.22322398407 | 621.528493473381 | -5.85688872225449 | 7.06466393250301e-08 |

Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.01
R-squared: 0.999,  Adjusted R-Squared: 0.999
F-statistic vs. constant model: 2.11e+04, p-value = 7.58e-146

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 14.8796738812575 | 100 | 0.148796738812575 | | |
| Model | 14.8702885326437 | 7 | 2.12432693323482 | 21050.0869941955 | 7.58131869353859e-146 |
| Residual | 0.00938534861377605 | 93 | 0.00010091772702985 | | |

A = 0.500000, B = 0.700000, C = 0.700000
List of powers: [1.200000, 2.189949, 2.702385, 3.112436, 3.465248, 3.779890, 4.066669]
Fitting arccos(x) in range (0.000000, 1.000000)

 

```
MSS of errors squared = 9.591882e-04
Corrected MSS of errors squared = 1.661363e-03
Particle swarm AICc =  -6.174655e+02
AIC =  -6.350307e+02
AICc =  -6.174655e+02
```

## Comments

The following table shows the summary results for the seven models that fit the arc cosine function.

| *Power* | *A* | *B* | *C* | *F* | *AICc* | *MSSE* | *Pwr Rng* |
|---|---|---|---|---|---|---|---|
| A+B*i | 0.500 | 0.500 | | 17084.5 | -5.963987e+02 | 1.064625e-03 | 1, 4 |
| A+B/i | 2.000 | 2.000 | | 19572.0 | -6.101171e+02 | 9.947243e-04 | 2.28, 4 |
| A+B*√(i) | 1.400 | 1.000 | | 25722.5 | -6.377009e+02 | 8.677575e-04 | 2.4, 4.04 |
| A+B*√(i+C) | 1.000 | 1.000 | 2.000 | 26789.5 | -6.418036e+02 | 8.503105e-04 | 2.73, 4 |
| A+B*i+C*(i-1) | 0.500 | 0.270 | 0.270 | 16150.0 | -5.907216e+02 | 1.094971e-03 | 0.77, 4.01 |
| A+B*i-C*(i-1) | 0.750 | 0.250 | 0.000 | 11367.6 | -5.552901e+02 | 1.304906e-03 | 1, 2.5 |
| A+B*i+C*√(i-1) | 0.340 | 0.390 | 0.360 | 16659.4 | -5.938555e+02 | 1.078114e-03 | 0.73, 3.95 |
| A+B*√(i)+C*(i-1) | 0.550 | 0.400 | 0.400 | 17240.0 | -5.973128e+02 | 1.059819e-03 | 0.95, 4 |
| A+B*√(i)+C*√(i-1) | 0.500 | 0.700 | 0.700 | 21050.0 | -6.174655e+02 | 9.591882e-04 | 1.2, 4.06 |

The above table shows that the top two Shammas polynomials use the following power equations:

$1+\sqrt{(i+2)}$

And,

$1.4+\sqrt{(i)}$

 

# RESULTS FOR THE ARC SINE

## Using Power A+B* i

```
Fitting arcsin(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
```

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | -0.00936147397659409 | 0.0104052126698733 | -0.899690787070489 | 0.370608541844128 |
| x1 | 8.39503512549006 | 2.53182216781053 | 3.31580757614976 | 0.00130421184389125 |
| x2 | -89.6959076272847 | 25.1871013114892 | -3.56118421560362 | 0.000584209953587542 |
| x3 | 435.32795103503 | 105.658734578208 | 4.12013216676189 | 8.20801709330763e-05 |
| x4 | -1075.19339372595 | 231.893044108922 | -4.63659182989949 | 1.15383074772844e-05 |
| x5 | 1430.48695111679 | 278.761006062874 | 5.13158913909984 | 1.56612704778352e-06 |
| x6 | -976.806713526839 | 173.907707140765 | -5.61681094867283 | 2.01064779489067e-07 |
| x7 | 268.984822589822 | 44.0514793890309 | 6.1061473149254 | 2.34057400004462e-08 |

```
Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.0112
R-squared: 0.999,  Adjusted R-Squared: 0.999
F-statistic vs. constant model: 1.71e+04, p-value = 1.24e-141
```

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 14.87967388121 | 100 | 0.1487967388121 | | |
| Model | 14.8681117894943 | 7 | 2.12401596992776 | 17084.5803736441 | 1.23548709285622e-141 |
| Residual | 0.0115620917156392 | 93 | 0.00012432356683483 | | |

```
A = 0.500000, B = 0.500000
List of powers: [1.000000, 1.500000, 2.000000, 2.500000, 3.000000, 3.500000, 4.000000]
Fitting arcsin(x) in range (0.000000, 1.000000)
MSS of errors squared = 1.064625e-03
Corrected MSS of errors squared = 1.505608e-03
Particle swarm AICc =  -5.963987e+02
AIC =  -6.139640e+02
AICc =  -5.963987e+02
```

## Using Power A+B/i

```
Fitting arcsin(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B/i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
```

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | -0.00923217816035739 | 0.00802566894891529 | -1.15033129563675 | 0.252956771182148 |
| x1 | 449.524818312958 | 71.3318679137439 | 6.30187925061115 | 9.7111250959816e-09 |
| x2 | -34304.7929190504 | 6196.09732128445 | -5.53651615529161 | 2.84060510707857e-07 |
| x3 | 471342.719845178 | 88787.8743769923 | 5.30863840532844 | 7.4805103399305e-07 |
| x4 | -2249062.4720282 | 432176.998943779 | -5.2040309352992 | 1.15922576667724e-06 |
| x5 | 4666165.32624878 | 906825.926792598 | 5.14560202612733 | 1.47782532562438e-06 |

 

| | | | | |
|---|---|---|---|---|
| x6 | -4350327.34390837 | 851512.546221502 | -5.10894098180056 | 1.71984932410706e-06 |
| x7 | 1495738.54253379 | 294200.52214956 | 5.08407847683361 | 1.9055762658503e-06 |

Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.0104
R-squared: 0.999,  Adjusted R-Squared: 0.999
F-statistic vs. constant model: 1.96e+04, p-value = 2.23e-144

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 14.8796738317706 | 100 | 0.148796738317706 | | |
| Model | 14.8695801817345 | 7 | 2.12422574024779 | 19572.0074636878 | 2.23353866997482e-144 |
| Residual | 0.0100936500361328 | 93 | 0.000108533871356267 | | |

A = 2.000000, B = 2.000000
List of powers: [4.000000, 3.000000, 2.666667, 2.500000, 2.400000, 2.333333, 2.285714]
Fitting arcsin(x) in range (0.000000, 1.000000)
MSS of errors squared = 9.947243e-04
Corrected MSS of errors squared = 1.406753e-03
Particle swarm AICc =  -6.101171e+02
AIC =  -6.276823e+02
AICc =  -6.101171e+02

## Using Power A+B*sqrt(i)

Fitting arcsin(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*sqrt(i)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | -0.00135485397372937 | 0.00540239197780393 | -0.250787795349888 | 0.802531422041736 |
| x1 | 2354.79962383764 | 342.439897031788 | 6.87653408451716 | 6.95473453840326e-10 |
| x2 | -34959.9818877615 | 5179.07195268535 | -6.75024062363813 | 1.24914478739179e-09 |
| x3 | 173848.247812633 | 25708.5606586364 | 6.76227075179456 | 1.18153575410864e-09 |
| x4 | -407596.845097215 | 59807.54070425 | -6.81514137343973 | 9.24886543457495e-10 |
| x5 | 495707.161434956 | 72006.8319902728 | 6.8841684564309 | 6.71215790396029e-10 |
| x6 | -303385.051381148 | 43587.7866353713 | -6.96032248480707 | 4.70755792645779e-10 |
| x7 | 74033.1776175598 | 10516.5764855017 | 7.03966521040595 | 3.24921952303427e-10 |

Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.00909
R-squared: 0.999,  Adjusted R-Squared: 0.999
F-statistic vs. constant model: 2.57e+04, p-value = 6.82e-150

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 14.8796738819405 | 100 | 0.148796738819405 | | |
| Model | 14.8719924983448 | 7 | 2.12457035690639 | 25722.5850953238 | 6.82114241423634e-150 |
| Residual | 0.00768138359578075 | 93 | 8.25955225352768e-05 | | |

A = 1.400000, B = 1.000000
List of powers: [2.400000, 2.814214, 3.132051, 3.400000, 3.636068, 3.849490, 4.045751]
Fitting arcsin(x) in range (0.000000, 1.000000)
MSS of errors squared = 8.677575e-04
Corrected MSS of errors squared = 1.227194e-03

 

```
Particle swarm AICc =  -6.377009e+02
AIC =  -6.552661e+02
AICc =  -6.377009e+02
```

## Using Power A+B*sqrt(i+C)

```
Fitting arcsin(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*sqrt(i+C)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                        Estimate                SE                tStat               pValue
                    ____________________    ____________________    _________________    ____________________

    (Intercept)    0.000607120280344195    0.00503281795807506    0.120632275079626      0.904242545656595
    x1                 25462.7293959234       3462.49404953596     7.3538695032085    7.40536712664693e-11
    x2                -248019.695286829       34068.3345702239    -7.28006515186691    1.04959589127219e-10
    x3                 947687.053825414       130442.446324309     7.26517387959169    1.12600613853721e-10
    x4                -1845626.74662571       253508.538303294    -7.28033366835805    1.04826642982827e-10
    x5                 1951332.97320753       266848.288555042     7.31251822439564    9.00450934349133e-11
    x6                -1069254.45322208       145377.506708882    -7.35501988875944    7.36516911100697e-11
    x7                 238419.646509964       32201.1599693594     7.40407012470448    5.83865857598313e-11


Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.00891
R-squared: 1,  Adjusted R-Squared: 0.999
F-statistic vs. constant model: 2.68e+04, p-value = 1.03e-150
                        SumSq           DF            MeanSq                F                   pValue
                    ____________________    ___    ____________________    _________________    ____________________

    Total            14.879673881064    100        0.14879673881064
    Model           14.8722982729606      7        2.12461403899437    26789.5342126184    1.03167665463462e-150
    Residual    0.00737560810345883     93    7.93076140156863e-05

A = 1.000000, B = 1.000000, C = 2.000000
List of powers: [2.732051, 3.000000, 3.236068, 3.449490, 3.645751, 3.828427, 4.000000]
Fitting arcsin(x) in range (0.000000, 1.000000)
MSS of errors squared = 8.503105e-04
Corrected MSS of errors squared = 1.472781e-03
Particle swarm AICc =  -6.418036e+02
AIC =  -6.593689e+02
AICc =  -6.418036e+02
```

## Using Power A+B*i+C*(i–1)

```
Fitting arcsin(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*i+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                        Estimate                SE                tStat               pValue
                    ____________________    ____________________    _________________    ____________________

    (Intercept)    -0.00732611471509338    0.0111110739582369    -0.659352529074147      0.511298834896565
    x1                 2.72243765690123     0.941970406099157     2.89015200400537     0.00479195349911499
    x2                -32.9629698896172      10.1442075993477    -3.24943762899105     0.00161013391424268
    x3                 186.748723826689      46.6057882687051     4.00698562912382    0.000123862429020203
    x4                -510.729443960469      110.748982837111    -4.61159489574407    1.27258835353068e-05
```

| | | | | |
|---|---|---|---|---|
| x5 | 736.810106223016 | 142.402653814551 | 5.17413184716734 | 1.31281687442885e-06 |
| x6 | -538.069863915491 | 94.061696860817 | -5.72039291096005 | 1.28334771831725e-07 |
| x7 | 156.975054796415 | 25.0202860773749 | 6.27391126987803 | 1.10187037073336e-08 |

Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.0115
R-squared: 0.999,  Adjusted R-Squared: 0.999
F-statistic vs. constant model: 1.62e+04, p-value = 1.69e-140

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 14.8796738812173 | 100 | 0.148796738812173 | | |
| Model | 14.8674432814195 | 7 | 2.12392046877421 | 16150.0340834957 | 1.68623459030343e-140 |
| Residual | 0.0122305997977961 | 93 | 0.000131511825782753 | | |

A = 0.500000, B = 0.270000, C = 0.270000
List of powers: [0.770000, 1.310000, 1.850000, 2.390000, 2.930000, 3.470000, 4.010000]
Fitting arcsin(x) in range (0.000000, 1.000000)
MSS of errors squared = 1.094971e-03
Corrected MSS of errors squared = 1.896545e-03
Particle swarm AICc =  -5.907216e+02
AIC =  -6.082868e+02
AICc =  -5.907216e+02

## Using Power A+B*i–C*(i–1)

Fitting arcsin(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*i-C*(i-1)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | -0.00438421829191862 | 0.0136019257329963 | -0.322323351706233 | 0.747931204932117 |
| x1 | 207.119071729484 | 53.2090661465633 | 3.89255227969945 | 0.00018645377141334 |
| x2 | -1930.00437712015 | 454.790660875307 | -4.24372033806849 | 5.19587574790241e-05 |
| x3 | 7358.14825122482 | 1604.3126668187 | 4.58648018145727 | 1.40380357173124e-05 |
| x4 | -14647.2006548286 | 2980.37936133726 | -4.91454237163171 | 3.80861693402851e-06 |
| x5 | 16087.4473957902 | 3073.09712174156 | 5.23492970071614 | 1.01898118974322e-06 |
| x6 | -9260.95347740218 | 1667.92142429027 | -5.55239194276964 | 2.65347215341927e-07 |
| x7 | 2186.91720095097 | 372.489836156573 | 5.87107885550927 | 6.63734147130779e-08 |

Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.0137
R-squared: 0.999,  Adjusted R-Squared: 0.999
F-statistic vs. constant model: 1.14e+04, p-value = 2.05e-133

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 14.8796738812105 | 100 | 0.148796738812105 | | |
| Model | 14.8623038196742 | 7 | 2.12318625995346 | 11367.6236416333 | 2.0464115479751e-133 |
| Residual | 0.0173700615362122 | 93 | 0.000186774855228089 | | |

A = 0.750000, B = 0.250000, C = 0.000000
List of powers: [1.000000, 1.250000, 1.500000, 1.750000, 2.000000, 2.250000, 2.500000]
Fitting arcsin(x) in range (0.000000, 1.000000)
MSS of errors squared = 1.304906e-03

 

```
Corrected MSS of errors squared = 2.260164e-03
Particle swarm AICc =  -5.552901e+02
AIC =  -5.728553e+02
AICc =  -5.552901e+02
```

## Using Power A+B*i+C*sqrt(i–1)

```
Fitting arcsin(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*i+C*sqrt(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                       Estimate                SE                tStat                pValue
                   ____________________    ____________________    ____________________    ____________________

    (Intercept)    -0.00799614634820268    0.0108303478322756    -0.738309283509185       0.462185605283823
    x1                1.56620313347045     0.534011902951065      2.93289929459488      0.00422789992968553
    x2               -41.8965163154419      12.7464211959149     -3.28692388800625      0.00142996253142093
    x3                299.503717337522      74.2570300917301      4.03333821683339      0.00011261292327324
    x4               -894.65785467174       194.542902664803     -4.59876892149201     1.33803985394494e-05
    x5                1332.09732996283      260.936044683929      5.10507213204827     1.74754249446129e-06
    x6               -977.314306916937      175.026158232231     -5.58381853768509     2.31801577473079e-07
    x7                282.197672393625      46.6039509583402      6.05523065299517     2.93739806278425e-08


Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.0113
R-squared: 0.999,  Adjusted R-Squared: 0.999
F-statistic vs. constant model: 1.67e+04, p-value = 3.98e-141
                     SumSq           DF            MeanSq                F                pValue
                 ____________________    ___    ____________________    ____________    ____________________

    Total          14.8796738812035     100       0.148796738812035
    Model          14.8678169518651       7        2.12397385026644    16659.4201953533    3.98412394734221e-141
    Residual     0.0118569293383857      93    0.000127493863853609

A = 0.340000, B = 0.390000, C = 0.360000
List of powers: [0.730000, 1.480000, 2.019117, 2.523538, 3.010000, 3.484984, 3.951816]
Fitting arcsin(x) in range (0.000000, 1.000000)
MSS of errors squared = 1.078114e-03
Corrected MSS of errors squared = 1.867348e-03
Particle swarm AICc =  -5.938555e+02
AIC =  -6.114207e+02
AICc =  -5.938555e+02
```

## Using Power A+B*sqrt(i)+C*(i–1)

```
Fitting arcsin(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*sqrt(i)+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                       Estimate                SE                tStat                pValue
                   ____________________    ____________________    ____________________    ____________________

    (Intercept)    -0.009303142154984    0.0103563845267833    -0.898300186800185       0.371345354928313
    x1                5.55040791283471     1.69511280166876      3.27435903225472      0.00148813674378169
    x2               -72.489731490747      20.6379454303116     -3.51244903401475     0.000687270037537597
    x3                405.063825764167      98.3600127775393      4.1181758148029     8.26710621603494e-05
```

| | | | | |
|---|---|---|---|---|
| x4 | -1088.32494137562 | 234.356851256809 | -4.64387934698368 | 1.12128152467178e-05 |
| x5 | 1527.40936917228 | 297.575064953785 | 5.13285402259592 | 1.55794936589483e-06 |
| x6 | -1079.75357974667 | 192.717901844168 | -5.60276740984737 | 2.13625472318927e-07 |
| x7 | 304.043774977868 | 50.0976367450058 | 6.06902430398932 | 2.76230635780243e-08 |

Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.0111
R-squared: 0.999,  Adjusted R-Squared: 0.999
F-statistic vs. constant model: 1.72e+04, p-value = 8.11e-142

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 14.8796738812051 | 100 | 0.148796738812051 | | |
| Model | 14.8682159533334 | 7 | 2.12403085047621 | 17240.0168082502 | 8.11120723058046e-142 |
| Residual | 0.0114579278716107 | 93 | 0.00012320352550119 | | |

A = 0.550000, B = 0.400000, C = 0.400000
List of powers: [0.950000, 1.515685, 2.042820, 2.550000, 3.044427, 3.529796, 4.008301]
Fitting arcsin(x) in range (0.000000, 1.000000)
MSS of errors squared = 1.059819e-03
Corrected MSS of errors squared = 1.835660e-03
Particle swarm AICc =  -5.973128e+02
AIC =  -6.148780e+02
AICc =  -5.973128e+02

## Using Power A+B*sqrt(i)+C*sqrt(i–1)

Fitting arcsin(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*sqrt(i)+C*sqrt(i-1)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | -0.00944384159800688 | 0.00792321197777522 | -1.19192085539262 | 0.236324955138344 |
| x1 | 6.91367632843 | 1.47770181814215 | 4.6786680800882 | 9.777824804009e-06 |
| x2 | -408.084081255195 | 92.485951040757 | -4.41238995396565 | 2.74871058657899e-05 |
| x3 | 3455.6434257429 | 721.573533902576 | 4.78903848794636 | 6.30921329457103e-06 |
| x4 | -11256.270304665 | 2205.21464437908 | -5.10438760841551 | 1.75248742976916e-06 |
| x5 | 17367.7276238946 | 3228.23904149718 | 5.37993853634823 | 5.5365256599281e-07 |
| x6 | -12804.6453552442 | 2275.10565761095 | -5.62815415293292 | 1.91451236547625e-07 |
| x7 | 3640.22322398389 | 621.52849347244 | 5.85688872226307 | 7.06466393223678e-08 |

Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.01
R-squared: 0.999,  Adjusted R-Squared: 0.999
F-statistic vs. constant model: 2.11e+04, p-value = 7.58e-146

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 14.8796738812805 | 100 | 0.148796738812805 | | |
| Model | 14.8702885326667 | 7 | 2.12432693323811 | 21050.0869942918 | 7.58131869192595e-146 |
| Residual | 0.00938534861374762 | 93 | 0.000100917727029544 | | |

A = 0.500000, B = 0.700000, C = 0.700000
List of powers: [1.200000, 2.189949, 2.702385, 3.112436, 3.465248, 3.779890, 4.066669]
Fitting arcsin(x) in range (0.000000, 1.000000)

 

```
MSS of errors squared = 9.591882e-04
Corrected MSS of errors squared = 1.661363e-03
Particle swarm AICc =  -6.174655e+02
AIC =  -6.350307e+02
AICc =  -6.174655e+02
```

## Comments

The following table shows the summary results for the seven models that fit the arc sine function.

| *Power* | *A* | *B* | *C* | *F* | *AICc* | *MSSE* | *Pwr Rng* |
|---|---|---|---|---|---|---|---|
| A+B*i | 0.500 | 0.500 | | 17084.5 | -5.963987e+02 | 1.064625e-03 | 1, 4 |
| A+B/i | 2.000 | 2.000 | | 19572.0 | -6.101171e+02 | 9.947243e-04 | 2.28, 4 |
| A+B*√(i) | 1.400 | 1.000 | | 25722.5 | -6.377009e+02 | 8.677575e-04 | 2.4, 4.04 |
| A+B*√(i+C) | 1.000 | 1.000 | 2.000 | 26789.5 | -6.418036e+02 | 8.503105e-04 | 2.73, 4 |
| A+B*i+C*(i-1) | 0.500 | 0.270 | 0.270 | 16150.0 | -5.907216e+02 | 1.094971e-03 | 0.77, 4.01 |
| A+B*i-C*(i-1) | 0.750 | 0.250 | 0.000 | 11367.6 | -5.552901e+02 | 1.304906e-03 | 1, 2.5 |
| A+B*i+C*√(i-1) | 0.340 | 0.390 | 0.360 | 16659.4 | -5.938555e+02 | 1.078114e-03 | 0.73, 3.95 |
| A+B*√(i)+C*(i-1) | 0.550 | 0.400 | 0.400 | 17240.0 | -5.973128e+02 | 1.059819e-03 | 0.95, 4 |
| A+B*√(i)+C*√(i-1) | 0.500 | 0.700 | 0.700 | 21050.0 | -6.174655e+02 | 9.591882e-04 | 1.2, 4.06 |

The above table shows that the top two Shammas polynomials use the following power equations:

$1+\sqrt{(i+2)}$

And,

$1.4+\sqrt{(i)}$

 

# RESULTS FOR THE ARC TANGENT

## Using Power A+B* i

```
Fitting arctan(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7
```

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | -5.96456211089194e-07 | 1.00005048989183e-06 | -0.596426097600043 | 0.552339650938527 |
| x1 | 0.887475429672544 | 0.00022972950903834 | 3863.132052071 | 7.33643781347519e-244 |
| x2 | 0.530391649517621 | 0.00229431822629984 | 231.176147858534 | 3.71145894724562e-130 |
| x3 | -1.74312878588787 | 0.00966639812330362 | -180.328677099028 | 3.80098821802896e-120 |
| x4 | 3.86725557100945 | 0.0212970463663946 | 181.586474691239 | 1.99500537974213e-120 |
| x5 | -5.43324123845714 | 0.0256860742671616 | -211.524781169199 | 1.41547064185885e-126 |
| x6 | 3.51744050220056 | 0.0160698711508233 | 218.884175808738 | 5.91973149193157e-128 |
| x7 | -0.840794286334462 | 0.00408049000391579 | -206.05228429125 | 1.61200173107103e-125 |

```
Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 1.07e-06
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 6.82e+11, p-value = 0
```

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 5.42875862162706 | 100 | 0.0542875862162706 | | |
| Model | 5.42875862152124 | 7 | 0.775536945931605 | 681551169062.837 | 0 |
| Residual | 1.05824682350431e-10 | 93 | 1.13789981021968e-12 | | |

```
A = 0.483308, B = 0.500000
List of powers: [0.983308, 1.483308, 1.983308, 2.483308, 2.983308, 3.483308, 3.983308]
Fitting arctan(x) in range (0.000000, 1.000000)
MSS of errors squared = 1.018527e-07
Corrected MSS of errors squared = 1.440414e-07
Particle swarm AICc =  -2.465829e+03
AIC =  -2.483394e+03
AICc =  -2.465829e+03
```

## Using Power A+B/i

```
Fitting arctan(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B/i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7
```

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 6.82051639998846e-08 | 6.66850455219813e-06 | 0.0102279549284258 | 0.991861323657391 |
| x1 | 39637.0837747738 | 572.40986711212 | 69.2459827339244 | 8.13663715563822e-82 |
| x2 | -6227578.13490785 | 94405.9300603947 | -65.9659634826314 | 6.78702324670152e-80 |
| x3 | 110753348.304285 | 1702566.32763938 | 65.0508273929304 | 2.42144570181468e-79 |
| x4 | -602565705.852567 | 9322590.3054003 | -64.6350087382386 | 4.34045176491588e-79 |
| x5 | 1353000741.52167 | 21009258.3645226 | 64.4002143267667 | 6.0442566994876e-79 |

 

| | | | | |
|---|---|---|---|---|
| x6 | -1329696284.28071 | 20695612.1181956 | -64.2501548969232 | 7.47334348635622e-79 |
| x7 | 474695842.143867 | 7400211.25163514 | 64.1462555597962 | 8.65873529155688e-79 |

Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 6.69e-06
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.73e+10, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 5.42875879939313 | 100 | 0.0542875879939313 | | |
| Model | 5.42875879523153 | 7 | 0.775536970747361 | 17331048852.7541 | 0 |
| Residual | 4.16160261806908e-09 | 93 | 4.47484152480546e-11 | | |

A = 1.495287, B = 0.223062
List of powers: [1.718349, 1.606818, 1.569641, 1.551053, 1.539899, 1.532464, 1.527153]
Fitting arctan(x) in range (0.000000, 1.000000)
MSS of errors squared = 6.375193e-07
Corrected MSS of errors squared = 9.015884e-07
Particle swarm AICc =  -2.095349e+03
AIC =  -2.112914e+03
AICc =  -2.095349e+03

## Using Power A+B*sqrt(i)

Fitting arctan(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*sqrt(i)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | -7.85842106955192e-07 | 6.67864752900609e-06 | -0.117664857075058 | 0.906586966532663 |
| x1 | 520002.587746388 | 8213.61667144434 | 63.3098193581691 | 2.85649059420972e-78 |
| x2 | -6162884.655082 | 96299.5153658774 | -63.9970474583068 | 1.07016411207279e-78 |
| x3 | 25525720.6283221 | 394867.695195211 | 64.6437298845197 | 4.28748880780407e-79 |
| x4 | -51215667.8897515 | 784818.150651471 | -65.2580063894264 | 1.81287635656369e-79 |
| x5 | 54328204.4805859 | 825083.074221129 | 65.84573866319 | 8.01362641954499e-80 |
| x6 | -29417888.3808711 | 442966.647599105 | -66.4110685089208 | 3.67835967430884e-80 |
| x7 | 6422514.01446838 | 95919.9275366641 | 66.9570357214195 | 1.74450638850471e-80 |

Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 6.7e-06
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.73e+10, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 5.42875873108915 | 100 | 0.0542875873108915 | | |
| Model | 5.42875872691768 | 7 | 0.775536960988239 | 17290015020.8742 | 0 |
| Residual | 4.17147916209615e-09 | 93 | 4.48546146461951e-11 | | |

A = 1.386504, B = 0.100000
List of powers: [1.486504, 1.527925, 1.559709, 1.586504, 1.610111, 1.631453, 1.651079]
Fitting arctan(x) in range (0.000000, 1.000000)
MSS of errors squared = 6.393359e-07
Corrected MSS of errors squared = 9.041576e-07

 

```
Particle swarm AICc =  -2.094775e+03
AIC =  -2.112340e+03
AICc =  -2.094775e+03
```

## Using Power A+B*sqrt(i+C)

```
Fitting arctan(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*sqrt(i+C)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                        Estimate                SE                tStat              pValue
                    ____________________    ____________________    _______________    ____________________

    (Intercept)    -8.51220834131712e-07    6.70397507700549e-06    -0.126972553500592      0.899236249960752
    x1                  17455.1127528711        285.190321526823      61.2051371849569     6.1616310778121e-77
    x2                 -156483.675053885        2527.01831905237     -61.9242345312975     2.13357640478693e-77
    x3                  556554.676176658        8837.60211649545      62.9757561881909     4.62059391855403e-78
    x4                 -1017308.12406236        15850.7715782485     -64.1803535582061     8.25010900564409e-79
    x5                  1015975.21821622        15520.9634930529      65.4582570644514     1.37160211326785e-79
    x6                 -528546.536344148        7916.07981478186     -66.7687224877624     2.25492423824983e-80
    x7                  112354.113732806        1650.11378900445      68.0887066585828     3.78555028715021e-81


Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 6.72e-06
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.72e+10, p-value = 0
                        SumSq             DF            MeanSq                 F            pValue
                    ____________________    ___    ____________________    _____________    ______

    Total            5.42875862221212     100      0.0542875862221212
    Model            5.42875861800978       7       0.775536945429968    17163049558.773       0
    Residual    4.20233803311021e-09      93    4.51864304635506e-11

A = 1.000000, B = 0.239681, C = 2.000000
List of powers: [1.415140, 1.479362, 1.535943, 1.587096, 1.634136, 1.677920, 1.719043]
Fitting arctan(x) in range (0.000000, 1.000000)
MSS of errors squared = 6.418345e-07
Corrected MSS of errors squared = 1.111690e-06
Particle swarm AICc =  -2.093987e+03
AIC =  -2.111552e+03
AICc =  -2.093987e+03
```

## Using Power A+B*i+C*(i–1)

```
Fitting arctan(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*i+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                        Estimate                SE                tStat              pValue
                    ____________________    ____________________    _______________    ____________________

    (Intercept)    -1.2466593089308e-06    3.67646430189442e-06    -0.339091911837255      0.735304974214405
    x1                0.0173858774948243    0.000142164007641393      122.29450887935    1.57859901027252e-104
    x2                   1.49671776645632     0.00167654405383077      892.739897312239    1.07457535092155e-184
    x3                  -1.6994427282925     0.00837995508390535     -202.798548593234    7.06031142745649e-125
    x4                  3.27797883473343      0.0212513022729352      154.248374647051    7.38786942427428e-114
```

| | | | | |
|---|---|---|---|---|
| x5 | -4.23139644733717 | 0.0287629366508807 | -147.112810444117 | 5.94032154276027e-112 |
| x6 | 2.43872422461638 | 0.0198055556179406 | 123.133340546493 | 8.39187716637754e-105 |
| x7 | -0.514562171667314 | 0.00545321154394917 | -94.359473774361 | 3.87528073720535e-94 |

Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 3.71e-06
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 5.64e+10, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 5.42875862162705 | 100 | 0.0542875862162705 | | |
| Model | 5.428758620349 | 7 | 0.775536945764143 | 56433437117.0596 | 0 |
| Residual | 1.27805321881169e-09 | 93 | 1.3742507729158e-11 | | |

A = 0.289665, B = 0.270000, C = 0.270000
List of powers: [0.559665, 1.099665, 1.639665, 2.179665, 2.719665, 3.259665, 3.799665]
Fitting arctan(x) in range (0.000000, 1.000000)
MSS of errors squared = 3.539595e-07
Corrected MSS of errors squared = 6.130759e-07
Particle swarm AICc =  -2.214207e+03
AIC =  -2.231772e+03
AICc =  -2.214207e+03

## Using Power A+B*i–C*(i–1)

Fitting arctan(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*i-C*(i-1)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | -8.04471911156096e-07 | 7.13533312268397e-06 | -0.112744828773109 | 0.910475868509615 |
| x1 | 4.89979908054197 | 0.119016563239542 | 41.1690519972439 | 1.69231383912903e-61 |
| x2 | -36.5088053524097 | 0.950331669981005 | -38.4169090704296 | 7.40149509393066e-59 |
| x3 | 140.584603403759 | 3.13431287075645 | 44.8534046219291 | 8.53136431884088e-65 |
| x4 | -284.954092151319 | 5.46232560762516 | -52.1671743173889 | 1.15235720927219e-70 |
| x5 | 320.255117721826 | 5.30544763545988 | 60.3634489918147 | 2.16467721411273e-76 |
| x6 | -188.60820798312 | 2.72382538846486 | -69.243868855125 | 8.15933428079444e-82 |
| x7 | 45.1170040707477 | 0.577713445934001 | 78.0958178977575 | 1.36209272176466e-86 |

Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 7.15e-06
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.52e+10, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 5.42875862162741 | 100 | 0.0542875862162741 | | |
| Model | 5.42875861687777 | 7 | 0.775536945268253 | 15185334388.1395 | 0 |
| Residual | 4.74964423347046e-09 | 93 | 5.10714433706501e-11 | | |

A = 0.750000, B = 0.250000, C = 0.062857
List of powers: [1.000000, 1.187143, 1.374286, 1.561428, 1.748571, 1.935714, 2.122857]
Fitting arctan(x) in range (0.000000, 1.000000)
MSS of errors squared = 6.823525e-07

 

```
Corrected MSS of errors squared = 1.181869e-06
Particle swarm AICc =  -2.081621e+03
AIC =  -2.099187e+03
AICc =  -2.081621e+03
```

## Using Power A+B*i+C*sqrt(i–1)

```
Fitting arctan(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*i+C*sqrt(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                        Estimate                SE                tStat              pValue
                    ____________________    ____________________    _______________    ____________________

    (Intercept)    -2.27009972766482e-06    4.70802550336017e-06    -0.48217659952025       0.630813380712063
    x1                 0.105514946187995    0.000524181216316793     201.294786809426    1.40848770196836e-124
    x2                  2.61077253672843     0.00781072865961169     334.254670787427    4.95628351964352e-145
    x3                 -6.06159302263922      0.0383450835039101    -158.080057956351     7.6066236947727e-115
    x4                  11.1399532749843      0.0912176983973359     122.124910743305    1.79464902130046e-104
    x5                 -12.1626753927219       0.114979291632228    -105.781443076074     1.03637435446623e-98
    x6                  6.44814451361716      0.0738959495097115      87.259782929912     5.16051054281543e-91
    x7                 -1.29470792990703       0.019082230568252    -67.8488777963461     5.22228945842902e-81


Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 4.77e-06
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 3.41e+10, p-value = 0
                    SumSq            DF            MeanSq                F            pValue
                ____________________    ___    ____________________    _______________    ______

    Total           5.42875862162705    100      0.0542875862162705
    Model           5.42875861951364      7       0.775536945644805    34127224912.4891       0
    Residual    2.11341344424909e-09     93    2.27248757446139e-11

A = 0.340000, B = 0.390000, C = 0.128023
List of powers: [0.730000, 1.248023, 1.691052, 2.121743, 2.546047, 2.966269, 3.383592]
Fitting arctan(x) in range (0.000000, 1.000000)
MSS of errors squared = 4.551662e-07
Corrected MSS of errors squared = 7.883710e-07
Particle swarm AICc =  -2.163408e+03
AIC =  -2.180973e+03
AICc =  -2.163408e+03
```

## Using Power A+B*sqrt(i)+C*(i–1)

```
Fitting arctan(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*sqrt(i)+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                        Estimate                SE                tStat              pValue
                    ____________________    ____________________    _______________    ____________________

    (Intercept)    -3.01000908692395e-06    2.29731543356003e-06    -1.31022890585795       0.19334530724109
    x1                 0.693018910839272    0.000565146895738783      1226.2633238626    1.62728914093141e-197
    x2                  1.4745086915022     0.00685122997337653     215.218099119728    2.83909770653028e-127
    x3                 -4.82969472455565      0.0319957505576897    -150.948005293625    5.47834571331974e-113
```

| | | | | |
|---|---|---|---|---|
| x4 | 10.2214154366232 | 0.0745190762775667 | 137.165085065611 | 3.87977072508964e-109 |
| x5 | -12.5885697467032 | 0.0925517899075087 | -136.016491515546 | 8.44842942751974e-109 |
| x6 | 7.53819379939588 | 0.0587151565075372 | 128.38582484964 | 1.76389948257274e-106 |
| x7 | -1.72346880615085 | 0.0149760685732695 | -115.08152474856 | 4.34149283564962e-102 |

Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 2.4e-06
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.35e+11, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 5.42875862162701 | 100 | 0.0542875862162701 | | |
| Model | 5.4287586210916 | 7 | 0.775536945870229 | 134711085189.2 | 0 |
| Residual | 5.35404609536272e-10 | 93 | 5.75703881221798e-12 | | |

A = 0.550000, B = 0.400000, C = 0.336515
List of powers: [0.950000, 1.452201, 1.915851, 2.359546, 2.790489, 3.212373, 3.627393]
Fitting arctan(x) in range (0.000000, 1.000000)
MSS of errors squared = 2.290969e-07
Corrected MSS of errors squared = 3.968076e-07
Particle swarm AICc =  -2.302084e+03
AIC =  -2.319650e+03
AICc =  -2.302084e+03

## Using Power A+B*sqrt(i)+C*sqrt(i–1)

Fitting arctan(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*sqrt(i)+C*sqrt(i-1)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | -2.23154765683016e-07 | 5.03300933722138e-07 | -0.443382379668335 | 0.658518939793136 |
| x1 | 0.986773133758476 | 5.59560001450049e-05 | 17634.8046894228 | 3.45550442083755e-305 |
| x2 | 0.478666228884316 | 0.00354795029350422 | 134.913454047167 | 1.79478168435765e-108 |
| x3 | -4.11385367057894 | 0.028283626522837 | -145.450006817807 | 1.70200728831031e-111 |
| x4 | 14.3844751826791 | 0.0879635900839092 | 163.527604648214 | 3.29340845395217e-116 |
| x5 | -24.6503186718897 | 0.130645865219724 | -188.68043493326 | 5.70502424075177e-122 |
| x6 | 19.2869395641789 | 0.0932066110571061 | 206.926733473467 | 1.08815601474554e-125 |
| x7 | -5.58728495878206 | 0.0257334597719806 | -217.121405683105 | 1.25389491986641e-127 |

Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 5.72e-07
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 2.37e+12, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 5.42875862162705 | 100 | 0.0542875862162705 | | |
| Model | 5.42875862159657 | 7 | 0.775536945942368 | 2366402223237.73 | 0 |
| Residual | 3.04787306504295e-11 | 93 | 3.27728286563758e-13 | | |

A = 0.297728, B = 0.699979, C = 0.659823
List of powers: [0.997706, 1.947470, 2.443257, 2.840532, 3.182574, 3.487728, 3.765927]
Fitting arctan(x) in range (0.000000, 1.000000)

 

```
MSS of errors squared = 5.465061e-08
Corrected MSS of errors squared = 9.465764e-08
Particle swarm AICc =  -2.591588e+03
AIC =  -2.609153e+03
AICc =  -2.591588e+03
```

## Comments

The following table shows the summary results for the seven models that fit the arc tangent function.

| Power | A | B | C | F | AICc | MSSE | Pwr Rng |
|---|---|---|---|---|---|---|---|
| A+B*i | 0.483 | 0.500 | | 681551169062.8 | -2.465829e+03 | 1.018527e-07 | 0.98, 3.98 |
| A+B/i | 1.495 | 0.223 | | 17331048852.7 | -2.095349e+03 | 6.375193e-07 | 1.52, 1.71 |
| A+B*√(i) | 1.386 | 0.100 | | 17290015020.8 | -2.094775e+03 | 6.393359e-07 | 1.48, 1.65 |
| A+B*√(i+C) | 1.000 | 0.239 | 2.000 | 17163049558.7 | -2.093987e+03 | 6.418345e-07 | 1.41, 1.71 |
| A+B*i+C*(i-1) | 0.289 | 0.270 | 0.270 | 56433437117.0 | -2.214207e+03 | 3.539595e-07 | 0.55, 3.79 |
| A+B*i-C*(i-1) | 0.750 | 0.250 | 0.062 | 15185334388.1 | -2.081621e+03 | 6.823525e-07 | 1, 2.12 |
| A+B*i+C*√(i-1) | 0.340 | 0.390 | 0.128 | 34127224912.4 | -2.163408e+03 | 4.551662e-07 | 0.73, 3.38 |
| A+B*√(i)+C*(i-1) | 0.550 | 0.400 | 0.336 | 134711085189.2 | -2.302084e+03 | 2.290969e-07 | 0.95, 3.62 |
| A+B*√(i)+C*√(i-1) | 0.297 | 0.699 | 0.659 | 2366402223237.7 | -2.591588e+03 | 5.465061e-08 | 0.99, 3.76 |

The above table shows that the top two Shammas polynomials use the following power equations:

$0.297+0.699*\sqrt{(i)} + 0.659*\sqrt{(i-1)}$

And,

$0.483+0.5*i$

 

# RESULTS FOR INVERSE HYPERBOLIC COSINE

## Using Power A+B* i

```
Fitting acosh(x) in range (1.000000, 100.000000)
Shammas polynomial power is A+B*i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE              tStat              pValue
                ________________    ________________    ________________    ____________________

    (Intercept)    -21416.0800426452     847.91409220176      -25.25737010342    3.72074987174796e-43
    x1               112635.72233092    4634.05811023245     24.3060660120359    8.06786450060879e-42
    x2              -253282.205629563    10809.2424128745    -23.4320034610278    1.47459702826257e-40
    x3               315608.456310756     13950.582242004     22.6233178541097    2.32601122969978e-39
    x4              -235334.840932752    10760.0090248281    -21.8712494004169    3.22002167837068e-38
    x5               105006.428182587    4960.13372149876     21.1700800983362    3.94657516377135e-37
    x6              -25961.0232311501    1265.46121996173     -20.515068199352    4.31293612579468e-36
    x7               2743.54371605102    137.852068490865     19.9020859540662    4.22933780383823e-35


Number of observations: 100, Error degrees of freedom: 92
Root Mean Squared Error: 0.00182
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 3.96e+06, p-value = 4.18e-249
                    SumSq            DF          MeanSq                F                  pValue
                ________________    ____    ____________________    ________________    ____________________

    Total          91.5520963395277    99       0.924768649894219
    Model          91.5517927931639     7        13.0788275418806    3963981.38008388    4.18176517569804e-249
    Residual    0.00030354636373886    92    3.29941699716152e-06

A = 0.000000, B = 0.100000
List of powers: [0.100000, 0.200000, 0.300000, 0.400000, 0.500000, 0.600000, 0.700000]
Fitting acosh(x) in range (1.000000, 100.000000)
MSS of errors squared = 1.742258e-04
Corrected MSS of errors squared = 2.463925e-04
Particle swarm AICc =  -9.531445e+02
AIC =  -9.707269e+02
AICc =  -9.531445e+02
```

## Using Power A+B/i

```
Fitting acosh(x) in range (1.000000, 100.000000)
Shammas polynomial power is A+B/i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE              tStat              pValue
                ________________    ________________    ________________    ____________________

    (Intercept)    -101789563.797428    4213005.65294993    -24.1607944974285    1.30070832795203e-41
    x1               57077.5355839394    2629.98641461501     21.7025971186603    5.85421895305217e-38
    x2              -34610703.9139141    1513711.69215477    -22.8647926109666    1.01339636143649e-39
    x3               1146007014.82207    49225141.6878144     23.2809287191095    2.45591063786781e-40
    x4              -9255098717.26538    393921250.453716    -23.4947942173859    1.19370340196048e-40
    x5               27697266047.5496    1172369791.63998     23.6250253504103    7.71092921803469e-41
```

| | | | | |
|---|---|---|---|---|
| x6 | -34087181623.2687 | 1437510042.73244 | -23.7126563362823 | 5.75205629800501e-41 |
| x7 | 14635350468.3384 | 615560447.216702 | 23.7756511720549 | 4.66157969930594e-41 |

Number of observations: 100, Error degrees of freedom: 92
Root Mean Squared Error: 0.00107
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.13e+07, p-value = 4.47e-270

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 91.5520541911146 | 99 | 0.924768224152672 | | |
| Model | 91.5519479413822 | 7 | 13.0788497059117 | 11324773.6768355 | 4.4678687107011e-270 |
| Residual | 0.000106249732425567 | 92 | 1.15488839593008e-06 | | |

A = 0.000000, B = 0.314545
List of powers: [0.314545, 0.157272, 0.104848, 0.078636, 0.062909, 0.052424, 0.044935]
Fitting acosh(x) in range (1.000000, 100.000000)
MSS of errors squared = 1.031154e-04
Corrected MSS of errors squared = 1.458272e-04
Particle swarm AICc =  -1.058045e+03
AIC =  -1.075628e+03
AICc =  -1.058045e+03

## Using Power A+B*sqrt(i)

Fitting acosh(x) in range (1.000000, 100.000000)
Shammas polynomial power is A+B*sqrt(i)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | -1010432.43482752 | 41308.8530168744 | -24.4604330799203 | 4.86793835584461e-42 |
| x1 | 60308687.838344 | 2556197.07745684 | 23.5931291723192 | 8.58065106997977e-41 |
| x2 | -418888119.358277 | 18015167.0288841 | -23.2519697811664 | 2.70891733788345e-40 |
| x3 | 1224304162.42667 | 53237732.94501 | 22.9969252013656 | 6.44865690472187e-40 |
| x4 | -1879295566.41587 | 82474780.5053118 | -22.7863057640369 | 1.32668083207262e-39 |
| x5 | 1597484715.89609 | 70672692.5309819 | 22.6039883112671 | 2.48663550401915e-39 |
| x6 | -714637438.573322 | 31844154.2474388 | -22.441715142452 | 4.36271835216187e-39 |
| x7 | 131733990.621515 | 5908788.08079403 | 22.2945871167227 | 7.28079384700631e-39 |

Number of observations: 100, Error degrees of freedom: 92
Root Mean Squared Error: 0.00125
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 8.41e+06, p-value = 3.85e-264

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 91.5521853720665 | 99 | 0.924769549212793 | | |
| Model | 91.5520423632768 | 7 | 13.0788631947538 | 8413856.34876772 | 3.85190996572553e-264 |
| Residual | 0.000143008789791566 | 92 | 1.55444336729962e-06 | | |

A = 0.000000, B = 0.100000
List of powers: [0.100000, 0.141421, 0.173205, 0.200000, 0.223607, 0.244949, 0.264575]
Fitting acosh(x) in range (1.000000, 100.000000)
MSS of errors squared = 1.195787e-04
Corrected MSS of errors squared = 1.691098e-04

 

```
Particle swarm AICc =  -1.028420e+03
AIC =  -1.046002e+03
AICc =  -1.028420e+03
```

## Using Power A+B*sqrt(i+C)

```
Fitting acosh(x) in range (1.000000, 100.000000)
Shammas polynomial power is A+B*sqrt(i+C)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat               pValue
                ________________    ________________    ________________    ____________________

    (Intercept)    -1010400.0349036    41304.0992732139    -24.4624638397296    4.83576732556447e-42
    x1              60308298.5929023    2555970.62919422     23.5950671357733    8.52508931440848e-41
    x2             -418885248.934755    18013567.4866077    -23.2538751275214    2.69149043151455e-40
    x3              1224294831.38066    53232973.8834863     22.9988058540621    6.40738667032798e-40
    x4             -1879280082.67595    82467368.5960868    -22.7881659699898     1.318227850455e-39
    x5              1597470782.71128    70666315.9417369     22.6058308180147    2.47085380709278e-39
    x6             -714630930.892407    31841272.3670495    -22.4435419117212    4.33512791420551e-39
    x7              131732749.853514    5908252.09419184     22.296399637892    7.23489894819594e-39


Number of observations: 100, Error degrees of freedom: 92
Root Mean Squared Error: 0.00125
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 8.41e+06, p-value = 3.83e-264
                    SumSq            DF           MeanSq                F                  pValue
                ____________    ____    ________________    _____________    ____________________

    Total          91.5520465665777    99       0.924768147137149
    Model          91.5519035774296     7        13.0788433682042    8414999.35990595    3.82791594906183e-264
    Residual    0.000142989148116612    92    1.55422987083274e-06

A = 0.000000, B = 0.100000, C = 0.000012
List of powers: [0.100001, 0.141422, 0.173205, 0.200000, 0.223607, 0.244949, 0.264575]
Fitting acosh(x) in range (1.000000, 100.000000)
MSS of errors squared = 1.195749e-04
Corrected MSS of errors squared = 2.071098e-04
Particle swarm AICc =  -1.028426e+03
AIC =  -1.046009e+03
AICc =  -1.028426e+03
```

## Using Power A+B*i+C*(i–1)

```
Fitting acosh(x) in range (1.000000, 100.000000)
Shammas polynomial power is A+B*i+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat               pValue
                ________________    ________________    ________________    ____________________

    (Intercept)    -21416.0800426452    847.91409220176     -25.25737010342    3.72074987174796e-43
    x1               112635.72233092    4634.05811023245     24.3060660120359    8.06786450060879e-42
    x2             -253282.205629563    10809.2424128745    -23.4320034610278    1.47459702826257e-40
    x3              315608.456310756    13950.582242004      22.6233178541097    2.32601122969978e-39
    x4             -235334.840932752    10760.0090248281    -21.8712494004169    3.22002167837068e-38
```

| | | | | |
|---|---|---|---|---|
| x5 | 105006.428182587 | 4960.13372149876 | 21.1700800983362 | 3.94657516377135e-37 |
| x6 | -25961.0232311501 | 1265.46121996173 | -20.515068199352 | 4.31293612579468e-36 |
| x7 | 2743.54371605102 | 137.852068490865 | 19.9020859540662 | 4.22933780383823e-35 |

Number of observations: 100, Error degrees of freedom: 92
Root Mean Squared Error: 0.00182
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 3.96e+06, p-value = 4.18e-249

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 91.5520963395277 | 99 | 0.924768649894219 | | |
| Model | 91.5517927931639 | 7 | 13.0788275418806 | 3963981.38008388 | 4.18176517569804e-249 |
| Residual | 0.00030354636373886 | 92 | 3.29941699716152e-06 | | |

A = 0.000000, B = 0.100000, C = 0.000000
List of powers: [0.100000, 0.200000, 0.300000, 0.400000, 0.500000, 0.600000, 0.700000]
Fitting acosh(x) in range (1.000000, 100.000000)
MSS of errors squared = 1.742258e-04
Corrected MSS of errors squared = 3.017680e-04
Particle swarm AICc =  -9.531445e+02
AIC =  -9.707269e+02
AICc =  -9.531445e+02

## Using Power A+B*i–C*(i–1)

Fitting acosh(x) in range (1.000000, 100.000000)
Shammas polynomial power is A+B*i-C*(i-1)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | -26197.6000658637 | 1197.96349384341 | -21.8684460757767 | 3.25209234269292e-38 |
| x1 | 2186.51454295874 | 103.067430056685 | 21.2144082932523 | 3.36282669402478e-37 |
| x2 | 40215.0480639118 | 1808.89363012262 | 22.2318479064939 | 9.06429658180212e-39 |
| x3 | -38347.9865427135 | 1639.38124630117 | -23.3917440676083 | 1.6889255519698e-40 |
| x4 | 44276.0344063136 | 1795.44871272326 | 24.6601499071269 | 2.54086513622105e-42 |
| x5 | -35003.218865047 | 1342.9793516732 | -26.0638548324938 | 2.93193225206999e-44 |
| x6 | 16118.0351948492 | 583.46934147484 | 27.6244766419218 | 2.53639513684538e-46 |
| x7 | -3246.82669202273 | 110.555172916085 | -29.368383282139 | 1.60283770623304e-48 |

Number of observations: 100, Error degrees of freedom: 92
Root Mean Squared Error: 0.000418
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 7.47e+07, p-value = 9.02e-308

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 91.552096346782 | 99 | 0.924768649967495 | | |
| Model | 91.552080245148 | 7 | 13.0788686064497 | 74728808.0089561 | 9.01631429609108e-308 |
| Residual | 1.61016339461639e-05 | 92 | 1.75017760284391e-07 | | |

A = 0.000000, B = 0.100000, C = 0.250000
List of powers: [0.100000, -0.050000, -0.200000, -0.350000, -0.500000, -0.650000, -0.800000]
Fitting acosh(x) in range (1.000000, 100.000000)
MSS of errors squared = 4.012684e-05

 

```
Corrected MSS of errors squared = 6.950173e-05
Particle swarm AICc =  -1.246806e+03
AIC =  -1.264388e+03
AICc =  -1.246806e+03
```

## Using Power A+B*i+C*sqrt(i–1)

```
Fitting acosh(x) in range (1.000000, 100.000000)
Shammas polynomial power is A+B*i+C*sqrt(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat                pValue
                ________________    ________________    ________________    ____________________

    (Intercept)    -21416.0800426452     847.91409220176      -25.25737010342    3.72074987174796e-43
    x1               112635.72233092    4634.05811023245     24.3060660120359    8.06786450060879e-42
    x2             -253282.205629563    10809.2424128745    -23.4320034610278    1.47459702826257e-40
    x3              315608.456310756     13950.582242004     22.6233178541097    2.32601122969978e-39
    x4             -235334.840932752    10760.0090248281    -21.8712494004169    3.22002167837068e-38
    x5              105006.428182587    4960.13372149876     21.1700800983362    3.94657516377135e-37
    x6             -25961.0232311501    1265.46121996173     -20.515068199352    4.31293612579468e-36
    x7              2743.54371605102    137.852068490865     19.9020859540662    4.22933780383823e-35


Number of observations: 100, Error degrees of freedom: 92
Root Mean Squared Error: 0.00182
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 3.96e+06, p-value = 4.18e-249
                    SumSq           DF           MeanSq                F                pValue
                ________________    __    ____________________    ________________    ____________________

    Total          91.5520963395277    99       0.924768649894219
    Model          91.5517927931639     7        13.0788275418806    3963981.38008388    4.18176517569804e-249
    Residual    0.00030354636373886    92    3.29941699716152e-06

A = 0.000000, B = 0.100000, C = 0.000000
List of powers: [0.100000, 0.200000, 0.300000, 0.400000, 0.500000, 0.600000, 0.700000]
Fitting acosh(x) in range (1.000000, 100.000000)
MSS of errors squared = 1.742258e-04
Corrected MSS of errors squared = 3.017680e-04
Particle swarm AICc =  -9.531445e+02
AIC =  -9.707269e+02
AICc =  -9.531445e+02
```

## Using Power A+B*sqrt(i)+C*(i–1)

```
Fitting acosh(x) in range (1.000000, 100.000000)
Shammas polynomial power is A+B*sqrt(i)+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat                pValue
                ________________    ________________    ________________    ____________________

    (Intercept)    -1010432.43482752    41308.8530168744    -24.4604330799203    4.86793835584461e-42
    x1               60308687.838344    2556197.07745684     23.5931291723192    8.58065106997977e-41
    x2             -418888119.358277    18015167.0288841    -23.2519697811664    2.70891733788345e-40
    x3              1224304162.42667     53237732.94501     22.9969252013656    6.44865690472187e-40
```

| | | | | |
|---|---|---|---|---|
| x4 | -1879295566.41587 | 82474780.5053118 | -22.7863057640369 | 1.32668083207262e-39 |
| x5 | 1597484715.89609 | 70672692.5309819 | 22.6039883112671 | 2.48663550401915e-39 |
| x6 | -714637438.573322 | 31844154.2474388 | -22.441715142452 | 4.36271835216187e-39 |
| x7 | 131733990.621515 | 5908788.08079403 | 22.2945871167227 | 7.28079384700631e-39 |

Number of observations: 100, Error degrees of freedom: 92
Root Mean Squared Error: 0.00125
R-squared: 1, Adjusted R-Squared: 1
F-statistic vs. constant model: 8.41e+06, p-value = 3.85e-264

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 91.5521853720665 | 99 | 0.924769549212793 | | |
| Model | 91.5520423632768 | 7 | 13.0788631947538 | 8413856.34876772 | 3.85190996572553e-264 |
| Residual | 0.000143008789791566 | 92 | 1.55444336729962e-06 | | |

A = 0.000000, B = 0.100000, C = 0.000000
List of powers: [0.100000, 0.141421, 0.173205, 0.200000, 0.223607, 0.244949, 0.264575]
Fitting acosh(x) in range (1.000000, 100.000000)
MSS of errors squared = 1.195787e-04
Corrected MSS of errors squared = 2.071163e-04
Particle swarm AICc = -1.028420e+03
AIC = -1.046002e+03
AICc = -1.028420e+03

## Using Power A+B*sqrt(i)+C*sqrt(i–1)

Fitting acosh(x) in range (1.000000, 100.000000)
Shammas polynomial power is A+B*sqrt(i)+C*sqrt(i-1)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | -1010432.43482752 | 41308.8530168744 | -24.4604330799203 | 4.86793835584461e-42 |
| x1 | 60308687.838344 | 2556197.07745684 | 23.5931291723192 | 8.58065106997977e-41 |
| x2 | -418888119.358277 | 18015167.0288841 | -23.2519697811664 | 2.70891733788345e-40 |
| x3 | 1224304162.42667 | 53237732.94501 | 22.9969252013656 | 6.44865690472187e-40 |
| x4 | -1879295566.41587 | 82474780.5053118 | -22.7863057640369 | 1.32668083207262e-39 |
| x5 | 1597484715.89609 | 70672692.5309819 | 22.6039883112671 | 2.48663550401915e-39 |
| x6 | -714637438.573322 | 31844154.2474388 | -22.441715142452 | 4.36271835216187e-39 |
| x7 | 131733990.621515 | 5908788.08079403 | 22.2945871167227 | 7.28079384700631e-39 |

Number of observations: 100, Error degrees of freedom: 92
Root Mean Squared Error: 0.00125
R-squared: 1, Adjusted R-Squared: 1
F-statistic vs. constant model: 8.41e+06, p-value = 3.85e-264

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 91.5521853720665 | 99 | 0.924769549212793 | | |
| Model | 91.5520423632768 | 7 | 13.0788631947538 | 8413856.34876772 | 3.85190996572553e-264 |
| Residual | 0.000143008789791566 | 92 | 1.55444336729962e-06 | | |

A = 0.000000, B = 0.100000, C = 0.000000
List of powers: [0.100000, 0.141421, 0.173205, 0.200000, 0.223607, 0.244949, 0.264575]
Fitting acosh(x) in range (1.000000, 100.000000)

 

```
MSS of errors squared = 1.195787e-04
Corrected MSS of errors squared = 2.071163e-04
Particle swarm AICc =  -1.028420e+03
AIC =  -1.046002e+03
AICc =  -1.028420e+03
```

## Comments

The following table shows the summary results for the seven models that fit the inverse hyperbolic cosine function.

| *Power* | *A* | *B* | *C* | *F* | *AICc* | *MSSE* | *Pwr Rng* |
|---|---|---|---|---|---|---|---|
| A+B*i | 0 | 0.1 | | 3963981.3 | -9.531445E+02 | 1.742258E-04 | 0.10, 0.70 |
| A+B/i | 0 | 0.314 | | 11324773.6 | -1.058045E+03 | 1.031154E-04 | 0.04, 0.31 |
| A+B*sqrt(i) | 0 | 0.1 | | 8413856.3 | -1.028420E+03 | 1.195787E-04 | 0.10, 0.26 |
| A+B*sqrt(i+C) | 0 | 0.1 | 0 | 8414999.3 | -1.028426E+03 | 1.195749E-04 | 0.10, 0.26 |
| A+B*i+C*(i-1) | 0 | 0.1 | 0 | 3963981.3 | -9.531445E+02 | 1.742258E-04 | 0.10, 0.70 |
| A+B*i-C*(i-1) | 0 | 0.1 | 0.25 | 74728808 | -1.246806E+03 | 4.012684E-05 | -0.80, 0.10 |
| A+B*i+C*sqrt(i-1) | 0 | 0.1 | 0 | 3963981.3 | -9.531445E+02 | 1.742258E-04 | 0.10, 0.70 |
| A+B*sqrt(i)+C*(i-1) | 0 | 0.1 | 0 | 8413856.3 | -1.028420E+03 | 1.195787E-04 | 0.10, 0.26 |
| A+B*sqrt(i)+C*sqrt(i-1) | 0 | 0.1 | 0 | 8413856.3 | -1.028420E+03 | 1.195787E-04 | 0.10, 0.26 |

The above table shows that the top two Shammas polynomials use the following power equations:

0.314/i

And,

0.1*√(i)

 

# RESULTS FOR INVERSE HYPERBOLIC SINE

## Using Power A+B* i

```
Fitting asinh(x) in range (0.000000, 100.000000)
Shammas polynomial power is A+B*i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                        Estimate                SE                tStat                pValue
                    ____________________    ____________________    ____________________    ____________________

    (Intercept)    4.90382845251735e-09    0.000239093798564182    2.05100612477867e-05       0.999983679269895
    x1                  413.048111238561         9.09650827260524         45.4073254110574    2.86221501899489e-65
    x2                 -1881.52199128997         42.447943534514        -44.3253979962569    2.44426159697362e-64
    x3                  3510.20336997517         82.1255855235592         42.7418976388084    6.15743946613062e-63
    x4                 -3448.80337010273         84.3356747933231        -40.8937662330269    3.05790495652855e-61
    x5                  1892.63867334852         48.4887447991385         39.0325359253709    1.83789207209278e-59
    x6                 -551.379852108721         14.8014999381032        -37.251620066512     1.09214029184984e-57
    x7                  66.6962077325962         1.87438018520545         35.5830734122307    5.87536127158345e-56


Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.000239
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 2.55e+08, p-value = 0
                    SumSq             DF            MeanSq                 F            pValue
                ____________________    ___    ____________________    ________________    ______

    Total           101.858189538069    100        1.01858189538069
    Model           101.858184221646      7        14.5511691745208    254543063.234917       0
    Residual    5.31642353962525e-06     93    5.71658445120994e-08

A = 0.000000, B = 0.100000
List of powers: [0.100000, 0.200000, 0.300000, 0.400000, 0.500000, 0.600000, 0.700000]
Fitting asinh(x) in range (0.000000, 100.000000)
MSS of errors squared = 2.282908e-05
Corrected MSS of errors squared = 3.228520e-05
Particle swarm AICc =  -1.372552e+03
AIC =  -1.390117e+03
AICc =  -1.372552e+03
```

## Using Power A+B/i

```
Fitting asinh(x) in range (0.000000, 100.000000)
Shammas polynomial power is A+B/i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                        Estimate                SE                tStat                pValue
                    ____________________    ____________________    ____________________    ____________________

    (Intercept)    -6.55046394690208e-07    2.54325968924637e-05    -0.0257561741516186       0.979507004367092
    x1                  115976.443492283         1152.30205275006         100.647606428798    1.01778061296246e-96
    x2                 -27446205.9871308         259084.366486587        -105.935399959965     9.0626268010625e-99
    x3                  558650334.649859         5182214.7850796         107.801462852968     1.80976385439759e-99
    x4                 -3250138530.95758         29884968.1464161        -108.754960521761    8.03009153765496e-100
    x5                  7596095067.83093         69476223.5376672         109.333735788224    4.92029580327447e-100
    x6                 -7666689283.55698         69873515.5454796        -109.722392292783    3.5461872027021e-100
```

 

```
    x7                2789412642.45878        25357977.433592       110.001385156357    2.80523529594539e-100


Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 2.54e-05
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 2.25e+10, p-value = 0
                        SumSq             DF            MeanSq                  F             pValue
                ______________________    ___    ______________________    ________________    ______

    Total           101.858145956726      100        1.01858145956726
    Model           101.858145896572        7        14.5511636995103    22496570195.5152       0
    Residual    6.01539795752615e-08       93    6.46816984680231e-10

A = 0.000000, B = 0.125345
List of powers: [0.125345, 0.062672, 0.041782, 0.031336, 0.025069, 0.020891, 0.017906]
Fitting asinh(x) in range (0.000000, 100.000000)
MSS of errors squared = 2.419673e-06
Corrected MSS of errors squared = 3.421935e-06
Particle swarm AICc =  -1.825919e+03
AIC =  -1.843484e+03
AICc =  -1.825919e+03
```

## Using Power A+B*sqrt(i)

```
Fitting asinh(x) in range (0.000000, 100.000000)
Shammas polynomial power is A+B*sqrt(i)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                        Estimate                  SE                  tStat                  pValue
                ____________________    ____________________    ____________________    ____________________

    (Intercept)    1.67585147007928e-10    8.32807415129017e-05    2.01229172511589e-06       0.999998398733688
    x1                  164702.15399303         2486.56147344996         66.2369121984806    4.67237508787273e-80
    x2                 -1596371.23105439         24787.5029114124        -64.4022609602763    6.02680587948598e-79
    x3                  5654932.15788646         89716.9792562909         63.0307908799765    4.26791733880435e-78
    x4                   -9934538.634835         160483.450440623        -61.9038200360147    2.19843385452457e-77
    x5                  9368004.24036093         153738.023789554         60.9348553431676    9.20761005147444e-77
    x6                 -4558509.89972484         75875.6208581707        -60.0787163013238    3.32380078167079e-76
    x7                  901782.094696079         15205.0097227901         59.3082221673584    1.07114178376883e-75


Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 8.33e-05
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 2.1e+09, p-value = 0
                        SumSq             DF            MeanSq                  F             pValue
                ______________________    ___    ______________________    ________________    ______

    Total           101.858189344594      100        1.01858189344594
    Model           101.858188699576        7        14.5511698142251    2098015740.84368       0
    Residual    6.45018417344545e-07       93    6.93568190693059e-09

A = 0.000000, B = 0.100000
List of powers: [0.100000, 0.141421, 0.173205, 0.200000, 0.223607, 0.244949, 0.264575]
Fitting asinh(x) in range (0.000000, 100.000000)
MSS of errors squared = 7.951792e-06
Corrected MSS of errors squared = 1.124553e-05
Particle swarm AICc =  -1.585589e+03
```

 

```
AIC =  -1.603154e+03
AICc =  -1.585589e+03
```

## Using Power A+B*sqrt(i+C)

```
Fitting asinh(x) in range (0.000000, 100.000000)
Shammas polynomial power is A+B*sqrt(i+C)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                        Estimate                SE                tStat               pValue
                    ____________________    ____________________    ____________________    ____________________

    (Intercept)    1.67585147007928e-10    8.32807415129017e-05    2.01229172511589e-06       0.999998398733688
    x1                   164702.15399303         2486.56147344996         66.2369121984806    4.67237508787273e-80
    x2                  -1596371.23105439         24787.5029114124        -64.4022609602763    6.02680587948598e-79
    x3                   5654932.15788646         89716.9792562909         63.0307908799765    4.26791733880435e-78
    x4                    -9934538.634835         160483.450440623        -61.9038200360147    2.19843385452457e-77
    x5                   9368004.24036093         153738.023789554         60.9348553431676    9.20761005147444e-77
    x6                  -4558509.89972484         75875.6208581707        -60.0787163013238    3.32380078167079e-76
    x7                   901782.094696079         15205.0097227901         59.3082221673584    1.07114178376883e-75


Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 8.33e-05
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 2.1e+09, p-value = 0
                      SumSq            DF            MeanSq                F            pValue
                  ____________________    ___    ____________________    ________________    ______

    Total            101.858189344594    100        1.01858189344594
    Model            101.858188699576      7        14.5511698142251    2098015740.84368       0
    Residual    6.45018417344545e-07     93    6.93568190693059e-09

A = 0.000000, B = 0.100000, C = 0.000000
List of powers: [0.100000, 0.141421, 0.173205, 0.200000, 0.223607, 0.244949, 0.264575]
Fitting asinh(x) in range (0.000000, 100.000000)
MSS of errors squared = 7.951792e-06
Corrected MSS of errors squared = 1.377291e-05
Particle swarm AICc =  -1.585589e+03
AIC =  -1.603154e+03
AICc =  -1.585589e+03
```

## Using Power A+B*i+C*(i–1)

```
Fitting asinh(x) in range (0.000000, 100.000000)
Shammas polynomial power is A+B*i+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                        Estimate                SE                tStat               pValue
                    ____________________    ____________________    ____________________    ____________________

    (Intercept)    4.90382845251735e-09    0.000239093798564182    2.05100612477867e-05       0.999983679269895
    x1                   413.048111238561         9.09650827260524         45.4073254110574    2.86221501899489e-65
    x2                  -1881.52199128997          42.447943534514        -44.3253979962569    2.44426159697362e-64
    x3                   3510.20336997517         82.1255855235592         42.7418976388084    6.15743946613062e-63
    x4                  -3448.80337010273         84.3356747933231        -40.8937662330269    3.05790495652855e-61
    x5                   1892.63867334852         48.4887447991385         39.0325359253709    1.83789207209278e-59
```

 

| | | | | |
|---|---|---|---|---|
| x6 | -551.379852108721 | 14.8014999381032 | -37.251620066512 | 1.09214029184984e-57 |
| x7 | 66.6962077325962 | 1.87438018520545 | 35.5830734122307 | 5.87536127158345e-56 |

Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.000239
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 2.55e+08, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 101.858189538069 | 100 | 1.01858189538069 | | |
| Model | 101.858184221646 | 7 | 14.5511691745208 | 254543063.234917 | 0 |
| Residual | 5.31642353962525e-06 | 93 | 5.71658445120994e-08 | | |

A = 0.000000, B = 0.100000, C = 0.000000
List of powers: [0.100000, 0.200000, 0.300000, 0.400000, 0.500000, 0.600000, 0.700000]
Fitting asinh(x) in range (0.000000, 100.000000)
MSS of errors squared = 2.282908e-05
Corrected MSS of errors squared = 3.954113e-05
Particle swarm AICc =  -1.372552e+03
AIC =  -1.390117e+03
AICc =  -1.372552e+03

## Using Power A+B*i–C*(i–1)

Fitting asinh(x) in range (0.000000, 100.000000)
Shammas polynomial power is A+B*i-C*(i-1)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | -1.31711418156341e-07 | 2.65248929274198e-05 | -0.00496557775055844 | 0.996048694507166 |
| x1 | 4623026.69229361 | 45961.5977272352 | 100.584551471199 | 1.07829542865206e-96 |
| x2 | -29322879.7869305 | 287587.839571702 | -101.961473164513 | 3.07944796526684e-97 |
| x3 | 77494780.6185395 | 749682.617243677 | 103.370118015355 | 8.69141483672167e-98 |
| x4 | -109228074.387761 | 1042138.98678351 | -104.811427048598 | 2.42400470723231e-98 |
| x5 | 86599658.1669302 | 814776.911776381 | 106.286342820055 | 6.67968581569966e-99 |
| x6 | -36617934.8501188 | 339697.731195206 | -107.79564149952 | 1.81880395072449e-99 |
| x7 | 6451424.4284114 | 59003.4308128225 | 109.339818711176 | 4.89509723953757e-100 |

Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 2.65e-05
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 2.07e+10, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 101.858190736326 | 100 | 1.01858190736326 | | |
| Model | 101.858190670894 | 7 | 14.5511700958419 | 20681909793.9778 | 0 |
| Residual | 6.54320047033252e-08 | 93 | 7.03569943046508e-10 | | |

A = 0.000003, B = 0.100000, C = 0.116666
List of powers: [0.100003, 0.083337, 0.066671, 0.050005, 0.033338, 0.016672, 0.000006]
Fitting asinh(x) in range (0.000000, 100.000000)
MSS of errors squared = 2.532558e-06
Corrected MSS of errors squared = 4.386518e-06
Particle swarm AICc =  -1.816711e+03
AIC =  -1.834276e+03

 

```
AICc =  -1.816711e+03
```

## Using Power A+B*i+C*sqrt(i–1)

```
Fitting asinh(x) in range (0.000000, 100.000000)
Shammas polynomial power is A+B*i+C*sqrt(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                        Estimate                SE                tStat               pValue
                    ____________________    ____________________    ____________________    ____________________

    (Intercept)    4.90382845251735e-09    0.000239093798564182    2.05100612477867e-05       0.999983679269895
    x1                  413.048111238561        9.09650827260524        45.4073254110574    2.86221501899489e-65
    x2                 -1881.52199128997        42.447943534514       -44.3253979962569    2.44426159697362e-64
    x3                  3510.20336997517        82.1255855235592        42.7418976388084    6.15743946613062e-63
    x4                 -3448.80337010273        84.3356747933231       -40.8937662330269    3.05790495652855e-61
    x5                  1892.63867334852        48.4887447991385        39.0325359253709    1.83789207209278e-59
    x6                 -551.379852108721        14.8014999381032       -37.251620066512    1.09214029184984e-57
    x7                  66.6962077325962        1.87438018520545        35.5830734122307    5.87536127158345e-56


Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.000239
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 2.55e+08, p-value = 0
                        SumSq            DF            MeanSq                F            pValue
                    ____________________    ___    ____________________    ________________    ______

    Total            101.858189538069    100        1.01858189538069
    Model            101.858184221646      7        14.5511691745208    254543063.234917       0
    Residual    5.31642353962525e-06     93    5.71658445120994e-08

A = 0.000000, B = 0.100000, C = 0.000000
List of powers: [0.100000, 0.200000, 0.300000, 0.400000, 0.500000, 0.600000, 0.700000]
Fitting asinh(x) in range (0.000000, 100.000000)
MSS of errors squared = 2.282908e-05
Corrected MSS of errors squared = 3.954113e-05
Particle swarm AICc =  -1.372552e+03
AIC =  -1.390117e+03
AICc =  -1.372552e+03
```

## Using Power A+B*sqrt(i)+C*(i–1)

```
Fitting asinh(x) in range (0.000000, 100.000000)
Shammas polynomial power is A+B*sqrt(i)+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                        Estimate                SE                tStat               pValue
                    ____________________    ____________________    ____________________    ____________________

    (Intercept)    1.67585147007928e-10    8.32807415129017e-05    2.01229172511589e-06       0.999998398733688
    x1                  164702.15399303        2486.56147344996        66.2369121984806    4.67237508787273e-80
    x2                 -1596371.23105439        24787.5029114124       -64.4022609602763    6.02680587948598e-79
    x3                  5654932.15788646        89716.9792562909        63.0307908799765    4.26791733880435e-78
    x4                  -9934538.634835        160483.450440623       -61.9038200360147    2.19843385452457e-77
    x5                  9368004.24036093        153738.023789554        60.9348553431676    9.20761005147444e-77
    x6                 -4558509.89972484        75875.6208581707       -60.0787163013238    3.32380078167079e-76
```

 

```
    x7                901782.094696079        15205.0097227901        59.3082221673584    1.07114178376883e-75


Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 8.33e-05
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 2.1e+09, p-value = 0
                      SumSq             DF            MeanSq                 F            pValue
                ____________________    ___    ____________________    ________________    ______

    Total           101.858189344594    100        1.01858189344594
    Model           101.858188699576      7        14.5511698142251    2098015740.84368       0
    Residual    6.45018417344545e-07     93    6.93568190693059e-09

A = 0.000000, B = 0.100000, C = 0.000000
List of powers: [0.100000, 0.141421, 0.173205, 0.200000, 0.223607, 0.244949, 0.264575]
Fitting asinh(x) in range (0.000000, 100.000000)
MSS of errors squared = 7.951792e-06
Corrected MSS of errors squared = 1.377291e-05
Particle swarm AICc =  -1.585589e+03
AIC =  -1.603154e+03
AICc =  -1.585589e+03
```

## Using Power A+B*sqrt(i)+C*sqrt(i–1)

```
Fitting asinh(x) in range (0.000000, 100.000000)
Shammas polynomial power is A+B*sqrt(i)+C*sqrt(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                        Estimate                  SE                 tStat                 pValue
                ____________________    ____________________    ____________________    ____________________

    (Intercept)    1.67585147007928e-10    8.32807415129017e-05    2.01229172511589e-06      0.999998398733688
    x1                   164702.15399303         2486.56147344996         66.2369121984806    4.67237508787273e-80
    x2                  -1596371.23105439         24787.5029114124        -64.4022609602763    6.02680587948598e-79
    x3                   5654932.15788646         89716.9792562909         63.0307908799765    4.26791733880435e-78
    x4                    -9934538.634835         160483.450440623        -61.9038200360147    2.19843385452457e-77
    x5                   9368004.24036093         153738.023789554         60.9348553431676    9.20761005147444e-77
    x6                  -4558509.89972484         75875.6208581707        -60.0787163013238    3.32380078167079e-76
    x7                   901782.094696079         15205.0097227901         59.3082221673584    1.07114178376883e-75


Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 8.33e-05
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 2.1e+09, p-value = 0
                      SumSq             DF            MeanSq                 F            pValue
                ____________________    ___    ____________________    ________________    ______

    Total           101.858189344594    100        1.01858189344594
    Model           101.858188699576      7        14.5511698142251    2098015740.84368       0
    Residual    6.45018417344545e-07     93    6.93568190693059e-09

A = 0.000000, B = 0.100000, C = 0.000000
List of powers: [0.100000, 0.141421, 0.173205, 0.200000, 0.223607, 0.244949, 0.264575]
Fitting asinh(x) in range (0.000000, 100.000000)
MSS of errors squared = 7.951792e-06
Corrected MSS of errors squared = 1.377291e-05
Particle swarm AICc =  -1.585589e+03
```

 

```
AIC =  -1.603154e+03
AICc =  -1.585589e+03
```

## Comments

The following table shows the summary results for the seven models that fit the inverse hyperbolic sine function.

| Power | A | B | C | F | AICc | MSSE | Pwr Rng |
|---|---|---|---|---|---|---|---|
| A+B*i | 0 | 0.1 | | 254543063.2 | -1.372552E+03 | 2.282908E-05 | 0.10, 0.70 |
| A+B/i | 0 | 0.125 | | 22496570196 | -1.825919E+03 | 2.419673E-06 | 0.01, 0.12 |
| A+B*sqrt(i) | 0 | 0.1 | | 2098015741 | -1.585589E+03 | 7.951792E-06 | 0.10, 0.26 |
| A+B*sqrt(i+C) | 0 | 0.1 | 0 | 2098015741 | -1.585589E+03 | 7.951792E-06 | 0.10, 0.26 |
| A+B*i+C*(i-1) | 0 | 0.1 | 0 | 254543063.2 | -1.372552E+03 | 2.282908E-05 | 0.10, 0.70 |
| A+B*i-C*(i-1) | 0 | 0.1 | 0.116 | 20681909794 | -1.816711E+03 | 2.532558E-06 | 0.00, 0.10 |
| A+B*i+C*sqrt(i-1) | 0 | 0.1 | 0 | 254543063.2 | -1.372552E+03 | 2.282908E-05 | 0.10, 0.70 |
| A+B*sqrt(i)+C*(i-1) | 0 | 0.1 | 0 | 2098015741 | -1.585589E+03 | 7.951792E-06 | 0.10, 0.26 |
| A+B*sqrt(i)+C*sqrt(i-1) | 0 | 0.1 | 0 | 2098015741 | -1.585589E+03 | 7.951792E-06 | 0.10, 0.26 |

The above table shows that the top two Shammas polynomials use the following power equations:

0.125/i

And,

0.1*i+0.116*(i–1)

 

# RESULTS FOR INVERSE HYPERBOLIC TANGENT

## Using Power A+B* i

```
Fitting atanh(x) in range (0.000000, 0.990000)
Shammas polynomial power is A+B*i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat               pValue
                ________________    ________________    ________________    ________________

    (Intercept)    -0.0336622118162722    0.031529650392396    -1.06763669743673       0.288477952544728
    x1                28.3075841844249     7.76011304727422     3.64783141843121    0.000438274442793201
    x2               -333.959852113552     77.6297994679933    -4.30195433199905    4.21383810470257e-05
    x3                1633.49433869027     327.404358454753     4.98922600297642     2.8518039670431e-06
    x4               -4065.02863969668     722.345771247568    -5.62753850233794    1.96044732430748e-07
    x5                5447.98903294113     872.845324745008     6.24164313938749    1.31211735115063e-08
    x6               -3747.14975968832     547.334065531085    -6.84618406868649    8.32324970693735e-10
    x7                1038.96779535695     139.350807891836     7.45577159598092    4.80150239148689e-11


Number of observations: 100, Error degrees of freedom: 92
Root Mean Squared Error: 0.0337
R-squared: 0.997,  Adjusted R-Squared: 0.996
F-statistic vs. constant model: 3.78e+03, p-value = 3.29e-110
                    SumSq          DF          MeanSq                F                pValue
                ________________    __    ________________    ________________    ________________

    Total        30.2001278931121    99      0.305051796900122
    Model        30.0953864235941     7       4.29934091765629    3776.3396508046    3.28939337887778e-110
    Residual    0.104741469518004    92    0.00113849423389135

A = 0.500000, B = 0.500000
List of powers: [1.000000, 1.500000, 2.000000, 2.500000, 3.000000, 3.500000, 4.000000]
Fitting atanh(x) in range (0.000000, 0.990000)
MSS of errors squared = 3.236379e-03
Corrected MSS of errors squared = 4.576931e-03
Particle swarm AICc =  -3.687729e+02
AIC =  -3.863553e+02
AICc =  -3.687729e+02
```

## Using Power A+B/i

```
Fitting atanh(x) in range (0.000000, 0.990000)
Shammas polynomial power is A+B/i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat               pValue
                ________________    ________________    ________________    ________________

    (Intercept)    -0.0377532377839088    0.0238087278593531    -1.58568899636012       0.116241270101437
    x1                1620.83818999227     220.508387487348     7.35046049023991    7.89276660182047e-11
    x2               -118660.781724739      18960.746689413    -6.25823358480891    1.21788080116881e-08
    x3                1597890.00982472     270779.941194806     5.90106491187674    5.96031624638922e-08
    x4               -7531488.21685371     1315787.31098492    -5.72394045297191    1.29207598217839e-07
    x5                15495899.5796287     2758062.57649487     5.61840029000428    2.03915500941053e-07
```

 

| | | | | |
|---|---|---|---|---|
| x6 | -14359773.2228845 | 2588064.34463383 | -5.54846066816634 | 2.75349720625205e-07 |
| x7 | 4914514.45021724 | 893749.419008157 | 5.49876100134549 | 3.40511614660253e-07 |

Number of observations: 100, Error degrees of freedom: 92
Root Mean Squared Error: 0.0308
R-squared: 0.997,  Adjusted R-Squared: 0.997
F-statistic vs. constant model: 4.53e+03, p-value = 7.59e-114

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 30.2001279209864 | 99 | 0.305051797181681 | | |
| Model | 30.112822160464 | 7 | 4.30183173720915 | 4533.13180545183 | 7.59007586317338e-114 |
| Residual | 0.0873057605223977 | 92 | 0.000948975657852149 | | |

A = 2.000000, B = 2.000000
List of powers: [4.000000, 3.000000, 2.666667, 2.500000, 2.400000, 2.333333, 2.285714]
Fitting atanh(x) in range (0.000000, 0.990000)
MSS of errors squared = 2.954755e-03
Corrected MSS of errors squared = 4.178654e-03
Particle swarm AICc =  -3.869808e+02
AIC =  -4.045632e+02
AICc =  -3.869808e+02

## Using Power A+B*sqrt(i)

Fitting atanh(x) in range (0.000000, 0.990000)
Shammas polynomial power is A+B*sqrt(i)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | -0.0207325220104394 | 0.0151538219911944 | -1.36813815171425 | 0.174601074983338 |
| x1 | 5984.47574116439 | 984.379191441086 | 6.07944153350438 | 2.70870276447867e-08 |
| x2 | -94919.3898967178 | 14950.2330256617 | -6.34902410777083 | 8.08973679992687e-09 |
| x3 | 490591.452567465 | 74450.3223735865 | 6.58951414751048 | 2.71104219527473e-09 |
| x4 | -1181847.7307006 | 173667.192195965 | -6.80524465073981 | 1.00573612250271e-09 |
| x5 | 1467599.05465709 | 209588.883848432 | 7.00227525291088 | 4.03320863699292e-10 |
| x6 | -913508.776555278 | 127143.00403766 | -7.18489218867831 | 1.71852200659244e-10 |
| x7 | 226103.616510225 | 30736.8765754283 | 7.35610256153931 | 7.6856792528043e-11 |

Number of observations: 100, Error degrees of freedom: 92
Root Mean Squared Error: 0.0254
R-squared: 0.998,  Adjusted R-Squared: 0.998
F-statistic vs. constant model: 6.68e+03, p-value = 1.42e-121

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 30.200127899463 | 99 | 0.305051796964272 | | |
| Model | 30.1408290043943 | 7 | 4.30583271491347 | 6680.33711780045 | 1.42365316265896e-121 |
| Residual | 0.0592988950687072 | 92 | 0.000644553207268557 | | |

A = 1.400000, B = 1.000000
List of powers: [2.400000, 2.814214, 3.132051, 3.400000, 3.636068, 3.849490, 4.045751]
Fitting atanh(x) in range (0.000000, 0.990000)
MSS of errors squared = 2.435136e-03
Corrected MSS of errors squared = 3.443803e-03

 

```
Particle swarm AICc =  -4.256634e+02
AIC =  -4.432458e+02
AICc =  -4.256634e+02
```

## Using Power A+B*sqrt(i+C)

```
Fitting atanh(x) in range (0.000000, 0.990000)
Shammas polynomial power is A+B*sqrt(i+C)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                       Estimate                SE                tStat               pValue
                   ____________________    ____________________    ________________    ____________________

    (Intercept)    -0.0174013962765457     0.0140022930740054    -1.24275332508577       0.217116877200122
    x1                64288.6366653489        9904.9791540823     6.49053730101518    4.25847470101136e-09
    x2               -650992.093946225       97721.266820171    -6.66172385120832    1.94742777585095e-09
    x3                2561014.97296171       375050.949427083     6.82844551353315    9.03488965762674e-10
    x4               -5104645.38693958       730461.561623753    -6.98824641175149    4.30535584178392e-10
    x5                5501617.35667732       770419.289333325     7.14106906829669    2.11001603481412e-10
    x6               -3064326.44775299       420492.414915701    -7.28747140032791    1.06170545256005e-10
    x7                693045.669982408       93300.0568068717     7.42813770646454    5.47119776688298e-11


Number of observations: 100, Error degrees of freedom: 92
Root Mean Squared Error: 0.0247
R-squared: 0.998,  Adjusted R-Squared: 0.998
F-statistic vs. constant model: 7.07e+03, p-value = 1.03e-122
                       SumSq           DF           MeanSq                F                  pValue
                   ____________________    __    ____________________    ________________    ____________________

    Total           30.200127873321     99       0.305051796700212
    Model          30.1441227776778      7        4.30630325395397    7073.99737137961    1.02764630703967e-122
    Residual     0.0560050956431866     92    0.000608751039599854

A = 1.000000, B = 1.000000, C = 2.000000
List of powers: [2.732051, 3.000000, 3.236068, 3.449490, 3.645751, 3.828427, 4.000000]
Fitting atanh(x) in range (0.000000, 0.990000)
MSS of errors squared = 2.366540e-03
Corrected MSS of errors squared = 4.098967e-03
Particle swarm AICc =  -4.313782e+02
AIC =  -4.489606e+02
AICc =  -4.313782e+02
```

## Using Power A+B*i+C*(i–1)

```
Fitting atanh(x) in range (0.000000, 0.990000)
Shammas polynomial power is A+B*i+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                       Estimate                SE                tStat               pValue
                   ____________________    ____________________    ________________    ____________________

    (Intercept)    -0.0263007801770186     0.0340418565419158    -0.77260122827421       0.441738757159598
    x1                9.54929004995351        2.91426199504847     3.27674384327093     0.00148203686319858
    x2               -129.145817210135        31.5834253023997    -4.08903771435844    9.26480688120563e-05
    x3                  715.99353425131        145.958364322161     4.90546422314629    4.00479643774293e-06
    x4               -1960.71498969623        348.809321032541    -5.62116569560741    2.01501464279288e-07
```

 

| | | | | |
|---|---|---|---|---|
| x5 | 2841.89405858322 | 451.005663501425 | 6.30123807430689 | 1.00359999519194e-08 |
| x6 | -2087.62048160688 | 299.549462464419 | -6.96920122784349 | 4.70417009535982e-10 |
| x7 | 612.642976962956 | 80.1172090883433 | 7.64683373190659 | 1.94144652705777e-11 |

Number of observations: 100, Error degrees of freedom: 92
Root Mean Squared Error: 0.0351
R-squared: 0.996, Adjusted R-Squared: 0.996
F-statistic vs. constant model: 3.49e+03, p-value = 1.28e-108

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 30.2001278930496 | 99 | 0.305051796899491 | | |
| Model | 30.0867139931753 | 7 | 4.29810199902505 | 3486.56896860751 | 1.2764393328098e-108 |
| Residual | 0.113413899874246 | 92 | 0.0012327597812418 | | |

A = 0.500000, B = 0.270000, C = 0.270000
List of powers: [0.770000, 1.310000, 1.850000, 2.390000, 2.930000, 3.470000, 4.010000]
Fitting atanh(x) in range (0.000000, 0.990000)
MSS of errors squared = 3.367698e-03
Corrected MSS of errors squared = 5.833024e-03
Particle swarm AICc = -3.608180e+02
AIC = -3.784004e+02
AICc = -3.608180e+02

## Using Power A+B*i–C*(i–1)

Fitting atanh(x) in range (0.000000, 0.990000)
Shammas polynomial power is A+B*i-C*(i-1)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | -0.0162034940614831 | 0.0445388821439196 | -0.363805584727616 | 0.716837909989598 |
| x1 | 790.952465378347 | 176.974534777804 | 4.46930100068581 | 2.23219748333658e-05 |
| x2 | -7416.52682671442 | 1516.78396141529 | -4.88963953692797 | 4.2686938861922e-06 |
| x3 | 28348.9206435164 | 5364.58388902186 | 5.28445844635405 | 8.42430904976081e-07 |
| x4 | -56573.7493686274 | 9991.38989067324 | -5.6622501961852 | 1.68775652934436e-07 |
| x5 | 62287.2548398756 | 10328.1550521458 | 6.03082104455188 | 3.36133681399292e-08 |
| x6 | -35938.79551925 | 5619.60090956076 | -6.39525761662299 | 6.5632180657066e-09 |
| x7 | 8504.437547149 | 1258.12194640263 | 6.75962896241168 | 1.24134760404192e-09 |

Number of observations: 100, Error degrees of freedom: 92
Root Mean Squared Error: 0.0447
R-squared: 0.994, Adjusted R-Squared: 0.993
F-statistic vs. constant model: 2.14e+03, p-value = 6.18e-99

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 30.200127892943 | 99 | 0.305051796898414 | | |
| Model | 30.0159379951637 | 7 | 4.28799114216625 | 2141.78513499186 | 6.18124265719799e-99 |
| Residual | 0.184189897779262 | 92 | 0.00200206410629633 | | |

A = 0.750000, B = 0.250000, C = 0.000000
List of powers: [1.000000, 1.250000, 1.500000, 1.750000, 2.000000, 2.250000, 2.500000]
Fitting atanh(x) in range (0.000000, 0.990000)
MSS of errors squared = 4.291735e-03

 

```
Corrected MSS of errors squared = 7.433503e-03
Particle swarm AICc =  -3.123257e+02
AIC =  -3.299081e+02
AICc =  -3.123257e+02
```

## Using Power A+B*i+C*sqrt(i–1)

```
Fitting atanh(x) in range (0.000000, 0.990000)
Shammas polynomial power is A+B*i+C*sqrt(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                       Estimate                SE                tStat                pValue
                   ____________________    ____________________    ____________________    ____________________

    (Intercept)    -0.0284195614968326     0.0329346092216053    -0.862908720295037       0.390431562779037
    x1                5.23078671201142      1.63845946288232      3.19250297642982      0.00193088882572467
    x2               -167.327119560884       39.448436806488     -4.24166666937139     5.27953465263007e-05
    x3                1162.75314705845      231.148965011707      5.03031950413258     2.41163782571879e-06
    x4               -3462.7206622611       608.778529318443     -5.68798092491497     1.51002031861701e-07
    x5                5165.27604348299      820.644894416447      6.29416703695691     1.03608092315505e-08
    x6               -3803.46558722232      553.136631919443     -6.87617736331057     7.2441558781226e-10
    x7                1102.86267029992      147.983489286838      7.45260620367065     4.87388068988738e-11


Number of observations: 100, Error degrees of freedom: 92
Root Mean Squared Error: 0.0343
R-squared: 0.996,  Adjusted R-Squared: 0.996
F-statistic vs. constant model: 3.65e+03, p-value = 1.52e-109
                       SumSq          DF          MeanSq                F                pValue
                   ____________________    __    ____________________    ____________________    ____________________

    Total          30.2001278930059     99       0.30505179689905
    Model           30.091848565233      7         4.298835509319     3652.52421668713     1.5157506157489e-109
    Residual      0.108279327772962     92      0.0011769492149235

A = 0.340000, B = 0.390000, C = 0.360000
List of powers: [0.730000, 1.480000, 2.019117, 2.523538, 3.010000, 3.484984, 3.951816]
Fitting atanh(x) in range (0.000000, 0.990000)
MSS of errors squared = 3.290582e-03
Corrected MSS of errors squared = 5.699456e-03
Particle swarm AICc =  -3.654510e+02
AIC =  -3.830334e+02
AICc =  -3.654510e+02
```

## Using Power A+B*sqrt(i)+C*(i–1)

```
Fitting atanh(x) in range (0.000000, 0.990000)
Shammas polynomial power is A+B*sqrt(i)+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                       Estimate                SE                tStat                pValue
                   ____________________    ____________________    ____________________    ____________________

    (Intercept)    -0.0333233317893246     0.0313079834052963    -1.06437170858112       0.289945962320064
    x1                18.4776624778042      5.18048863881652      3.56677984762946     0.000576012048180923
    x2               -272.689050377577       63.4711371690427     -4.29626854882596     4.30473935483316e-05
    x3                1527.61848847459      304.213983680899      5.02152619676077     2.49986757006781e-06
```

| | | | | |
|---|---|---|---|---|
| x4 | -4128.39081994774 | 728.694107290455 | -5.66546480703484 | 1.66447510671371e-07 |
| x5 | 5830.73744127546 | 930.00286644911 | 6.26959082775529 | 1.15725512156725e-08 |
| x6 | -4148.64384922215 | 605.30114039079 | -6.85385103775575 | 8.03312857865303e-10 |
| x7 | 1175.51278769018 | 158.119737331341 | 7.43432039244335 | 5.31372870770912e-11 |

Number of observations: 100, Error degrees of freedom: 92
Root Mean Squared Error: 0.0335
R-squared: 0.997,  Adjusted R-Squared: 0.996
F-statistic vs. constant model: 3.83e+03, p-value = 1.75e-110

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 30.2001278930909 | 99 | 0.305051796899908 | | |
| Model | 30.0968160228352 | 7 | 4.29954514611931 | 3828.77739473566 | 1.74830857833415e-110 |
| Residual | 0.103311870255723 | 92 | 0.00112295511147525 | | |

A = 0.550000, B = 0.400000, C = 0.400000
List of powers: [0.950000, 1.515685, 2.042820, 2.550000, 3.044427, 3.529796, 4.008301]
Fitting atanh(x) in range (0.000000, 0.990000)
MSS of errors squared = 3.214216e-03
Corrected MSS of errors squared = 5.567186e-03
Particle swarm AICc =  -3.701472e+02
AIC =  -3.877296e+02
AICc =  -3.701472e+02

## Using Power A+B*sqrt(i)+C*sqrt(i–1)

Fitting atanh(x) in range (0.000000, 0.990000)
Shammas polynomial power is A+B*sqrt(i)+C*sqrt(i-1)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | -0.035134595765426 | 0.0229688788970727 | -1.52966089128119 | 0.129531921394942 |
| x1 | 18.6234184737898 | 4.33824763718105 | 4.2928435698731 | 4.36040531260449e-05 |
| x2 | -1400.67365314514 | 274.328070535587 | -5.10583423129292 | 1.76904998471633e-06 |
| x3 | 12177.7530806024 | 2151.57962818987 | 5.65991280129734 | 1.7048854120137e-07 |
| x4 | -40268.4904859972 | 6603.05602253415 | -6.09846264344472 | 2.48889565642834e-08 |
| x5 | 62803.3266316993 | 9701.04687859882 | 6.47387105924082 | 4.59390323100414e-09 |
| x6 | -46698.7393736478 | 6858.69774427803 | -6.80868892532944 | 9.89862197681282e-10 |
| x7 | 13370.8746568244 | 1879.1586030386 | 7.11535185758335 | 2.37972899578289e-10 |

Number of observations: 100, Error degrees of freedom: 92
Root Mean Squared Error: 0.029
R-squared: 0.997,  Adjusted R-Squared: 0.997
F-statistic vs. constant model: 5.1e+03, p-value = 3.28e-116

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 30.2001278932149 | 99 | 0.30505179690116 | | |
| Model | 30.122570314662 | 7 | 4.303224330666 | 5104.5512999283 | 3.27590854111441e-116 |
| Residual | 0.0775575785528559 | 92 | 0.000843017158183217 | | |

A = 0.500000, B = 0.700000, C = 0.700000
List of powers: [1.200000, 2.189949, 2.702385, 3.112436, 3.465248, 3.779890, 4.066669]
Fitting atanh(x) in range (0.000000, 0.990000)

 

```
MSS of errors squared = 2.784916e-03
Corrected MSS of errors squared = 4.823616e-03
Particle swarm AICc =  -3.988204e+02
AIC =  -4.164028e+02
AICc =  -3.988204e+02
```

## Comments

The following table shows the summary results for the seven models that fit the inverse hyperbolic tangent function.

| *Power* | *A* | *B* | *C* | *F* | *AICc* | *MSSE* | *Pwr Rng* |
|---|---|---|---|---|---|---|---|
| A+B*i | 0.5 | 0.5 | | 3776.3 | -3.687729E+02 | 3.236379E-03 | 1.00, 4.00 |
| A+B/i | 2 | 2 | | 4533.1 | -3.869808E+02 | 2.954755E-03 | 2.28, 4.00 |
| A+B*sqrt(i) | 1.4 | 1 | | 6680.3 | -4.256634E+02 | 2.435136E-03 | 2.40, 4.04 |
| A+B*sqrt(i+C) | 1 | 1 | 2 | 7073.9 | -4.313782E+02 | 2.366540E-03 | 2.73, 4.00 |
| A+B*i+C*(i-1) | 0.5 | 0.27 | 0.27 | 3486.5 | -3.608180E+02 | 3.367698E-03 | 0.77, 4.01 |
| A+B*i-C*(i-1) | 0.75 | 0.25 | 0 | 2141.7 | -3.123257E+02 | 4.291735E-03 | 1.00, 2.50 |
| A+B*i+C*sqrt(i-1) | 0.34 | 0.39 | 0.36 | 3652.5 | -3.654510E+02 | 3.290582E-03 | 0.73, 3.95 |
| A+B*sqrt(i)+C*(i-1) | 0.55 | 0.4 | 0.4 | 3828.7 | -3.701472E+02 | 3.214216E-03 | 0.95, 4.00 |
| A+B*sqrt(i)+C*sqrt(i-1) | 0.5 | 0.7 | 0.7 | 5104.5 | -3.988204E+02 | 2.784916E-03 | 1.20, 4.06 |

The above table shows that the top two Shammas polynomials use the following power equations:

$1.4+\sqrt{(i)}$

And,

$1+\sqrt{(i+2)}$

 

# RESULTS FOR COSINE

## Using Power A+B* i

```
Fitting cos(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7
```

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 0.999999957968129 | 2.13617640913892e-07 | 4681261.12473653 | 0 |
| x1 | 0.000177849399403357 | 8.84835193366645e-06 | 20.0997203475452 | 1.30139182237817e-35 |
| x2 | -0.00388630426173497 | 0.00010730352442371 | -36.2178622054303 | 1.26596031720869e-56 |
| x3 | 0.0366458362743107 | 0.00053399401456448 | 68.6259307685288 | 1.84856262399216e-81 |
| x4 | -0.516267824135685 | 0.001340543939806 | -385.118166443986 | 9.51086721770916e-151 |
| x5 | 0.0724318433837343 | 0.00179538286881205 | 40.343396743927 | 1.00878532758335e-60 |
| x6 | -0.2010502256986 | 0.00122436306546222 | -164.208012614869 | 2.24137353831125e-116 |
| x7 | 0.152251805233707 | 0.000334231994931368 | 455.527320970487 | 1.58559655849016e-157 |

```
Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 2.14e-07
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 6.18e+12, p-value = 0
```

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 1.98338186886268 | 100 | 0.0198338186886268 | | |
| Model | 1.98338186885842 | 7 | 0.283340266979774 | 6180872616322.54 | 0 |
| Residual | 4.2632564145606e-12 | 93 | 4.58414668232323e-14 | | |

```
A = 0.000000, B = 0.483276
List of powers: [0.483276, 0.966553, 1.449829, 1.933105, 2.416382, 2.899658, 3.382934]
Fitting cos(x) in range (0.000000, 1.000000)
MSS of errors squared = 2.040413e-08
Corrected MSS of errors squared = 2.885580e-08
Particle swarm AICc =  -2.790602e+03
AIC =  -2.808167e+03
AICc =  -2.790602e+03
```

## Using Power A+B/i

```
Fitting cos(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B/i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7
```

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 0.999999856617248 | 1.01241081183569e-07 | 9877411.86607901 | 0 |
| x1 | 11.4075560597609 | 0.0817478260144246 | 139.545681101635 | 7.89148459553342e-110 |
| x2 | -974.449717496638 | 11.1805498630586 | -87.1557955048608 | 5.75807983906689e-91 |
| x3 | 14222.2025437807 | 189.066026770235 | 75.2234697408879 | 4.21098366097406e-85 |
| x4 | -70115.0195549412 | 1002.13525672784 | -69.9656249834777 | 3.1669747605192e-82 |
| x5 | 148386.306446468 | 2214.55389740393 | 67.0050553388731 | 1.63417556538902e-80 |

 

```
    x6             -140173.639679296        2153.08352604777    -65.1036701472518     2.24892946913425e-79
    x7              48642.7327082051         762.69766078631     63.7772150212923     1.46340599555934e-78


Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 1.11e-07
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 2.29e+13, p-value = 0
                      SumSq            DF            MeanSq                 F            pValue
                  _____________________    ___    _____________________    _________________    ______

    Total           1.98338186869708    100      0.0198338186869708
    Model           1.98338186869593      7       0.283340266956561    22892120799319.1       0
    Residual    1.15107923193136e-12     93    1.23771960422727e-14

A = 2.000000, B = 0.697074
List of powers: [2.697074, 2.348537, 2.232358, 2.174269, 2.139415, 2.116179, 2.099582]
Fitting cos(x) in range (0.000000, 1.000000)
MSS of errors squared = 1.052822e-08
Corrected MSS of errors squared = 1.488915e-08
Particle swarm AICc =  -2.924246e+03
AIC =  -2.941811e+03
AICc =  -2.924246e+03
```

## Using Power A+B*sqrt(i)

```
Fitting cos(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*sqrt(i)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                      Estimate                SE                tStat              pValue
                  _____________________    _____________________    _________________    ____________________

    (Intercept)     0.999999906585402    2.09615970031285e-08     47706284.3272941                    0
    x1             -0.823102366688516    0.000586819970376727     -1402.6488671817    6.07840949741933e-203
    x2               2.19184136390634     0.0091388948473014      239.83658861701    1.22059563464436e-131
    x3              -8.22952140315259     0.0463967766099673     -177.372697080527     1.7601025003763e-119
    x4               17.3610085963219      0.109947182172844      157.903169987833    8.43812489951431e-115
    x5              -20.2042068290124      0.134473288858994     -150.246989572763    8.43137591379066e-113
    x6               12.1182617662153     0.0825272441936438      146.839530201453    7.05650208059842e-112
    x7              -2.87397866184443     0.0201564707114756     -142.583426582125    1.07489682298466e-110


Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 3.03e-08
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 3.09e+14, p-value = 0
                      SumSq            DF            MeanSq                 F            pValue
                  _____________________    ___    _____________________    _________________    ______

    Total           1.98338186886271    100      0.0198338186886271
    Model           1.98338186886263      7       0.283340266980375    309043630816782       0
    Residual    8.5265128291212e-14      93    9.16829336464645e-16

A = 1.061374, B = 0.999994
List of powers: [2.061368, 2.475579, 2.793414, 3.061362, 3.297428, 3.510849, 3.707109]
Fitting cos(x) in range (0.000000, 1.000000)
MSS of errors squared = 2.664964e-09
Corrected MSS of errors squared = 3.768829e-09
```

 

```
Particle swarm AICc =  -3.200513e+03
AIC =  -3.218078e+03
AICc =  -3.200513e+03
```

## Using Power A+B*sqrt(i+C)

```
Fitting cos(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*sqrt(i+C)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                       Estimate                SE                tStat                pValue
                  ________________    ____________________    _______________    ____________________

    (Intercept)    0.999999899630052    2.11124434279495e-08     47365427.0782421                    0
    x1            -0.926018876875993    0.000735565152158212    -1258.92162530943    1.41233341692696e-198
    x2               2.7866940889228      0.010717082501729     260.023573437382    6.71952773433797e-135
    x3              -10.0785417809089     0.0525944441453944    -191.627498772442    1.35477491961994e-122
    x4               20.6758268931743      0.122010408178067     169.459533837467     1.211270043935e-117
    x5              -23.5464394157323      0.147027870850402    -160.149496007396    2.27921271468093e-115
    x6               13.8707948839626      0.089238871028333     155.434450527267    3.63345931987178e-114
    x7              -3.24201331491532     0.0216085166171437    -150.034052422793    9.61496992070465e-113


Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 3.03e-08
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 3.09e+14, p-value = 0
                       SumSq           DF            MeanSq                F           pValue
                  ________________    ____    ____________________    _____________    ______

    Total          1.98338186886268    100      0.0198338186886268
    Model          1.98338186886259      7       0.283340266980371    309043630816777       0
    Residual    8.5265128291212e-14     93    9.16829336464645e-16

A = 0.997683, B = 0.998646, C = 0.161237
List of powers: [2.073831, 2.465807, 2.773263, 3.034832, 3.266443, 3.476507, 3.670109]
Fitting cos(x) in range (0.000000, 1.000000)
MSS of errors squared = 2.865028e-09
Corrected MSS of errors squared = 4.962374e-09
Particle swarm AICc =  -3.187144e+03
AIC =  -3.204710e+03
AICc =  -3.187144e+03
```

## Using Power A+B*i+C*(i–1)

```
Fitting cos(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*i+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                       Estimate                SE                tStat                pValue
                  ________________    ____________________    _______________    ____________________

    (Intercept)    0.999999929322354    1.74124285004523e-07     5743023.89409022                    0
    x1            0.00071643629414154    3.63445319954239e-05     19.7123543709889    5.65642342289087e-35
    x2            -0.0116542331324046    0.000373692701902918    -31.1866757714532    4.86502876267707e-51
    x3             0.0753703954388857     0.00159448994512739     47.2692823615542    7.94840668831562e-67
    x4             -0.772824961019901     0.00352845469881943    -219.026465403815    5.57320443777743e-128
```

| | | | | |
|---|---|---|---|---|
| x5 | 0.53071365251669 | 0.00425722539349661 | 124.661863881442 | 2.68249327790346e-105 |
| x6 | -0.56535228065019 | 0.00265921489210483 | -212.601201327773 | 8.83699794652317e-127 |
| x7 | 0.283333889546826 | 0.000673534647487267 | 420.667133611983 | 2.59538305311201e-154 |

Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 1.76e-07
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 9.18e+12, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 1.98338186886268 | 100 | 0.0198338186886268 | | |
| Model | 1.98338186885981 | 7 | 0.283340266979973 | 9179513786624.05 | 0 |
| Residual | 2.8705926524708e-12 | 93 | 3.08665876609764e-14 | | |

A = 0.500000, B = 0.270000, C = 0.141965
List of powers: [0.770000, 1.181965, 1.593930, 2.005895, 2.417860, 2.829825, 3.241790]
Fitting cos(x) in range (0.000000, 1.000000)
MSS of errors squared = 1.674866e-08
Corrected MSS of errors squared = 2.900954e-08
Particle swarm AICc =  -2.830481e+03
AIC =  -2.848046e+03
AICc =  -2.830481e+03

## Using Power A+B*i–C*(i–1)

Fitting cos(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*i-C*(i-1)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 0.999997596101049 | 2.81603575353666e-06 | 355108.274049841 | 0 |
| x1 | 0.246255756990362 | 0.0110159866788225 | 22.3543985818151 | 3.54590320090255e-39 |
| x2 | -2.51706636906072 | 0.0941562824661382 | -26.7328563016062 | 1.95520168573396e-45 |
| x3 | 10.5738546364411 | 0.332144280030413 | 31.8351248905231 | 8.4400715234975e-52 |
| x4 | -23.4605530399011 | 0.617034308625039 | -38.0214725696196 | 1.83004532371529e-58 |
| x5 | 28.6287780059278 | 0.636229864711113 | 44.9975387730768 | 6.41335388309046e-65 |
| x6 | -19.2937556688986 | 0.345313337029708 | -55.8731841488031 | 2.36857775307577e-73 |
| x7 | 5.36278544664907 | 0.0771173668373543 | 69.540567404999 | 5.52332090030908e-82 |

Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 2.83e-06
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 3.54e+10, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 1.98338186886267 | 100 | 0.0198338186886267 | | |
| Model | 1.98338186811815 | 7 | 0.283340266874022 | 35392754179.2423 | 0 |
| Residual | 7.44520889384148e-10 | 93 | 8.00560096111987e-12 | | |

A = 0.750000, B = 0.250000, C = 0.000000
List of powers: [1.000000, 1.250000, 1.500000, 1.750000, 2.000000, 2.250000, 2.500000]
Fitting cos(x) in range (0.000000, 1.000000)
MSS of errors squared = 2.701483e-07

 

```
Corrected MSS of errors squared = 4.679106e-07
Particle swarm AICc =  -2.268790e+03
AIC =  -2.286355e+03
AICc =  -2.268790e+03
```

## Using Power A+B*i+C*sqrt(i–1)

```
Fitting cos(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*i+C*sqrt(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                       Estimate                SE                tStat              pValue
                   ____________________    ____________________    _____________    ____________________

    (Intercept)       0.999999940267261    1.65969509724723e-07     6025202.71299144                   0
    x1              0.000147683820132616    9.72282314557576e-06     15.1893969396963    6.30902527032529e-27
    x2              -0.00825602997068081    0.000286476959978259    -28.8191761435452    3.79950753428054e-48
    x3                0.0562217030749348     0.00172184129566871     32.6520819406298    9.68523944578334e-53
    x4                -0.959667546097621     0.00451548003783349    -212.528355359105    9.12258816282556e-127
    x5                 0.975701677799351     0.00599650115417007     162.711830234675    5.23528523330508e-116
    x6                -0.94734488104741      0.00396463155511409     -238.9490341985    1.7221719762155e-131
    x7                 0.423500262043915     0.00103874124308346     407.705253703774    4.7599787480658e-153


Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 1.67e-07
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.01e+13, p-value = 0
                      SumSq            DF            MeanSq                F            pValue
                  ____________________    ___    ____________________    _____________    ______

    Total            1.98338186886267    100      0.0198338186886267
    Model            1.98338186886007      7       0.283340266980011    10132578059553.6       0
    Residual    2.60058641288197e-12     93    2.79632947621717e-14

A = 0.339989, B = 0.278751, C = 0.359995
List of powers: [0.618741, 1.257487, 1.685353, 2.078524, 2.453735, 2.817470, 3.173052]
Fitting cos(x) in range (0.000000, 1.000000)
MSS of errors squared = 1.596302e-08
Corrected MSS of errors squared = 2.764876e-08
Particle swarm AICc =  -2.840186e+03
AIC =  -2.857751e+03
AICc =  -2.840186e+03
```

## Using Power A+B*sqrt(i)+C*(i–1)

```
Fitting cos(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*sqrt(i)+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                       Estimate                SE                tStat              pValue
                   ____________________    ____________________    _____________    ____________________

    (Intercept)       0.999999909095415    1.50981083764069e-07     6623345.68122499                   0
    x1              0.00153690693345975    7.6771496785781e-05      20.019238881694    1.76340848940452e-35
    x2              -0.0284800072587711    0.000930794484152931    -30.5975247421983    2.4529790761294e-50
    x3                0.155289983000451     0.00423629441853314     36.6570326937336    4.43757666572182e-57
```

| | | | | |
|---|---|---|---|---|
| x4 | -1.3334695752024 | 0.00955965220982011 | -139.489339772486 | 8.19204653843005e-110 |
| x5 | 1.60898157212939 | 0.0114975083686616 | 139.941761339782 | 6.07041731745186e-110 |
| x6 | -1.4527663422554 | 0.00707182184556006 | -205.430280058242 | 2.13406347318627e-125 |
| x7 | 0.589210317449035 | 0.0017519041243395 | 336.325663752392 | 2.79180782219802e-145 |

Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 1.54e-07
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.2e+13, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 1.98338186886268 | 100 | 0.0198338186886268 | | |
| Model | 1.98338186886048 | 7 | 0.283340266980069 | 11962979257410.9 | 0 |
| Residual | 2.20268248085631e-12 | 93 | 2.368475785867e-14 | | |

A = 0.550000, B = 0.400000, C = 0.250449
List of powers: [0.950000, 1.366134, 1.743718, 2.101346, 2.446222, 2.782040, 3.110993]
Fitting cos(x) in range (0.000000, 1.000000)
MSS of errors squared = 1.464908e-08
Corrected MSS of errors squared = 2.537296e-08
Particle swarm AICc =  -2.857535e+03
AIC =  -2.875101e+03
AICc =  -2.857535e+03

## Using Power A+B*sqrt(i)+C*sqrt(i–1)

Fitting cos(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*sqrt(i)+C*sqrt(i-1)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 0.999999953665837 | 6.11606280685908e-08 | 16350387.2547605 | 0 |
| x1 | 0.000131022068129167 | 4.96112783009299e-06 | 26.4097343620978 | 5.31845956112681e-45 |
| x2 | -0.316721373653275 | 0.000334191277924655 | -947.724834771664 | 4.14447064852275e-187 |
| x3 | -0.661016853137225 | 0.00273095679677826 | -242.04588440104 | 5.20963582932518e-132 |
| x4 | 1.43398377619419 | 0.00863522577764027 | 166.062105742191 | 7.91686431600089e-117 |
| x5 | -1.92072795461364 | 0.0129852909618003 | -147.915665522164 | 3.58850369947111e-112 |
| x6 | 1.31433119615575 | 0.00935596937223513 | 140.480493668158 | 4.25356882815041e-110 |
| x7 | -0.309677264566606 | 0.00260426904503018 | -118.91139479524 | 2.10965095111151e-103 |

Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 6.88e-08
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 5.98e+13, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 1.98338186886268 | 100 | 0.0198338186886268 | | |
| Model | 1.98338186886224 | 7 | 0.28334026698032 | 59814896287107.4 | 0 |
| Residual | 4.40536496171262e-13 | 93 | 4.736951571734e-15 | | |

A = 0.234058, B = 0.700000, C = 0.700000
List of powers: [0.934058, 1.924008, 2.436443, 2.846494, 3.199306, 3.513949, 3.800727]
Fitting cos(x) in range (0.000000, 1.000000)

 

```
MSS of errors squared = 6.491073e-09
Corrected MSS of errors squared = 1.124287e-08
Particle swarm AICc =  -3.021925e+03
AIC =  -3.039490e+03
AICc =  -3.021925e+03
```

## Comments

The following table shows the summary results for the seven models that fit the cosine function.

| *Power* | *A* | *B* | *C* | *F* | *AICc* | *MSSE* | *Pwr Rng* |
|---|---|---|---|---|---|---|---|
| A+B*i | 0.000 | 0.483 | | 6180872616322.5 | -2.790602e+03 | 2.040413e-08 | 0.48, 3.38 |
| A+B/i | 2.000 | 0.697 | | 22892120799319.1 | -2.924246e+03 | 1.488915e-08 | 2.09, 2.69 |
| A+B*√(i) | 1.061 | 0.999 | | 309043630816782 | -3.200513e+03 | 2.664964e-09 | 2.06, 3.70 |
| A+B*√(i+C) | 0.997 | 0.998 | 0.161 | 309043630816777 | -3.187144e+03 | 2.865028e-09 | 2.07, 3.67 |
| A+B*i+C*(i-1) | 0.500 | 0.270 | 0.141 | 9179513786624.0 | -2.830481e+03 | 1.674866e-08 | 0.77, 3.24 |
| A+B*i-C*(i-1) | 0.750 | 0.250 | 0.000 | 35392754179.2 | -2.268790e+03 | 2.701483e-07 | 1, 2.5 |
| A+B*i+C*√(i-1) | 0.339 | 0.278 | 0.359 | 10132578059553.6 | -2.840186e+03 | 1.596302e-08 | 0.61, 3.17 |
| A+B*√(i)+C*(i-1) | 0.550 | 0.400 | 0.250 | 11962979257410.9 | -2.857535e+03 | 1.464908e-08 | 0.95, 3.11 |
| A+B*√(i)+C*√(i-1) | 0.234 | 0.700 | 0.700 | 59814896287107.4 | -3.021925e+03 | 6.491073e-09 | 0.93, 3.80 |

The above table shows that the top two Shammas polynomials use the following power equations:

$1.061+0.999*\sqrt{(i)}$

And,

$0.997+0.998*\sqrt{(i+0.161)}$

 

# RESULTS FOR THE HYPERBOLIC COSINE FUNCTION

## Using Power A+B* i

```
Fitting cosh(x) in range (0.000000, 5.000000)
Shammas polynomial power is A+B*i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat               pValue
                ________________    ________________    ________________    ____________________

    (Intercept)    0.842968039821023    0.0165673984556918     50.8811351447527    2.43403254896279e-198
    x1              15.1277171022618     0.726057020940964     20.8354394571605     1.2803807878466e-69
    x2             -80.7559167081994     3.08330322399544    -26.1913638852403     2.14787210591414e-95
    x3              179.140170001219     5.60062727752758     31.9857332267077    2.39078366732254e-122
    x4             -205.510892391179      5.3721344224448    -38.254979535239    1.14320150765583e-149
    x5              129.567910577991     2.83971587152917     45.6270684954898    4.35022512822573e-179
    x6             -42.6611683223115     0.782269663921979    -54.535117862587    5.94509096272655e-211
    x7              5.82170883243254    0.0877552748393687     66.3402723436154    7.28045188211524e-248


Number of observations: 501, Error degrees of freedom: 493
Root Mean Squared Error: 0.0276
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 3.13e+07, p-value = 0
                    SumSq          DF           MeanSq                F            pValue
                ________________    ___    ____________________    ________________    ______

    Total        167322.933901257    500        334.645867802514
    Model        167322.557990453      7        23903.2225700647    31348630.0787721       0
    Residual    0.375910803675652    493    0.000762496559179821

A = 0.500000, B = 0.500000
List of powers: [1.000000, 1.500000, 2.000000, 2.500000, 3.000000, 3.500000, 4.000000]
Fitting cosh(x) in range (0.000000, 5.000000)
MSS of errors squared = 1.223784e-03
Corrected MSS of errors squared = 1.730692e-03
Particle swarm AICc =  -2.150631e+03
AIC =  -2.166923e+03
AICc =  -2.150631e+03
```

## Using Power A+B/i

```
Fitting cosh(x) in range (0.000000, 5.000000)
Shammas polynomial power is A+B/i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat               pValue
                ________________    ________________    ________________    ____________________

    (Intercept)    0.922306067689812    0.00748429314319361     123.232221138823                   0
    x1              6.36080717637634     0.0979477542680944     64.940816907002    9.10487305313697e-244
    x2             -1935.52755867653      42.8617359889695    -45.1574700374861    2.67136829117271e-177
    x3              42465.7264911124      1054.14687378432     40.2844494891526    4.65631029213522e-158
    x4             -255772.696422397      6724.16207166219    -38.0378541885994    9.31153779387346e-149
    x5              609774.289940727      16596.0919238056     36.7420410021989    2.8750556412296e-143
```

| | | | | |
|---|---|---|---|---|
| x6 | -623407.660396974 | 17365.9326280894 | -35.8983115821036 | 1.20786777124078e-139 |
| x7 | 228870.142771319 | 6482.62659086978 | 35.3051559523207 | 4.49045109711274e-137 |

Number of observations: 501, Error degrees of freedom: 493
Root Mean Squared Error: 0.0187
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 6.83e+07, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 167322.93387025 | 500 | 334.645867740499 | | |
| Model | 167322.76137718 | 7 | 23903.2516253114 | 68317544.8284238 | 0 |
| Residual | 0.172493070131168 | 493 | 0.000349884523592632 | | |

A = 2.000000, B = 2.000000
List of powers: [4.000000, 3.000000, 2.666667, 2.500000, 2.400000, 2.333333, 2.285714]
Fitting cosh(x) in range (0.000000, 5.000000)
MSS of errors squared = 8.289878e-04
Corrected MSS of errors squared = 1.172366e-03
Particle swarm AICc =  -2.540907e+03
AIC =  -2.557200e+03
AICc =  -2.540907e+03

## Using Power A+B*sqrt(i)

Fitting cosh(x) in range (0.000000, 5.000000)
Shammas polynomial power is A+B*sqrt(i)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 0.980425008501921 | 0.00191061386587285 | 513.146599642216 | 0 |
| x1 | 89.4561583308673 | 2.46577915158905 | 36.279063464876 | 2.76773605796229e-141 |
| x2 | -776.909829432409 | 19.1088952980874 | -40.6569724368194 | 1.4199807202858e-159 |
| x3 | 2542.95391888598 | 56.809465697165 | 44.7628557614279 | 8.67434079011643e-176 |
| x4 | -4174.39822645095 | 85.8029738145851 | -48.6509737468024 | 2.4651029865984e-190 |
| x5 | 3703.24053996416 | 70.6166765275221 | 52.4414447417511 | 8.42980655000536e-204 |
| x6 | -1703.85773066282 | 30.3095036654411 | -56.2152963463257 | 1.49825385026272e-216 |
| x7 | 320.075327489764 | 5.33098891891679 | 60.0405163766126 | 7.48777580468381e-229 |

Number of observations: 501, Error degrees of freedom: 493
Root Mean Squared Error: 0.00665
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 5.41e+08, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 167322.933902145 | 500 | 334.645867804289 | | |
| Model | 167322.912116084 | 7 | 23903.2731594406 | 540910726.237856 | 0 |
| Residual | 0.021786060242448 | 493 | 4.41907915668316e-05 | | |

A = 1.400000, B = 1.000000
List of powers: [2.400000, 2.814214, 3.132051, 3.400000, 3.636068, 3.849490, 4.045751]
Fitting cosh(x) in range (0.000000, 5.000000)
MSS of errors squared = 2.946128e-04
Corrected MSS of errors squared = 4.166454e-04

 

```
Particle swarm AICc =  -3.577520e+03
AIC =  -3.593812e+03
AICc =  -3.577520e+03
```

## Using Power A+B*sqrt(i+C)

```
Fitting cosh(x) in range (0.000000, 5.000000)
Shammas polynomial power is A+B*sqrt(i+C)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat               pValue
                ________________    ____________________    ________________    ____________________

    (Intercept)    0.984278511454304    0.00155333935174226     633.653238972101                   0
    x1              523.025165893048       12.7886814557674     40.8975051651768    1.50510477369095e-160
    x2             -3583.96181311533       81.6850145317833    -43.8753893068211    2.32899145347176e-172
    x3              10029.7934740806       213.778267838547     46.9168057889565    6.09996586672194e-184
    x4             -14720.5022629785       294.572788396726    -49.9723764136459    4.16417006906746e-195
    x5              11991.1810428863       226.029338382276     53.0514362812762     6.6329167296105e-206
    x6             -5154.31344774509       91.7551745956421    -56.1746350596546    2.03999986618115e-216
    x7              915.333370426984       15.4179246180939     59.3681311265969    9.84563833032591e-227


Number of observations: 501, Error degrees of freedom: 493
Root Mean Squared Error: 0.00573
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 7.28e+08, p-value = 0
                    SumSq           DF            MeanSq                F           pValue
                ________________    ___    ____________________    ________________    ______

    Total         167322.933902221    500        334.645867804443
    Model         167322.917710157      7        23903.2739585939    727783316.17163       0
    Residual    0.0161920640384778    493    3.28439432829165e-05

A = 1.000000, B = 1.000000, C = 2.000000
List of powers: [2.732051, 3.000000, 3.236068, 3.449490, 3.645751, 3.828427, 4.000000]
Fitting cosh(x) in range (0.000000, 5.000000)
MSS of errors squared = 2.539881e-04
Corrected MSS of errors squared = 4.399203e-04
Particle swarm AICc =  -3.726191e+03
AIC =  -3.742483e+03
AICc =  -3.726191e+03
```

## Using Power A+B*i+C*(i–1)

```
Fitting cosh(x) in range (0.000000, 5.000000)
Shammas polynomial power is A+B*i+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat               pValue
                ________________    ____________________    ________________    ____________________

    (Intercept)    0.796140541574037    0.0239342990733288     33.2635829081461    4.38382551723852e-128
    x1              9.12481896373606     0.507924652803749     17.9649066320506     6.99605549374206e-56
    x2             -50.1005797374524      2.09613063500765    -23.9014586689963     2.02842756511218e-84
    x3              115.23188945284      3.82415122798795     30.1326706458386    6.92560694378514e-114
    x4             -135.343155623795      3.68002480661358    -36.7777835031335    2.02302817066963e-143
```

 

| | | | | |
|---|---|---|---|---|
| x5 | 86.6353713047536 | 1.93792494464865 | 44.7052253205093 | 1.44429774263853e-175 |
| x6 | -28.7003688543613 | 0.527891274581506 | -54.3679546079142 | 2.17738328831538e-210 |
| x7 | 3.93304318746138 | 0.0581694716548333 | 67.6135277762073 | 1.5633487730454e-251 |

Number of observations: 501, Error degrees of freedom: 493
Root Mean Squared Error: 0.0337
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 2.1e+07, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 167322.933901285 | 500 | 334.64586780257 | | |
| Model | 167322.373066394 | 7 | 23903.1961523421 | 21012023.145848 | 0 |
| Residual | 0.560834890638944 | 493 | 0.00113759612705668 | | |

A = 0.500000, B = 0.270000, C = 0.270000
List of powers: [0.770000, 1.310000, 1.850000, 2.390000, 2.930000, 3.470000, 4.010000]
Fitting cosh(x) in range (0.000000, 5.000000)
MSS of errors squared = 1.494789e-03
Corrected MSS of errors squared = 2.589050e-03
Particle swarm AICc =  -1.950193e+03
AIC =  -1.966486e+03
AICc =  -1.950193e+03

## Using Power A+B*i–C*(i–1)

Fitting cosh(x) in range (0.000000, 5.000000)
Shammas polynomial power is A+B*i-C*(i-1)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 0.289186789395451 | 0.109307581938282 | 2.64562424918276 | 0.00841445802039274 |
| x1 | 1158.76763007941 | 54.5332687981946 | 21.2488203186121 | 1.29626266930601e-71 |
| x2 | -7248.85435507695 | 296.116201322885 | -24.479762750883 | 3.34542721543422e-87 |
| x3 | 18715.5765490362 | 674.109317510684 | 27.7634147205503 | 7.66561463357163e-103 |
| x4 | -25451.9437601169 | 816.392682501622 | -31.1761047173108 | 1.13611680993823e-118 |
| x5 | 19222.6584911079 | 552.497798344599 | 34.7922807089965 | 7.76034706501429e-135 |
| x6 | -7650.55407888795 | 197.7380576089 | -38.69034707532 | 1.73530804810593e-151 |
| x7 | 1255.58134879315 | 29.2171245543051 | 42.9741587492442 | 7.79186450687662e-169 |

Number of observations: 501, Error degrees of freedom: 493
Root Mean Squared Error: 0.125
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.53e+06, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 167322.933902029 | 500 | 334.645867804059 | | |
| Model | 167315.222077229 | 7 | 23902.1745824613 | 1528013.45650556 | 0 |
| Residual | 7.7118248003535 | 493 | 0.015642646653861 | | |

A = 0.750000, B = 0.250000, C = 0.000000
List of powers: [1.000000, 1.250000, 1.500000, 1.750000, 2.000000, 2.250000, 2.500000]
Fitting cosh(x) in range (0.000000, 5.000000)
MSS of errors squared = 5.542949e-03

 

```
Corrected MSS of errors squared = 9.600669e-03
Particle swarm AICc =  -6.370304e+02
AIC =  -6.533231e+02
AICc =  -6.370304e+02
```

## Using Power A+B*i+C*sqrt(i–1)

```
Fitting cosh(x) in range (0.000000, 5.000000)
Shammas polynomial power is A+B*i+C*sqrt(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat                pValue
                ________________    ________________    ________________    ____________________

    (Intercept)    0.82127221243576    0.0203013273528972     40.4541140665149    9.47898030706848e-159
    x1              4.85367724580712    0.275458928947329      17.6203300592053     2.96023361960748e-54
    x2             -45.5632005090071     1.76769498706245     -25.7754877637141     2.06779522775848e-93
    x3              133.014026824988     4.13942335586591      32.1334677296285      5.141596782191e-123
    x4             -180.529905062368     4.68474217757386     -38.5357183425324     7.65870389997008e-151
    x5              129.130933879783     2.81790128351295      45.8252156082635     7.71572650794505e-180
    x6             -47.2045471988304     0.867928483811807     -54.3875999915516     1.86902208539264e-210
    x7              7.05254690486854     0.107887522683329      65.3694396669869     4.97765154340044e-245


Number of observations: 501, Error degrees of freedom: 493
Root Mean Squared Error: 0.03
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 2.66e+07, p-value = 0
                    SumSq          DF            MeanSq                F            pValue
                ________________    ___    ____________________    ________________    ______

    Total        167322.933901227    500        334.645867802454
    Model        167322.490310123      7        23903.2129014461    26565645.3747725       0
    Residual    0.443591104005463    493    0.000899779115629743

A = 0.340000, B = 0.390000, C = 0.360000
List of powers: [0.730000, 1.480000, 2.019117, 2.523538, 3.010000, 3.484984, 3.951816]
Fitting cosh(x) in range (0.000000, 5.000000)
MSS of errors squared = 1.329394e-03
Corrected MSS of errors squared = 2.302578e-03
Particle swarm AICc =  -2.067689e+03
AIC =  -2.083982e+03
AICc =  -2.067689e+03
```

## Using Power A+B*sqrt(i)+C*(i–1)

```
Fitting cosh(x) in range (0.000000, 5.000000)
Shammas polynomial power is A+B*sqrt(i)+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat                pValue
                ________________    ________________    ________________    ____________________

    (Intercept)    0.849009577288261    0.0159818724492305     53.1232857717583     3.7582366110052e-206
    x1              10.3891114927185     0.51275576772785      20.2613254625205     7.47309344875874e-67
    x2             -62.5950958444375     2.38020648077621     -26.2981788974983     6.65870576316346e-96
    x3              152.365213873298     4.69624757593707      32.4440335415869     2.04913186758939e-124
```

| | | | | |
|---|---|---|---|---|
| x4 | -187.833951930539 | 4.83544193511672 | -38.8452502275792 | 3.93331951260067e-152 |
| x5 | 125.95896543104 | 2.72580711653778 | 46.20978669651 | 2.72450386293053e-181 |
| x6 | -43.8675036075889 | 0.798547523613135 | -54.9341176453777 | 2.71317939586684e-212 |
| x7 | 6.30566854202365 | 0.0952163800508993 | 66.2246195313544 | 1.57807251779799e-247 |

Number of observations: 501, Error degrees of freedom: 493
Root Mean Squared Error: 0.0266
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 3.38e+07, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 167322.933901267 | 500 | 334.645867802534 | | |
| Model | 167322.58546835 | 7 | 23903.2264954786 | 33820830.610973 | 0 |
| Residual | 0.348432916915044 | 493 | 0.000706760480557899 | | |

A = 0.550000, B = 0.400000, C = 0.400000
List of powers: [0.950000, 1.515685, 2.042820, 2.550000, 3.044427, 3.529796, 4.008301]
Fitting cosh(x) in range (0.000000, 5.000000)
MSS of errors squared = 1.178208e-03
Corrected MSS of errors squared = 2.040716e-03
Particle swarm AICc =  -2.188660e+03
AIC =  -2.204952e+03
AICc =  -2.188660e+03

## Using Power A+B*sqrt(i)+C*sqrt(i–1)

Fitting cosh(x) in range (0.000000, 5.000000)
Shammas polynomial power is A+B*sqrt(i)+C*sqrt(i-1)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 0.945365962077869 | 0.00537029486818317 | 176.036136801124 | 0 |
| x1 | 3.15514212962763 | 0.138358730670447 | 22.8040696408437 | 3.9865678572224e-79 |
| x2 | -56.0553814717533 | 1.71271643648314 | -32.7289329848766 | 1.07642409617302e-125 |
| x3 | 228.842155462452 | 5.80659515746799 | 39.4107302569791 | 1.78914773396156e-154 |
| x4 | -412.899908507707 | 9.12369648302183 | -45.2557698818749 | 1.12581751554522e-177 |
| x5 | 384.530216231149 | 7.5419745393343 | 50.985350616828 | 1.04245083466955e-198 |
| x6 | -181.594674623531 | 3.19459878867524 | -56.8442820636128 | 1.29136035743154e-218 |
| x7 | 34.6276214710134 | 0.548932419739532 | 63.0817569263701 | 3.24525458207718e-238 |

Number of observations: 501, Error degrees of freedom: 493
Root Mean Squared Error: 0.0128
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.45e+08, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 167322.933901215 | 500 | 334.645867802429 | | |
| Model | 167322.852779208 | 7 | 23903.264682744 | 145266493.870053 | 0 |
| Residual | 0.0811220066971146 | 493 | 0.00016454768092721 | | |

A = 0.500000, B = 0.700000, C = 0.700000
List of powers: [1.200000, 2.189949, 2.702385, 3.112436, 3.465248, 3.779890, 4.066669]
Fitting cosh(x) in range (0.000000, 5.000000)

 

```
MSS of errors squared = 5.685015e-04
Corrected MSS of errors squared = 9.846735e-04
Particle swarm AICc =  -2.918863e+03
AIC =  -2.935156e+03
AICc =  -2.918863e+03
```

## Comments

The following table shows the summary results for the seven models that fit the hyperbolic cosine function.

| *Power* | *A* | *B* | *C* | *F* | *AICc* | *MSSE* | *Pwr Rng* |
|---|---|---|---|---|---|---|---|
| A+B*i | 0.500 | 0.500 | | 31348630.0 | -2.150631e+03 | 1.223784e-03 | 1, 4 |
| A+B/i | 2.000 | 2.000 | | 68317544.8 | -2.540907e+03 | 8.289878e-04 | 2.28, 4 |
| A+B*√(i) | 1.400 | 1.000 | | 540910726.2 | -3.577520e+03 | 2.946128e-04 | 2.40, 4.04 |
| A+B*√(i+C) | 1.000 | 1.000 | 2.000 | 727783316.1 | -3.726191e+03 | 2.539881e-04 | 2.732, 4 |
| A+B*i+C*(i-1) | 0.500 | 0.270 | 0.270 | 21012023.1 | -1.950193e+03 | 1.494789e-03 | 0.77, 4.01 |
| A+B*i-C*(i-1) | 0.750 | 0.250 | 0.000 | 1528013.4 | -6.370304e+02 | 5.542949e-03 | 1, 2.5 |
| A+B*i+C*√(i-1) | 0.340 | 0.390 | 0.360 | 26565645.3 | -2.067689e+03 | 1.329394e-03 | 0.73, 3.95 |
| A+B*√(i)+C*(i-1) | 0.550 | 0.400 | 0.400 | 33820830.6 | -2.188660e+03 | 1.178208e-03 | 0.95, 4.00 |
| A+B*√(i)+C*√(i-1) | 0.500 | 0.700 | 0.700 | 145266493.8 | -2.918863e+03 | 5.685015e-04 | 1.20, 4.06 |

The above table shows that the top two Shammas polynomials use the following power equations:

$1+\sqrt{(i+2)}$

And,

$1.4+\sqrt{(i)}$

 

# RESULTS FOR THE ERROR FUNCTION

## Using Power A+B* i

```
Fitting erf(x) in range (0.000000, 2.100000)
Shammas polynomial power is A+B*i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
```

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 7.43328795545811e-05 | 5.44722491436725e-05 | 1.36460088803246 | 0.173889249547516 |
| x1 | 0.263958217344449 | 0.00374096001564401 | 70.5589517772509 | 8.8982553320767e-145 |
| x2 | 2.75820697047733 | 0.0257697638826871 | 107.032683071279 | 1.57100460666227e-180 |
| x3 | -6.61155901666738 | 0.0764938319129105 | -86.4325769977736 | 4.33421289155636e-162 |
| x4 | 11.7668164198554 | 0.119285004214483 | 98.6445571875725 | 1.80284259988512e-173 |
| x5 | -12.4996648846621 | 0.101867659524504 | -122.704938377968 | 2.14801999794779e-192 |
| x6 | 6.39677883547837 | 0.0450870133102576 | 141.876304634777 | 4.79324854625366e-205 |
| x7 | -1.23195282832721 | 0.00808976931677844 | -152.285285288926 | 3.13787259280776e-211 |

```
Number of observations: 211, Error degrees of freedom: 203
Root Mean Squared Error: 6.08e-05
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 7.04e+08, p-value = 0
```

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 18.228004442316 | 210 | 0.0868000211538857 | | |
| Model | 18.2280036915711 | 7 | 2.6040005273673 | 704116838.055738 | 0 |
| Residual | 7.50744874267184e-07 | 203 | 3.69825061215362e-09 | | |

```
A = 0.336151, B = 0.487192
List of powers: [0.823343, 1.310535, 1.797726, 2.284918, 2.772109, 3.259301, 3.746493]
Fitting erf(x) in range (0.000000, 2.100000)
MSS of errors squared = 4.106424e-06
Corrected MSS of errors squared = 5.807360e-06
Particle swarm AICc =  -3.473301e+03
AIC =  -3.490014e+03
AICc =  -3.473301e+03
```

## Using Power A+B/i

```
Fitting erf(x) in range (0.000000, 2.100000)
Shammas polynomial power is A+B/i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
```

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 0.00141620491680569 | 0.000160110183024105 | 8.84518954420582 | 4.46772291438797e-16 |
| x1 | -0.659906591347379 | 0.0270284608911204 | -24.4152485783671 | 2.5537106791438e-62 |
| x2 | 48.3811452697361 | 4.28079494422834 | 11.3019067486442 | 2.72148144072862e-23 |
| x3 | -256.654466225047 | 74.5097752783063 | -3.44457442350886 | 0.000694818615212243 |
| x4 | -569.402092285209 | 399.456481651055 | -1.42544211557596 | 0.155565224753898 |
| x5 | 4271.38009792683 | 888.131316554033 | 4.80940151339317 | 2.94596328619293e-06 |

 

| | | | | |
|---|---|---|---|---|
| x6 | -6359.95072834835 | 866.804976330822 | -7.3372337515527 | 5.13149523857108e-12 |
| x7 | 2867.74707640302 | 307.874280948034 | 9.31466918110989 | 2.08050286865473e-17 |

Number of observations: 211, Error degrees of freedom: 203
Root Mean Squared Error: 0.000197
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 6.7e+07, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 18.2280044422529 | 210 | 0.0868000211535851 | | |
| Model | 18.2279965483153 | 7 | 2.60399950690219 | 66964287.2731222 | 0 |
| Residual | 7.89393752143042e-06 | 203 | 3.88863917311843e-08 | | |

A = 1.465884, B = 2.000000
List of powers: [3.465884, 2.465884, 2.132551, 1.965884, 1.865884, 1.799217, 1.751598]
Fitting erf(x) in range (0.000000, 2.100000)
MSS of errors squared = 1.331571e-05
Corrected MSS of errors squared = 1.883126e-05
Particle swarm AICc =  -2.976864e+03
AIC =  -2.993577e+03
AICc =  -2.976864e+03

## Using Power A+B*sqrt(i)

Fitting erf(x) in range (0.000000, 2.100000)
Shammas polynomial power is A+B*sqrt(i)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 0.00109887699487031 | 0.000128644307792447 | 8.5419791495417 | 3.12549652790066e-15 |
| x1 | 5.32331988111958 | 0.133204422582459 | 39.9635370801913 | 3.73245953741507e-98 |
| x2 | -7.9185284004124 | 1.66275265021991 | -4.76230087461603 | 3.63689589891572e-06 |
| x3 | -47.1869719160538 | 7.15487455722477 | -6.59508025453862 | 3.60799743259735e-10 |
| x4 | 203.974965900885 | 14.7355657162792 | 13.8423573161865 | 3.99950841182808e-31 |
| x5 | -306.974704050223 | 15.9005086179431 | -19.3059675904841 | 7.76774264718821e-48 |
| x6 | 204.89613584943 | 8.69875544967898 | 23.5546495167872 | 5.61560689733641e-60 |
| x7 | -51.2727462337673 | 1.90860526997761 | -26.8639865142826 | 9.20233932053932e-69 |

Number of observations: 211, Error degrees of freedom: 203
Root Mean Squared Error: 0.000157
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.05e+08, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 18.2280044423208 | 210 | 0.0868000211539087 | | |
| Model | 18.2279994084663 | 7 | 2.60399991549518 | 105011374.319937 | 0 |
| Residual | 5.03385453498595e-06 | 203 | 2.47973129802263e-08 | | |

A = 0.298120, B = 1.000000
List of powers: [1.298120, 1.712333, 2.030171, 2.298120, 2.534188, 2.747610, 2.943871]
Fitting erf(x) in range (0.000000, 2.100000)
MSS of errors squared = 1.063330e-05
Corrected MSS of errors squared = 1.503775e-05

 

```
Particle swarm AICc =  -3.071795e+03
AIC =  -3.088507e+03
AICc =  -3.071795e+03
```

## Using Power A+B*sqrt(i+C)

```
Fitting erf(x) in range (0.000000, 2.100000)
Shammas polynomial power is A+B*sqrt(i+C)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                       Estimate                SE                tStat              pValue
                   ____________________    ____________________    _____________    ____________________

    (Intercept)    0.00109887780884854    0.00012864427347281     8.54198775572237    3.12532527979537e-15
    x1                5.32333961239185      0.133204556678809      39.963644976709    3.73064388626466e-98
    x2               -7.91869121570518       1.66275419949959    -4.76239435635664    3.63538079197574e-06
    x3               -47.1864045290858       7.15488077871654    -6.5949952191307    3.60970060651175e-10
    x4                203.973942958233       14.7355777773235     13.8422765663201    4.00182062268502e-31
    x5               -306.973705595908       15.9005209500925    -19.3058898233219    7.77186674507412e-48
    x6                204.895634058637       8.69876187544603      23.554574431678    5.61827272084904e-60
    x7               -51.2726442583087       1.90860661829384    -26.8639141072154    9.20627522236944e-69


Number of observations: 211, Error degrees of freedom: 203
Root Mean Squared Error: 0.000157
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.05e+08, p-value = 0
                       SumSq            DF            MeanSq                F           pValue
                   ____________________    ___    ____________________    _____________    ______

    Total            18.2280044423244    210      0.0868000211539259
    Model              18.22799940847      7        2.60399991549571    105011375.50577       0
    Residual    5.03385447814253e-06    203    2.47973127002095e-08

A = 0.298120, B = 1.000000, C = 0.000000
List of powers: [1.298120, 1.712334, 2.030171, 2.298120, 2.534188, 2.747610, 2.943872]
Fitting erf(x) in range (0.000000, 2.100000)
MSS of errors squared = 1.063330e-05
Corrected MSS of errors squared = 1.841741e-05
Particle swarm AICc =  -3.071795e+03
AIC =  -3.088507e+03
AICc =  -3.071795e+03
```

## Using Power A+B*i+C*(i–1)

```
Fitting erf(x) in range (0.000000, 2.100000)
Shammas polynomial power is A+B*i+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                       Estimate                SE                tStat              pValue
                   ____________________    ____________________    _____________    ____________________

    (Intercept)    6.43483132274099e-05    5.55005050657235e-05     1.15941851612357      0.247647559763701
    x1                0.170297199093703     0.00296356516623028      57.4636255798364    1.52980941474642e-127
    x2                2.67424659781836      0.0205441281351003      130.170848830004    1.54814688968422e-197
    x3               -5.86691486510755       0.061807690723142    -94.9220848807876    3.76373645391828e-170
    x4                10.3582005952789      0.0974977785157995      106.24037545225    6.91493535931498e-180
```

| | | | | |
|---|---|---|---|---|
| x5 | -11.0711201978924 | 0.0839771235047821 | -131.834953804556 | 1.21038624347416e-198 |
| x6 | 5.66341054309479 | 0.0373879987059651 | 151.476696777333 | 9.16052227747736e-211 |
| x7 | -1.08552603146901 | 0.00673293272507715 | -161.22632971304 | 3.21931290732276e-216 |

Number of observations: 211, Error degrees of freedom: 203
Root Mean Squared Error: 6.11e-05
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 6.98e+08, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 18.2280044423159 | 210 | 0.0868000211538852 | | |
| Model | 18.2280036855109 | 7 | 2.60400052650156 | 698478605.843292 | 0 |
| Residual | 7.56805007995354e-07 | 203 | 3.72810348766184e-09 | | |

A = 0.500000, B = 0.270000, C = 0.230930
List of powers: [0.770000, 1.270930, 1.771861, 2.272791, 2.773721, 3.274651, 3.775582]
Fitting erf(x) in range (0.000000, 2.100000)
MSS of errors squared = 4.122964e-06
Corrected MSS of errors squared = 7.141183e-06
Particle swarm AICc =  -3.471605e+03
AIC =  -3.488318e+03
AICc =  -3.471605e+03

## Using Power A+B*i–C*(i–1)

Fitting erf(x) in range (0.000000, 2.100000)
Shammas polynomial power is A+B*i-C*(i-1)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 0.00111584832687163 | 0.000422483931677348 | 2.64116157611363 | 0.00890524375562516 |
| x1 | -8.27062390008695 | 0.585385172130667 | -14.128516221181 | 5.15701024580006e-32 |
| x2 | 73.1353631413465 | 4.05776798172317 | 18.0235448332087 | 5.34490370510973e-44 |
| x3 | -230.113327624986 | 11.7152644851255 | -19.6421794759438 | 7.876373946594e-49 |
| x4 | 373.139363453522 | 17.9091750187829 | 20.835095031579 | 2.59223698681339e-52 |
| x5 | -327.130587906028 | 15.2469428097274 | -21.4554873057779 | 4.27384770974265e-54 |
| x6 | 145.991639083647 | 6.84743290539386 | 21.3206381282899 | 1.03907928137831e-53 |
| x7 | -25.9107130582428 | 1.26718687495602 | -20.4474285287575 | 3.44919351566957e-51 |

Number of observations: 211, Error degrees of freedom: 203
Root Mean Squared Error: 0.000435
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.37e+07, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 18.2280044423187 | 210 | 0.0868000211538985 | | |
| Model | 18.2279659778711 | 7 | 2.60399513969588 | 13742846.8870979 | 0 |
| Residual | 3.84644475559526e-05 | 203 | 1.89480037221441e-07 | | |

A = 0.750000, B = 0.250000, C = 0.000000
List of powers: [1.000000, 1.250000, 1.500000, 1.750000, 2.000000, 2.250000, 2.500000]
Fitting erf(x) in range (0.000000, 2.100000)
MSS of errors squared = 2.939323e-05

 

```
Corrected MSS of errors squared = 5.091057e-05
Particle swarm AICc =  -2.642716e+03
AIC =  -2.659429e+03
AICc =  -2.642716e+03
```

## Using Power A+B*i+C*sqrt(i–1)

```
Fitting erf(x) in range (0.000000, 2.100000)
Shammas polynomial power is A+B*i+C*sqrt(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                        Estimate                SE                tStat              pValue
                    ____________________    ____________________    _____________    ____________________

    (Intercept)    2.49292267764148e-06    5.06552585670238e-05    0.0492135021745671       0.960797537533222
    x1                0.116734617427294     0.00142943789492577      81.6647003984431    3.16388743477298e-157
    x2                 4.36041449719699      0.0207479238282473      210.161485712633    1.90138258203955e-239
    x3                -13.1199066861256      0.083688142812732     -156.771392519533    9.08891747731869e-214
    x4                 24.9749606083567       0.15531121416235      160.805906663314    5.44727737634291e-216
    x5                -26.6336221674591      0.149573231914965     -178.06409493512     6.47966742994772e-225
    x6                 13.9545636665869      0.0727193136549647      191.89625101246    1.79826209705487e-231
    x7                -2.81048960906681      0.014134620757966     -198.837284508173    1.37655224713072e-234


Number of observations: 211, Error degrees of freedom: 203
Root Mean Squared Error: 5.49e-05
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 8.63e+08, p-value = 0
                        SumSq            DF            MeanSq                F           pValue
                    ____________________    ___    ____________________    _____________    ______

    Total             18.2280044423163    210      0.0868000211538872
    Model             18.2280038298095      7        2.60400054711565    863030606.26177       0
    Residual     6.12506795505396e-07    203    3.01727485470639e-09

A = 0.340000, B = 0.331300, C = 0.360000
List of powers: [0.671300, 1.362599, 1.843016, 2.288737, 2.716498, 3.132782, 3.540913]
Fitting erf(x) in range (0.000000, 2.100000)
MSS of errors squared = 3.709138e-06
Corrected MSS of errors squared = 6.424416e-06
Particle swarm AICc =  -3.516241e+03
AIC =  -3.532954e+03
AICc =  -3.516241e+03
```

## Using Power A+B*sqrt(i)+C*(i–1)

```
Fitting erf(x) in range (0.000000, 2.100000)
Shammas polynomial power is A+B*sqrt(i)+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                        Estimate                SE                tStat              pValue
                    ____________________    ____________________    _____________    ____________________

    (Intercept)    5.68865008103416e-05    5.26825792674803e-05     1.07979718535642       0.28151374815548
    x1                0.239384673965556     0.00302529410945891      79.1277361156702    1.57806075414728e-154
    x2                 3.08531968775678      0.0245384057302566      125.734317122016    1.62067401649625e-194
    x3                -8.1668097683384      0.0815132129082758     -100.190011863822    8.18265052848362e-175
```

| | | | | |
|---|---|---|---|---|
| x4 | 15.3120409024123 | 0.137694372039906 | 111.203099121397 | 7.63007774887187e-184 |
| x5 | -16.6508013257806 | 0.125106878960435 | -133.092612205972 | 1.80054975239764e-199 |
| x6 | 8.78086628740932 | 0.0583258152664238 | 150.548539224006 | 3.1551965826982e-210 |
| x7 | -1.75739917898415 | 0.0109620124067384 | -160.317203974688 | 1.00561619866973e-215 |

Number of observations: 211, Error degrees of freedom: 203
Root Mean Squared Error: 5.87e-05
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 7.56e+08, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 18.228004442316 | 210 | 0.0868000211538855 | | |
| Model | 18.2280037433807 | 7 | 2.60400053476867 | 756310554.44905 | 0 |
| Residual | 6.98935252785304e-07 | 203 | 3.44303080189805e-09 | | |

A = 0.399533, B = 0.400000, C = 0.367881
List of powers: [0.799533, 1.333100, 1.828115, 2.303176, 2.765483, 3.218733, 3.665118]
Fitting erf(x) in range (0.000000, 2.100000)
MSS of errors squared = 3.962197e-06
Corrected MSS of errors squared = 6.862726e-06
Particle swarm AICc =  -3.488389e+03
AIC =  -3.505102e+03
AICc =  -3.488389e+03

## Using Power A+B*sqrt(i)+C*sqrt(i–1)

Fitting erf(x) in range (0.000000, 2.100000)
Shammas polynomial power is A+B*sqrt(i)+C*sqrt(i-1)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | -1.51355120224791e-05 | 4.44608420501035e-05 | -0.34042342260236 | 0.733889700027904 |
| x1 | 0.354860837080212 | 0.00203002880750321 | 174.805813478413 | 2.68665796042798e-223 |
| x2 | 11.8107106077377 | 0.0755459444032814 | 156.338115844968 | 1.5867625259958e-213 |
| x3 | -65.683397305529 | 0.44519499783396 | -147.538489033128 | 1.83457233156456e-208 |
| x4 | 166.331853979124 | 1.08175524382158 | 153.761079439295 | 4.50450102399979e-212 |
| x5 | -210.943767545189 | 1.29701626572358 | -162.637719448728 | 5.56186000979882e-217 |
| x6 | 130.17753615437 | 0.763977164244595 | 170.394538275351 | 4.64914286125435e-221 |
| x7 | -31.2051237348928 | 0.177078290247224 | -176.222187888343 | 5.27703380155364e-224 |

Number of observations: 211, Error degrees of freedom: 203
Root Mean Squared Error: 5.08e-05
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.01e+09, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 18.2280044423157 | 210 | 0.0868000211538841 | | |
| Model | 18.2280039181498 | 7 | 2.60400055973568 | 1008482458.32078 | 0 |
| Residual | 5.24165898241336e-07 | 203 | 2.58209802089328e-09 | | |

A = 0.327297, B = 0.485840, C = 0.632420
List of powers: [0.813137, 1.646799, 2.063174, 2.394361, 2.678509, 2.931492, 3.161816]
Fitting erf(x) in range (0.000000, 2.100000)

 

```
MSS of errors squared = 3.431246e-06
Corrected MSS of errors squared = 5.943093e-06
Particle swarm AICc =  -3.549105e+03
AIC =  -3.565818e+03
AICc =  -3.549105e+03
```

## Comments

The following table shows the summary results for the seven models that fit the error function.

| *Power* | *A* | *B* | *C* | *F* | *AICc* | *MSSE* | *Pwr Rng* |
|---|---|---|---|---|---|---|---|
| A+B*i | 0.336 | 0.487 | | 704116838.0 | -3.473301e+03 | 4.106424e-06 | 0.82, 3.74 |
| A+B/i | 1.465 | 2.000 | | 66964287.2 | -2.976864e+03 | 1.331571e-05 | 1.75, 3.46 |
| A+B*√(i) | 0.298 | 1.000 | | 105011374.3 | -3.071795e+03 | 1.063330e-05 | 1.29, 2.94 |
| A+B*√(i+C) | 0.298 | 1.000 | 0.000 | 105011375.5 | -3.071795e+03 | 1.063330e-05 | 1.29, 2.94 |
| A+B*i+C*(i-1) | 0.500 | 0.270 | 0.230 | 698478605.8 | -3.471605e+03 | 4.122964e-06 | 0.77, 3.77 |
| A+B*i-C*(i-1) | 0.750 | 0.250 | 0.000 | 13742846.8 | -2.642716e+03 | 2.939323e-05 | 1, 2.5 |
| A+B*i+C*√(i-1) | 0.340 | 0.331 | 0.360 | 863030606.2 | -3.516241e+03 | 3.709138e-06 | 0.67, 3.54 |
| A+B*√(i)+C*(i-1) | 0.399 | 0.400 | 0.367 | 756310554.4 | -3.488389e+03 | 3.962197e-06 | 0.79, 3.66 |
| A+B*√(i)+C*√(i-1) | 0.327 | 0.485 | 0.632 | 1008482458.3 | -3.549105e+03 | 3.431246e-06 | 0.81, 3.16 |

The above table shows that the top two Shammas polynomials use the following power equations:

$0.327+0.485*\sqrt{(i)}+0.632*\sqrt{(i-1)}$

And,

$0.399+0.4*\sqrt{(i)}+0.367*(i-1)$

 

# RESULTS FOR THE EXPONENTIAL FUNCTION

## Using Power A+B* i

```
Fitting exp(x) in range (0.000000, 2.000000)
Shammas polynomial power is A+B*i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat              pValue
                ________________    ____________________    ____________    ____________________

    (Intercept)    0.999780693909613    4.16328831339635e-05    24014.2074881671                   0
    x1               1.0706880364457     0.00474128282676011    225.822435734621    7.59155944144525e-236
    x2             -0.636123638783174     0.0324184727128278    -19.6222581001314    7.47424116165567e-48
    x3               2.85208727594114     0.0943476522606717    30.2295521679878    4.00282977370468e-75
    x4              -4.52539625351136        0.1445006597993    -31.3174781332956    1.37459233047969e-77
    x5               4.97487977749445      0.121673716302319     40.887053742433    5.28970522500908e-97
    x6              -2.71447404350022     0.0533026816034846    -50.9256563054938    7.6430805457975e-114
    x7              0.696865519418789     0.00949744571037243    73.3739934578118    6.26206080146147e-143


Number of observations: 201, Error degrees of freedom: 193
Root Mean Squared Error: 5.12e-05
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 3.55e+10, p-value = 0
                    SumSq            DF            MeanSq                F            pValue
                ________________    ___    ____________________    ________________    ______

    Total          650.160904122342    200        3.25080452061171
    Model           650.16090361709      7        92.8801290881556    35479054544.9804       0
    Residual    5.05252046423266e-07    193    2.61788625089775e-09

A = 0.500000, B = 0.500000
List of powers: [1.000000, 1.500000, 2.000000, 2.500000, 3.000000, 3.500000, 4.000000]
Fitting exp(x) in range (0.000000, 2.000000)
MSS of errors squared = 3.536380e-06
Corrected MSS of errors squared = 5.001196e-06
Particle swarm AICc =  -3.376940e+03
AIC =  -3.393690e+03
AICc =  -3.376940e+03
```

## Using Power A+B/i

```
Fitting exp(x) in range (0.000000, 2.000000)
Shammas polynomial power is A+B/i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat              pValue
                ________________    ____________________    ____________    ____________________

    (Intercept)    1.00032121586938    6.38714746088143e-05    15661.470507702                   0
    x1              1.82582720886019      0.0141518489947211    129.01686624421    3.01611119845128e-189
    x2             -210.734088857839        2.16811515907588    -97.1969076345838    7.15736177979808e-166
    x3              3677.65219683562        37.3489791613927     98.4672748602772    6.13394923804416e-167
    x4             -20231.8599027802        199.216308860638    -101.557247087303    1.76468232344445e-169
    x5              46190.5530541428        441.579795296213     104.602958618517    6.51421884705634e-172
```

| | | | | |
|---|---|---|---|---|
| x6 | -46147.7498060299 | 430.099501367439 | -107.295520360544 | 5.24123184705952e-174 |
| x7 | 16722.0307056071 | 152.541058723516 | 109.623145699521 | 8.90098670838167e-176 |

Number of observations: 201, Error degrees of freedom: 193
Root Mean Squared Error: 7.98e-05
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.46e+10, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 650.16090411677 | 200 | 3.25080452058385 | | |
| Model | 650.160902888281 | 7 | 92.8801289840402 | 14591804024.6949 | 0 |
| Residual | 1.22848859973601e-06 | 193 | 6.36522590536793e-09 | | |

A = 1.514587, B = 2.000000
List of powers: [3.514587, 2.514587, 2.181254, 2.014587, 1.914587, 1.847920, 1.800301]
Fitting exp(x) in range (0.000000, 2.000000)
MSS of errors squared = 5.514290e-06
Corrected MSS of errors squared = 7.798384e-06
Particle swarm AICc =  -3.198356e+03
AIC =  -3.215106e+03
AICc =  -3.198356e+03

## Using Power A+B*sqrt(i)

Fitting exp(x) in range (0.000000, 2.000000)
Shammas polynomial power is A+B*sqrt(i)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 1.00029065753863 | 5.72132800557957e-05 | 17483.5397754353 | 0 |
| x1 | 12.8762142901765 | 0.0792518256153779 | 162.472147363101 | 2.14196356726634e-208 |
| x2 | -108.833602753653 | 0.997654508633522 | -109.089471166448 | 2.24917638080997e-175 |
| x3 | 428.375465838866 | 4.31686342325553 | 99.2330365448091 | 1.41556826428795e-167 |
| x4 | -867.274645138427 | 8.93369313576171 | -97.0790726700375 | 9.0033331121963e-166 |
| x5 | 951.42945006533 | 9.68384813230668 | 98.2491089354476 | 9.33338083409517e-167 |
| x6 | -539.221876832665 | 5.32108242042832 | -101.336877392187 | 2.66322984094264e-169 |
| x7 | 124.367005065466 | 1.17251982801115 | 106.068146648248 | 4.65350241942976e-173 |

Number of observations: 201, Error degrees of freedom: 193
Root Mean Squared Error: 7.24e-05
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.77e+10, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 650.160904122317 | 200 | 3.25080452061159 | | |
| Model | 650.16090311052 | 7 | 92.8801290157886 | 17716860171.3228 | 0 |
| Residual | 1.0117969395651e-06 | 193 | 5.24247118945648e-09 | | |

A = 0.378980, B = 1.000000
List of powers: [1.378980, 1.793194, 2.111031, 2.378980, 2.615048, 2.828470, 3.024732]
Fitting exp(x) in range (0.000000, 2.000000)
MSS of errors squared = 5.004386e-06
Corrected MSS of errors squared = 7.077270e-06

 

```
Particle swarm AICc =  -3.237361e+03
AIC =  -3.254111e+03
AICc =  -3.237361e+03
```

## Using Power A+B*sqrt(i+C)

```
Fitting exp(x) in range (0.000000, 2.000000)
Shammas polynomial power is A+B*sqrt(i+C)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat               pValue
                ________________    ____________________    ________________    ____________________

    (Intercept)    1.00029066797184    5.72131960670156e-05      17483.56562357                     0
    x1              12.8763039566109      0.0792520808653196     162.472755491339     2.1404279547108e-208
    x2             -108.83433282078       0.997657462704541     -109.08987993308     2.2475762060341e-175
    x3              428.377890242773       4.31687528028329      99.2333255953276    1.41478789658756e-167
    x4             -867.278744643648       8.93371609728954     -97.0792820365958    8.99966117811893e-166
    x5              951.433173427758       9.68387157966019      98.2492555380566     9.3307457490081e-167
    x6             -539.223603296144       5.32109462080718     -101.336969500149    2.66277126270269e-169
    x7              124.367323657008       1.17252238426848      106.068187120026    4.65316548929101e-173


Number of observations: 201, Error degrees of freedom: 193
Root Mean Squared Error: 7.24e-05
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.77e+10, p-value = 0
                    SumSq           DF           MeanSq                F            pValue
                ____________________    ___    ____________________    _________________    ______

    Total          650.160904122352    200       3.25080452061176
    Model          650.160903110556      7       92.8801290157937    17716876096.8561       0
    Residual    1.0117960300704e-06    193    5.2424664770487e-09

A = 0.378982, B = 1.000000, C = 0.000000
List of powers: [1.378982, 1.793196, 2.111033, 2.378982, 2.615050, 2.828472, 3.024733]
Fitting exp(x) in range (0.000000, 2.000000)
MSS of errors squared = 5.004386e-06
Corrected MSS of errors squared = 8.667850e-06
Particle swarm AICc =  -3.237361e+03
AIC =  -3.254111e+03
AICc =  -3.237361e+03
```

## Using Power A+B*i+C*(i–1)

```
Fitting exp(x) in range (0.000000, 2.000000)
Shammas polynomial power is A+B*i+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat               pValue
                ________________    ____________________    ________________    ____________________

    (Intercept)    0.999462600157228    9.74236628543664e-05      10258.9306424587                     0
    x1             0.277218761266233      0.00440870555109699     62.8798539737483    1.64073298992383e-130
    x2              1.32620001552364      0.0309878768382778      42.7973824229692    1.88075827612637e-100
    x3             -0.537656511825026     0.0951009880829309     -5.65353234139033     5.59052752724516e-08
    x4             -0.0775914392365459      0.152735969672609    -0.508010257196544        0.612026258311858
```

| | | | | |
|---|---|---|---|---|
| x5 | 1.49603921510072 | 0.133572082364499 | 11.2002387670968 | 9.35273044504598e-23 |
| x6 | -1.17327358805466 | 0.0602293624264345 | -19.4800931105277 | 1.90373706955322e-47 |
| x7 | 0.407959106693379 | 0.0109619687998254 | 37.2158609591989 | 5.3092463852158e-90 |

Number of observations: 201, Error degrees of freedom: 193
Root Mean Squared Error: 0.000109
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 7.8e+09, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 650.16090412234 | 200 | 3.2508045206117 | | |
| Model | 650.160901825526 | 7 | 92.8801288322181 | 7804666064.87668 | 0 |
| Residual | 2.29681381824776e-06 | 193 | 1.19005897318537e-08 | | |

A = 0.500000, B = 0.270000, C = 0.270000
List of powers: [0.770000, 1.310000, 1.850000, 2.390000, 2.930000, 3.470000, 4.010000]
Fitting exp(x) in range (0.000000, 2.000000)
MSS of errors squared = 7.539923e-06
Corrected MSS of errors squared = 1.305953e-05
Particle swarm AICc =  -3.072582e+03
AIC =  -3.089332e+03
AICc =  -3.072582e+03

## Using Power A+B*i–C*(i–1)

Fitting exp(x) in range (0.000000, 2.000000)
Shammas polynomial power is A+B*i-C*(i-1)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 0.998758742363385 | 0.0005233259700926 | 1908.48304773917 | 0 |
| x1 | 14.682618569526 | 0.771755400230409 | 19.0249638229191 | 3.85533382140163e-46 |
| x2 | -113.188220330569 | 5.42500306009331 | -20.8641763104596 | 2.34174180019795e-51 |
| x3 | 383.93241090947 | 15.8746869909851 | 24.1851956594482 | 2.51562594065408e-60 |
| x4 | -684.542780095885 | 24.5875726383357 | -27.8410069251237 | 1.6609907283511e-69 |
| x5 | 679.1182618918 | 21.2034642042006 | 32.0286466094188 | 3.61607791908591e-79 |
| x6 | -356.161282212241 | 9.64416195930494 | -36.9302468908258 | 1.9573846177877e-89 |
| x7 | 77.8790343998807 | 1.80733766884828 | 43.090472656119 | 5.70414970895265e-101 |

Number of observations: 201, Error degrees of freedom: 193
Root Mean Squared Error: 0.000538
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 3.21e+08, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 650.16090412249 | 200 | 3.25080452061245 | | |
| Model | 650.160848341224 | 7 | 92.8801211916034 | 321359924.191905 | 0 |
| Residual | 5.57812659280899e-05 | 193 | 2.89022103254352e-07 | | |

A = 0.750000, B = 0.250000, C = 0.000000
List of powers: [1.000000, 1.250000, 1.500000, 1.750000, 2.000000, 2.250000, 2.500000]
Fitting exp(x) in range (0.000000, 2.000000)
MSS of errors squared = 3.715764e-05

 

```
Corrected MSS of errors squared = 6.435892e-05
Particle swarm AICc =  -2.431410e+03
AIC =  -2.448160e+03
AICc =  -2.431410e+03
```

## Using Power A+B*i+C*sqrt(i–1)

```
Fitting exp(x) in range (0.000000, 2.000000)
Shammas polynomial power is A+B*i+C*sqrt(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat                pValue
                ________________    ____________________    ________________    ____________________

    (Intercept)    0.999285848545161    0.000128091435157514     7801.34789899373                    0
    x1             0.250405026115859     0.00427795941287873     58.5337545190397    8.32788297803459e-125
    x2              1.98679019464899     0.0526988271804045     37.7008427122598     5.89326198834871e-91
    x3             -1.90030160489233      0.201040308129631    -9.45234128703687     1.14684554637951e-17
    x4             0.542741851825429      0.362180095651727     1.49854135647291       0.135626485851821
    x5              2.3833715856099      0.342241713342481     6.96400085872893     5.05807160292163e-11
    x6             -2.38592020200221       0.16409989338133    -14.5394378560495     7.88752018678569e-33
    x7             0.842013061034489     0.0315381690245491     26.6982227274853     1.03019432894259e-66


Number of observations: 201, Error degrees of freedom: 193
Root Mean Squared Error: 0.000143
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 4.55e+09, p-value = 0
                    SumSq            DF            MeanSq                F            pValue
                ____________    ____    ____________________    ________________    ______

    Total           650.16090412234    200         3.2508045206117
    Model           650.160900181868      7        92.8801285974097    4549167203.79928       0
    Residual    3.94047174268053e-06    193    2.04169520346141e-08

A = 0.340000, B = 0.390000, C = 0.288644
List of powers: [0.730000, 1.408644, 1.918204, 2.399946, 2.867288, 3.325428, 3.777030]
Fitting exp(x) in range (0.000000, 2.000000)
MSS of errors squared = 9.875933e-06
Corrected MSS of errors squared = 1.710562e-05
Particle swarm AICc =  -2.964087e+03
AIC =  -2.980837e+03
AICc =  -2.964087e+03
```

## Using Power A+B*sqrt(i)+C*(i–1)

```
Fitting exp(x) in range (0.000000, 2.000000)
Shammas polynomial power is A+B*sqrt(i)+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat                pValue
                ________________    ____________________    ________________    ____________________

    (Intercept)    0.999650943076206    5.17988064365096e-05     19298.7254310867                    0
    x1             0.810484104854736     0.0041187140289154     196.780863921297    2.33824983945606e-224
    x2             0.277580142520167     0.0327871556766048     8.46612451711491     6.32277008144876e-15
    x3              1.16447273241401      0.106322572575455      10.95226257423     5.0815072900439e-22
```

| | | | | |
|---|---|---|---|---|
| x4 | -2.55941274221803 | 0.175948113126927 | -14.54640630543 | 7.51351787996051e-33 |
| x5 | 3.68436557055073 | 0.157158334501888 | 23.4436537026705 | 2.24556422121255e-58 |
| x6 | -2.31545489146781 | 0.0722260429687325 | -32.0584486743957 | 3.10851873008778e-79 |
| x7 | 0.656628793404029 | 0.01340927052959 | 48.9682710148219 | 8.58338404957798e-111 |

Number of observations: 201, Error degrees of freedom: 193
Root Mean Squared Error: 6.36e-05
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 2.29e+10, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 650.160904122342 | 200 | 3.25080452061171 | | |
| Model | 650.160903341124 | 7 | 92.8801290487321 | 22946067648.905 | 0 |
| Residual | 7.8121729529812e-07 | 193 | 4.04775800672601e-09 | | |

A = 0.550000, B = 0.400000, C = 0.400000
List of powers: [0.950000, 1.515685, 2.042820, 2.550000, 3.044427, 3.529796, 4.008301]
Fitting exp(x) in range (0.000000, 2.000000)
MSS of errors squared = 4.397339e-06
Corrected MSS of errors squared = 7.616415e-06
Particle swarm AICc =  -3.289346e+03
AIC =  -3.306096e+03
AICc =  -3.289346e+03

## Using Power A+B*sqrt(i)+C*sqrt(i–1)

Fitting exp(x) in range (0.000000, 2.000000)
Shammas polynomial power is A+B*sqrt(i)+C*sqrt(i-1)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 1.00006325696842 | 1.32242401118116e-05 | 75623.4950751676 | 0 |
| x1 | 1.10423006664913 | 0.000677484225800455 | 1629.89782580468 | 0 |
| x2 | -1.30391259739498 | 0.0217426295262384 | -59.9703267639022 | 9.85414477185988e-127 |
| x3 | 9.72143991206299 | 0.121378652812007 | 80.091842237858 | 4.85705829148956e-150 |
| x4 | -23.688047953369 | 0.283770176686784 | -83.4761715622958 | 2.06159459155164e-153 |
| x5 | 29.9185810440422 | 0.329705305268803 | 90.7434019590617 | 3.09455874949824e-160 |
| x6 | -18.8568276174217 | 0.188978434293906 | -99.7829603567087 | 4.97189233380476e-168 |
| x7 | 4.82275033276748 | 0.0427417096268603 | 112.83475497051 | 3.68209994262014e-178 |

Number of observations: 201, Error degrees of freedom: 193
Root Mean Squared Error: 1.89e-05
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 2.6e+11, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 650.160904122342 | 200 | 3.25080452061171 | | |
| Model | 650.160904053271 | 7 | 92.8801291504673 | 259529349574.991 | 0 |
| Residual | 6.90706656314433e-08 | 193 | 3.57879096535975e-10 | | |

A = 0.322041, B = 0.700000, C = 0.700000
List of powers: [1.022041, 2.011990, 2.524426, 2.934476, 3.287288, 3.601931, 3.888709]
Fitting exp(x) in range (0.000000, 2.000000)

 

```
MSS of errors squared = 1.307532e-06
Corrected MSS of errors squared = 2.264711e-06
Particle swarm AICc =  -3.776915e+03
AIC =  -3.793665e+03
AICc =  -3.776915e+03
```

## Comments

The following table shows the summary results for the seven models that fit the exponential function.

| *Power* | *A* | *B* | *C* | *F* | *AICc* | *MSSE* | *Pwr Rng* |
|---|---|---|---|---|---|---|---|
| A+B*i | 0.500 | 0.500 | | 35479054544.9 | -3.376940e+03 | 3.536380e-06 | 1, 4 |
| A+B/i | 1.514 | 2.000 | | 14591804024.6 | -3.198356e+03 | 5.514290e-06 | 1.80, 3.51 |
| A+B*√(i) | 0.378 | 1.000 | | 17716860171.3 | -3.237361e+03 | 5.004386e-06 | 1.37, 3.02 |
| A+B*√(i+C) | 0.378 | 1.000 | 0.000 | 17716876096.8 | -3.237361e+03 | 5.004386e-06 | 1.37, 3.02 |
| A+B*i+C*(i-1) | 0.500 | 0.270 | 0.270 | 7804666064.8 | -3.072582e+03 | 7.539923e-06 | 0.77, 4.01 |
| A+B*i-C*(i-1) | 0.750 | 0.250 | 0.000 | 321359924.1 | -2.431410e+03 | 3.715764e-05 | 1, 2.5 |
| A+B*i+C*√(i-1) | 0.340 | 0.399 | 0.288 | 4549167203.7 | -2.964087e+03 | 9.875933e-06 | 0.73, 3.77 |
| A+B*√(i)+C*(i-1) | 0.550 | 0.400 | 0.400 | 22946067648.9 | -3.289346e+03 | 4.397339e-06 | 0.95, 4.00 |
| A+B*√(i)+C*√(i-1) | 0.322 | 0.700 | 0.700 | 259529349574.9 | -3.776915e+03 | 1.307532e-06 | 1.02, 3.88 |

The above table shows that the top two Shammas polynomials use the following power equations:

$0.322+0.7*\sqrt{(i)}+0.7*(i–1)$

And,

$0.5+0.5*i$

 

# RESULTS FOR THE NATURAL LOGRITHM

## Using Power A+B* i

```
Fitting ln(x) in range (1.000000, 10.000000)
Shammas polynomial power is A+B*i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                       Estimate         SE    tStat    pValue
                    ________________    __    _____    ______

    (Intercept)     -24.737547363287    0     -Inf       0
    x1               62.1126102399041   0      Inf       0
    x2              -82.5976850963343   0     -Inf       0
    x3               81.2902166360491   0      Inf       0
    x4              -53.9547481796169   0     -Inf       0
    x5               22.8991585078452   0      Inf       0
    x6              -5.61934311773935   0     -Inf       0
    x7              0.607338374338497   0      Inf       0


Number of observations: 901, Error degrees of freedom: 893
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: Inf, p-value = 0
                     SumSq          DF          MeanSq            F     pValue
                  _______________    ___    _______________    ___    ______

    Total        312.393980393318    900    0.347104422659242
    Model        312.393980393318      7     44.6277114847597    Inf       0
    Residual                    0    893                    0

A = 0.000000, B = 0.100000
List of powers: [0.100000, 0.200000, 0.300000, 0.400000, 0.500000, 0.600000, 0.700000]
Fitting ln(x) in range (1.000000, 10.000000)
MSS of errors squared = 5.567860e-12
Corrected MSS of errors squared = 7.874144e-12
Particle swarm AICc =  -1.000000e+99
AIC =  -Inf
AICc =  -Inf
```

## Using Power A+B/i

```
Fitting ln(x) in range (1.000000, 10.000000)
Shammas polynomial power is A+B/i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                       Estimate         SE    tStat    pValue
                    ________________    __    _____    ______

    (Intercept)     -33.9827889286955   0     -Inf       0
    x1               51.9359269274838   0      Inf       0
    x2              -2480.38633058639   0     -Inf       0
    x3               10071.226930391    0      Inf       0
    x4                             0    0      NaN      NaN
    x5              -19376.0058904542   0     -Inf       0
    x6                             0    0      NaN      NaN
```

 

```
    x7              11767.2121526734    0      Inf        0


Number of observations: 901, Error degrees of freedom: 895
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: Inf, p-value = 0
                    SumSq          DF          MeanSq           F     pValue
                ________________    ___    ________________    ___    ______

    Total       312.39398039245    900    0.347104422658278
    Model       312.39398039245      5       62.47879607849    Inf       0
    Residual                  0    895                    0

A = 0.104030, B = 0.100000
List of powers: [0.204030, 0.154030, 0.137364, 0.129030, 0.124030, 0.120697, 0.118316]
Fitting ln(x) in range (1.000000, 10.000000)
MSS of errors squared = 1.187203e-10
Corrected MSS of errors squared = 1.678959e-10
Particle swarm AICc =  -1.000000e+99
AIC =  -Inf
AICc =  -Inf
```

## Using Power A+B*sqrt(i)

```
Fitting ln(x) in range (1.000000, 10.000000)
Shammas polynomial power is A+B*sqrt(i)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                     Estimate          SE    tStat    pValue
                  _____________________    __    _____    ______

    (Intercept)    -33.9818934854884    0     -Inf        0
    x1              352.041415463287    0      Inf        0
    x2             -1085.09961833925    0     -Inf        0
    x3              1249.05825386279    0      Inf        0
    x4                             0    0      NaN      NaN
    x5             -1210.3243658041     0     -Inf        0
    x6              975.538545871408    0      Inf        0
    x7             -247.232337567554    0     -Inf        0


Number of observations: 901, Error degrees of freedom: 894
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: Inf, p-value = 0
                    SumSq          DF          MeanSq           F     pValue
                ________________    ___    ________________    ___    ______

    Total       312.393980393285    900    0.347104422659206
    Model       312.393980393285      6     52.0656633988809    Inf       0
    Residual                  0    894                    0

A = 0.000000, B = 0.100000
List of powers: [0.100000, 0.141421, 0.173205, 0.200000, 0.223607, 0.244949, 0.264575]
Fitting ln(x) in range (1.000000, 10.000000)
MSS of errors squared = 5.488497e-12
Corrected MSS of errors squared = 7.761907e-12
Particle swarm AICc =  -1.000000e+99
AIC =  -Inf
AICc =  -Inf
```

 

## Using Power A+B*sqrt(i+C)

```
Fitting ln(x) in range (1.000000, 10.000000)
Shammas polynomial power is A+B*sqrt(i+C)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5

Estimated Coefficients:
                      Estimate          SE    tStat    pValue
                    ____________________    __    _____    ______

    (Intercept)    -27.6709623693291    0     -Inf       0
    x1              442.782553542175    0      Inf       0
    x2             -1378.43499059438    0     -Inf       0
    x3              1819.93276689515    0      Inf       0
    x4             -1123.80897236893    0     -Inf       0
    x5              267.199604958649    0      Inf       0


Number of observations: 901, Error degrees of freedom: 895
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: Inf, p-value = 0
                     SumSq          DF          MeanSq           F     pValue
                  ______________    ___    ________________    ___    ______

    Total        312.393980393252    900    0.347104422659168
    Model        312.393980393252      5     62.4787960786503    Inf       0
    Residual                    0    895                    0

A = 0.000000, B = 0.100000, C = 0.488877
List of powers: [0.122020, 0.157762, 0.186785, 0.211870, 0.234284]
Fitting ln(x) in range (1.000000, 10.000000)
MSS of errors squared = 3.338300e-10
Corrected MSS of errors squared = 5.782105e-10
Particle swarm AICc =  -1.000000e+99
AIC =  -Inf
AICc =  -Inf
```

## Using Power A+B*i+C*(i–1)

```
Fitting ln(x) in range (1.000000, 10.000000)
Shammas polynomial power is A+B*i+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                      Estimate          SE    tStat    pValue
                    ____________________    __    _____    ______

    (Intercept)    -10.3205051319854    0     -Inf       0
    x1              282.992254258008    0      Inf       0
    x2               -932.8899669656    0     -Inf       0
    x3              1403.34117142263    0      Inf       0
    x4             -1196.91999722651    0     -Inf       0
    x5              600.004732277997    0      Inf       0
    x6             -165.790461867275    0     -Inf       0
    x7              19.5827733331983    0      Inf       0


Number of observations: 901, Error degrees of freedom: 893
```

 

```
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: Inf, p-value = 0
                    SumSq          DF           MeanSq            F      pValue
                ___________________    ___    ___________________    ___    ______

    Total       312.39398039396    900    0.347104422659956
    Model       312.39398039396      7     44.6277114848514    Inf       0
    Residual                  0    893                    0

A = 0.272117, B = 0.100000, C = 0.000000
List of powers: [0.372117, 0.472117, 0.572117, 0.672117, 0.772117, 0.872117, 0.972117]
Fitting ln(x) in range (1.000000, 10.000000)
MSS of errors squared = 4.796230e-10
Corrected MSS of errors squared = 8.307315e-10
Particle swarm AICc =  -1.000000e+99
AIC =  -Inf
AICc =  -Inf
```

## Using Power A+B*i–C*(i–1)

```
Fitting ln(x) in range (1.000000, 10.000000)
Shammas polynomial power is A+B*i-C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                      Estimate          SE    tStat    pValue
                  ____________________    __    _____    ______

    (Intercept)    -19.4636062541967    0     -Inf        0
    x1              7039.75622437329    0      Inf        0
    x2             -13878.8853948417    0     -Inf        0
    x3                             0    0      NaN      NaN
    x4              12061.9479110746    0      Inf        0
    x5                             0    0      NaN      NaN
    x6             -8669.47700378932    0     -Inf        0
    x7              3466.12186974089    0      Inf        0


Number of observations: 901, Error degrees of freedom: 895
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: Inf, p-value = 0
                    SumSq          DF           MeanSq            F      pValue
                ___________________    ___    ___________________    ___    ______

    Total       312.393980393914    900    0.347104422659905
    Model       312.393980393914      5     62.4787960787829    Inf       0
    Residual                  0    895                    0

A = 0.096149, B = 0.111619, C = 0.099128
List of powers: [0.207767, 0.220258, 0.232749, 0.245239, 0.257730, 0.270220, 0.282711]
Fitting ln(x) in range (1.000000, 10.000000)
MSS of errors squared = 1.597884e-09
Corrected MSS of errors squared = 2.767617e-09
Particle swarm AICc =  -1.000000e+99
AIC =  -Inf
AICc =  -Inf
```

## Using Power A+B*i+C*sqrt(i–1)

```
Fitting ln(x) in range (1.000000, 10.000000)
Shammas polynomial power is A+B*i+C*sqrt(i-1)
```

```
Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                       Estimate         SE    tStat    pValue
                    ________________    __    _____    ______

    (Intercept)    -21.6103054832284    0     -Inf       0
    x1              54.0530825999491    0      Inf       0
    x2             -71.0973037023735    0     -Inf       0
    x3              68.9462684134273    0      Inf       0
    x4             -45.0394263598594    0     -Inf       0
    x5              18.8022617284005    0      Inf       0
    x6             -4.53647546530277    0     -Inf       0
    x7             0.481898271808515    0      Inf       0


Number of observations: 901, Error degrees of freedom: 893
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: Inf, p-value = 0
                     SumSq          DF          MeanSq           F     pValue
                  _____________    ___    _________________    ___    ______

    Total        312.393980393309    900    0.347104422659233
    Model        312.393980393309      7     44.6277114847585    Inf       0
    Residual                    0    893                    0

A = 0.000609, B = 0.113360, C = 0.000000
List of powers: [0.113969, 0.227330, 0.340690, 0.454050, 0.567411, 0.680771, 0.794131]
Fitting ln(x) in range (1.000000, 10.000000)
MSS of errors squared = 1.353255e-11
Corrected MSS of errors squared = 2.343906e-11
Particle swarm AICc =  -1.000000e+99
AIC =  -Inf
AICc =  -Inf
```

## Using Power A+B*sqrt(i)+C*(i–1)

```
Fitting ln(x) in range (1.000000, 10.000000)
Shammas polynomial power is A+B*sqrt(i)+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                       Estimate         SE    tStat    pValue
                    ________________    __    _____    ______

    (Intercept)     -9.933565579569    0     -Inf       0
    x1              142.156705604533    0      Inf       0
    x2             -654.270874527231    0     -Inf       0
    x3              1483.92869335806    0      Inf       0
    x4             -1888.55909368603    0     -Inf       0
    x5              1381.10379942271    0      Inf       0
    x6             -544.296280715919    0     -Inf       0
    x7              89.8706162525411    0      Inf       0


Number of observations: 901, Error degrees of freedom: 893
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: Inf, p-value = 0
```

 

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 312.393980393003 | 900 | 0.347104422658892 | | |
| Model | 312.393980393003 | 7 | 44.6277114847147 | Inf | 0 |
| Residual | 0 | 893 | 0 | | |

```
A = 0.000000, B = 0.361168, C = 0.000000
List of powers: [0.361168, 0.510768, 0.625561, 0.722335, 0.807595, 0.884676, 0.955559]
Fitting ln(x) in range (1.000000, 10.000000)
MSS of errors squared = 6.161463e-10
Corrected MSS of errors squared = 1.067197e-09
Particle swarm AICc =  -1.000000e+99
AIC =  -Inf
AICc =  -Inf
```

## Using Power A+B*sqrt(i)+C*sqrt(i–1)

```
Fitting ln(x) in range (1.000000, 10.000000)
Shammas polynomial power is A+B*sqrt(i)+C*sqrt(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
```

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | -14.4269905667126 | 0 | -Inf | 0 |
| x1 | 38.320392430738 | 0 | Inf | 0 |
| x2 | -171.630865585907 | 0 | -Inf | 0 |
| x3 | 483.228925715234 | 0 | Inf | 0 |
| x4 | -704.953200045666 | 0 | -Inf | 0 |
| x5 | 569.191527391733 | 0 | Inf | 0 |
| x6 | -242.368354096896 | 0 | -Inf | 0 |
| x7 | 42.6385647802004 | 0 | Inf | 0 |

```
Number of observations: 901, Error degrees of freedom: 893
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: Inf, p-value = 0
```

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 312.393980393395 | 900 | 0.347104422659328 | | |
| Model | 312.393980393395 | 7 | 44.6277114847707 | Inf | 0 |
| Residual | 0 | 893 | 0 | | |

```
A = 0.066204, B = 0.110981, C = 0.187289
List of powers: [0.177184, 0.410443, 0.523295, 0.612560, 0.688943, 0.756841, 0.818594]
Fitting ln(x) in range (1.000000, 10.000000)
MSS of errors squared = 1.087231e-10
Corrected MSS of errors squared = 1.883139e-10
Particle swarm AICc =  -1.000000e+99
AIC =  -Inf
AICc =  -Inf
```

## Comments

The following table shows the summary results for the seven models that fit the natural logarithm function.

 

| *Power* | *A* | *B* | *C* | *F* | *AICc* | *MSSE* | *Pwr Rng* |
|---|---|---|---|---|---|---|---|
| A+B*i | 0.000 | 0.100 | | Inifnity | Infinity | 5.567860e-12 | 0.1, 0.7 |
| A+B/i | 0.100 | 0.100 | | Inifnity | Infinity | 1.187203e-10 | 0.11, 0.20 |
| A+B*√(i) | 0.000 | 0.100 | | Inifnity | Infinity | 5.488497e-12 | 0.1, 0.26 |
| A+B*√(i+C) | 0.000 | 0.100 | 0.488 | Inifnity | Infinity | 3.338300e-10 | 0.12, 0.23 |
| A+B*i+C*(i-1) | 0.272 | 0.100 | 0.000 | Inifnity | Infinity | 4.796230e-10 | 0.37, 0.97 |
| A+B*i-C*(i-1) | 0.096 | 0.111 | 0.099 | Inifnity | Infinity | 1.597884e-09 | 0.20, 0.28 |
| A+B*i+C*√(i-1) | 0.0006 | 0.113 | 0.000 | Inifnity | Infinity | 1.353255e-11 | 0.11, 0.79 |
| A+B*√(i)+C*(i-1) | 0.000 | 0.361 | 0.000 | Inifnity | Infinity | 6.161463e-10 | 0.36, 0.95 |
| A+B*√(i)+C*√(i-1) | 0.066 | 0.110 | 0.187 | Inifnity | Infinity | 1.087231e-10 | 0.17, 0.81 |

The above table shows that the top two Shammas polynomials use the following power equations:

$0.1*\sqrt{(i)}$

And,

0.1* i

 

# RESULTS FOR THE COMMON LOGARITHM

## Using Power A+B* i

```
Fitting log(x) in range (1.000000, 10.000000)
Shammas polynomial power is A+B*i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                       Estimate         SE    tStat    pValue
                    ________________    __    _____    ______

    (Intercept)    -4.07228681846556    0     -Inf       0
    x1              45.6808689248787    0      Inf       0
    x2             -122.268784148046    0     -Inf       0
    x3              159.093640227107    0      Inf       0
    x4             -121.16235858931     0     -Inf       0
    x5              55.2236197058703    0      Inf       0
    x6             -14.0293519808898    0     -Inf       0
    x7              1.53465277435084    0      Inf       0


Number of observations: 901, Error degrees of freedom: 893
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: Inf, p-value = 0
                     SumSq          DF           MeanSq            F     pValue
                  _______________    ___    _________________    ___    ______

    Total        58.9211587781808    900    0.0654679541979787
    Model        58.9211587781808      7      8.41730839688297    Inf       0
    Residual                   0    893                     0

A = 0.225051, B = 0.136184
List of powers: [0.361235, 0.497420, 0.633604, 0.769788, 0.905972, 1.042156, 1.178340]
Fitting log(x) in range (1.000000, 10.000000)
MSS of errors squared = 4.552204e-10
Corrected MSS of errors squared = 6.437789e-10
Particle swarm AICc =  -1.000000e+99
AIC =  -Inf
AICc =  -Inf
```

## Using Power A+B/i

```
Fitting log(x) in range (1.000000, 10.000000)
Shammas polynomial power is A+B/i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                         Estimate           SE    tStat    pValue
                    ____________________    __    _____    ______

    (Intercept)       -7.95319268741776     0     -Inf       0
    x1             6.94365938702297e-05     0      Inf       0
    x2               -0.134143021206793     0     -Inf       0
    x3                7.71622438856209     0      Inf       0
    x4               -89.2743799949135     0     -Inf       0
    x5                348.483829330277     0      Inf       0
    x6               -529.360050306761     0     -Inf       0
```

 

```
    x7                270.521642857337    0      Inf       0


Number of observations: 901, Error degrees of freedom: 893
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: Inf, p-value = 0
                    SumSq          DF           MeanSq           F     pValue
                ________________    ___    ________________    ___    ______

    Total       58.9211587780752    900    0.0654679541978613
    Model       58.9211587780752      7      8.41730839686789    Inf       0
    Residual                   0    893                     0

A = 0.000000, B = 1.435601
List of powers: [1.435601, 0.717800, 0.478534, 0.358900, 0.287120, 0.239267, 0.205086]
Fitting log(x) in range (1.000000, 10.000000)
MSS of errors squared = 1.181949e-11
Corrected MSS of errors squared = 1.671528e-11
Particle swarm AICc =  -1.000000e+99
AIC =  -Inf
AICc =  -Inf
```

## Using Power A+B*sqrt(i)

```
Fitting log(x) in range (1.000000, 10.000000)
Shammas polynomial power is A+B*sqrt(i)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5

Estimated Coefficients:
                      Estimate          SE    tStat    pValue
                  _________________    __    _____    ______

    (Intercept)    -11.8579081329183    0     -Inf       0
    x1              97.6019340230752    0      Inf       0
    x2               -272.4981435193    0     -Inf       0
    x3               350.59059427593    0      Inf       0
    x4             -214.805230918033    0     -Inf       0
    x5              50.9687543028367    0      Inf       0


Number of observations: 901, Error degrees of freedom: 895
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: Inf, p-value = 0
                    SumSq          DF           MeanSq           F     pValue
                ________________    ___    ________________    ___    ______

    Total       58.9211587781783    900    0.0654679541979759
    Model       58.9211587781783      5      11.7842317556357    Inf       0
    Residual                   0    895                     0

A = 0.000000, B = 0.113356
List of powers: [0.113356, 0.160310, 0.196339, 0.226713, 0.253473]
Fitting log(x) in range (1.000000, 10.000000)
MSS of errors squared = 1.664764e-10
Corrected MSS of errors squared = 2.354331e-10
Particle swarm AICc =  -1.000000e+99
AIC =  -Inf
AICc =  -Inf
```

## Using Power A+B*sqrt(i+C)

 

```
Fitting log(x) in range (1.000000, 10.000000)
Shammas polynomial power is A+B*sqrt(i+C)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                       Estimate          SE    tStat    pValue
                    _________________    __    _____    ______

    (Intercept)    -14.7581488203393     0     -Inf       0
    x1              152.889643958496     0      Inf       0
    x2             -471.252775778988     0     -Inf       0
    x3              542.459106124598     0      Inf       0
    x4                             0     0      NaN      NaN
    x5             -525.637191996206     0     -Inf       0
    x6              423.671006130756     0      Inf       0
    x7             -107.371639617843     0     -Inf       0


Number of observations: 901, Error degrees of freedom: 894
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: Inf, p-value = 0
                     SumSq          DF           MeanSq           F     pValue
                  _______________    ___    _________________    ___    ______

    Total        58.9211587781858    900    0.0654679541979842
    Model        58.9211587781858      6      9.82019312969764    Inf       0
    Residual                    0    894                     0

A = 0.000000, B = 0.100000, C = 0.000000
List of powers: [0.100000, 0.141421, 0.173205, 0.200000, 0.223607, 0.244949, 0.264575]
Fitting log(x) in range (1.000000, 10.000000)
MSS of errors squared = 2.383659e-12
Corrected MSS of errors squared = 4.128619e-12
Particle swarm AICc =  -1.000000e+99
AIC =  -Inf
AICc =  -Inf
```

## Using Power A+B*i+C*(i–1)

```
Fitting log(x) in range (1.000000, 10.000000)
Shammas polynomial power is A+B*i+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                       Estimate          SE    tStat    pValue
                    _________________    __    _____    ______

    (Intercept)    -8.11938384983018     0     -Inf       0
    x1              11.7914030054079     0      Inf       0
    x2             -6.60873454808762     0     -Inf       0
    x3              4.75972909033536     0      Inf       0
    x4             -2.56551359077751     0     -Inf       0
    x5              0.917260611400924    0      Inf       0
    x6             -0.192743402725325    0     -Inf       0
    x7             0.0179826949328977    0      Inf       0


Number of observations: 901, Error degrees of freedom: 893
```

 

```
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: Inf, p-value = 0
                    SumSq          DF           MeanSq           F     pValue
                ___________________    ___    ___________________    ___    ______

    Total       58.9211587781896    900    0.0654679541979885
    Model       58.9211587781896      7      8.41730839688424    Inf       0
    Residual                   0    893                     0

A = 0.000000, B = 0.100000, C = 0.091445
List of powers: [0.100000, 0.291445, 0.482891, 0.674336, 0.865781, 1.057227, 1.248672]
Fitting log(x) in range (1.000000, 10.000000)
MSS of errors squared = 5.091279e-11
Corrected MSS of errors squared = 8.818354e-11
Particle swarm AICc =  -1.000000e+99
AIC =  -Inf
AICc =  -Inf
```

## Using Power A+B*i–C*(i–1)

```
Fitting log(x) in range (1.000000, 10.000000)
Shammas polynomial power is A+B*i-C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4

Estimated Coefficients:
                        Estimate          SE    tStat    pValue
                    ____________________    ___    _____    ______

    (Intercept)    0.0659122626082603    0      Inf       0
    x1             -0.169812004124193    0     -Inf       0
    x2               5.28490581082478    0      Inf       0
    x3              -5.41125764533236    0     -Inf       0
    x4               0.23025163498997    0      Inf       0


Number of observations: 901, Error degrees of freedom: 896
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: Inf, p-value = 0
                    SumSq          DF           MeanSq           F     pValue
                ___________________    ___    ___________________    ___    ______

    Total       58.9211587781904    900    0.0654679541979894
    Model       58.9211587781904      4      14.7302896945476    Inf       0
    Residual                   0    896                     0

A = 0.000000, B = 0.136181, C = 0.227637
List of powers: [0.136181, 0.044726, -0.046729, -0.138184]
Fitting log(x) in range (1.000000, 10.000000)
MSS of errors squared = 3.338336e-10
Corrected MSS of errors squared = 5.782167e-10
Particle swarm AICc =  -1.000000e+99
AIC =  -Inf
AICc =  -Inf
```

## Using Power A+B*i+C*sqrt(i–1)

```
Fitting log(x) in range (1.000000, 10.000000)
Shammas polynomial power is A+B*i+C*sqrt(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7
```

 

```
Estimated Coefficients:
                       Estimate         SE    tStat    pValue
                    ________________    __    _____    ______

    (Intercept)     -7.07639371882524    0     -Inf       0
    x1               8.65570326527023    0      Inf       0
    x2              -4.89026922934791    0     -Inf       0
    x3               7.10042396702222    0      Inf       0
    x4              -6.19974390753639    0     -Inf       0
    x5               3.21968680126287    0      Inf       0
    x6              -0.921865476429823    0     -Inf       0
    x7               0.112458337617327    0      Inf       0


Number of observations: 901, Error degrees of freedom: 893
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: Inf, p-value = 0
                      SumSq          DF           MeanSq           F     pValue
                  _____________    ___    _________________    ___    ______

    Total       58.9211587781908    900    0.0654679541979897
    Model       58.9211587781908      7       8.4173083968844    Inf       0
    Residual                   0    893                     0

A = 0.000000, B = 0.100000, C = 0.255905
List of powers: [0.100000, 0.455905, 0.661904, 0.843241, 1.011810, 1.172221, 1.326837]
Fitting log(x) in range (1.000000, 10.000000)
MSS of errors squared = 1.860714e-10
Corrected MSS of errors squared = 3.222851e-10
Particle swarm AICc =  -1.000000e+99
AIC =  -Inf
AICc =  -Inf
```

## Using Power A+B*sqrt(i)+C*(i–1)

```
Fitting log(x) in range (1.000000, 10.000000)
Shammas polynomial power is A+B*sqrt(i)+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                       Estimate         SE    tStat    pValue
                    ________________    __    _____    ______

    (Intercept)     -7.84129276888551    0     -Inf       0
    x1               10.8478595306292    0      Inf       0
    x2              -5.54318624201128    0     -Inf       0
    x3               4.21413969424651    0      Inf       0
    x4              -2.40007780177822    0     -Inf       0
    x5               0.90108052158313    0      Inf       0
    x6              -0.197694261517999    0     -Inf       0
    x7               0.019171343081009    0      Inf       0


Number of observations: 901, Error degrees of freedom: 893
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: Inf, p-value = 0
                      SumSq          DF           MeanSq           F     pValue
                  _____________    ___    _________________    ___    ______
```

 

```
    Total       58.9211587781887    900    0.0654679541979874
    Model       58.9211587781887      7       8.4173083968841    Inf       0
    Residual                   0    893                     0

A = 0.000000, B = 0.100000, C = 0.173328
List of powers: [0.100000, 0.314749, 0.519861, 0.719984, 0.916919, 1.111589, 1.304543]
Fitting log(x) in range (1.000000, 10.000000)
MSS of errors squared = 7.327415e-11
Corrected MSS of errors squared = 1.269146e-10
Particle swarm AICc =  -1.000000e+99
AIC =  -Inf
AICc =  -Inf
```

## Using Power A+B*sqrt(i)+C*sqrt(i–1)

```
Fitting log(x) in range (1.000000, 10.000000)
Shammas polynomial power is A+B*sqrt(i)+C*sqrt(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6

Estimated Coefficients:
                      Estimate          SE    tStat    pValue
                  _________________    __    _____    ______

    (Intercept)     -6.0517389803722    0     -Inf       0
    x1               37.2731062963977    0      Inf       0
    x2              -171.626359103842    0     -Inf       0
    x3               362.951719559082    0      Inf       0
    x4              -377.484325647549    0     -Inf       0
    x5               195.121401459373    0      Inf       0
    x6              -40.1838035080967    0     -Inf       0


Number of observations: 901, Error degrees of freedom: 894
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: Inf, p-value = 0
                     SumSq          DF           MeanSq           F     pValue
                 _____________    ___    _________________    ___    ______

    Total       58.9211587781907    900    0.0654679541979897
    Model       58.9211587781907      6      9.82019312969845    Inf       0
    Residual                   0    894                     0

A = 0.074327, B = 0.145358, C = 0.076836
List of powers: [0.219684, 0.356729, 0.434755, 0.498125, 0.553027, 0.602188]
Fitting log(x) in range (1.000000, 10.000000)
MSS of errors squared = 3.737431e-10
Corrected MSS of errors squared = 6.473421e-10
Particle swarm AICc =  -1.000000e+99
AIC =  -Inf
AICc =  -Inf
```

## Comments

The following table shows the summary results for the seven models that fit the common logarithm function.

 

| Power | A | B | C | F | AICc | MSSE | Pwr Rng |
|---|---|---|---|---|---|---|---|
| A+B*i | 0.225 | 0.136 | | Inifnity | Infinity | 4.552204e-10 | 0.36, 1.17 |
| A+B/i | 0.000 | 1.435 | | Inifnity | Infinity | 1.181949e-11 | 0.20, 1.43 |
| A+B*√(i) | 0.000 | 0.113 | | Inifnity | Infinity | 1.664764e-10 | 0.11, 0.25 |
| A+B*√(i+C) | 0.000 | 0.100 | 0.000 | Inifnity | Infinity | 2.383659e-12 | 0.1, 0.26 |
| A+B*i+C*(i-1) | 0.000 | 0.100 | 0.091 | Inifnity | Infinity | 5.091279e-11 | 0.10, 1.24 |
| A+B*i-C*(i-1) | 0.000 | 0.136 | 0.227 | Inifnity | Infinity | 3.338336e-10 | -0.13, 0.13 |
| A+B*i+C*√(i-1) | 0.000 | 0.100 | 0.255 | Inifnity | Infinity | 1.860714e-10 | 0.1, 1.32 |
| A+B*√(i)+C*(i-1) | 0.000 | 0.100 | 0.173 | Inifnity | Infinity | 7.327415e-11 | 0.1, 1.30 |
| A+B*√(i)+C*√(i-1) | 0.074 | 0.145 | 0.076 | Inifnity | Infinity | 3.737431e-10 | 0.21, 0.60 |

The above table shows that the top two Shammas polynomials use the following power equations:

1.43/i

And,

0.1*i+0.0.91* (i–1)

Also notice that not all of the fitted Shammas polynomials have the maximum order of 7.

 

# RESULTS FOR THE COMMON EXPONENT (10^X)

## Using Power A+B* i

```
Fitting 10^x in range (0.000000, 1.000000)
Shammas polynomial power is A+B*i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat               pValue
                ________________    ____________________    ________________    ____________________

    (Intercept)    0.999678462872992    0.000133120162173317     7509.59468913098    1.04380895521771e-270
    x1              2.62492506892167     0.0323911282033451     81.0383958361346     4.5893741016715e-88
    x2             -4.26688930378147      0.322233780090729    -13.2415952870617     4.04704248842258e-23
    x3              25.4088990558761       1.35175592545094     18.7969577772701     1.95759018052687e-33
    x4             -62.5282852547846       2.96674759258139    -21.0763751561279     3.46320966625117e-37
    x5              96.2772062069883       3.56635770089826     26.9959477656262     8.71611664380197e-46
    x6             -75.026252632888        2.22490619963948    -33.7210857001725     6.09909820277804e-54
    x7              26.5103047418086       0.563577147944419     47.0393535978203     1.22856851505174e-66


Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.000143
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 4.53e+09, p-value = 0
                    SumSq            DF            MeanSq                F           pValue
                ____________________    ___    ____________________    ________________    ______

    Total           645.004627337437    100        6.45004627337437
    Model           645.004625444997      7        92.1435179207138    4528199721.3302       0
    Residual    1.89244019566104e-06     93    2.03488193081832e-08

A = 0.500000, B = 0.500000
List of powers: [1.000000, 1.500000, 2.000000, 2.500000, 3.000000, 3.500000, 4.000000]
Fitting 10^x in range (0.000000, 1.000000)
MSS of errors squared = 1.362039e-05
Corrected MSS of errors squared = 1.926214e-05
Particle swarm AICc =  -1.476878e+03
AIC =  -1.494444e+03
AICc =  -1.476878e+03
```

## Using Power A+B/i

```
Fitting 10^x in range (0.000000, 1.000000)
Shammas polynomial power is A+B/i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat               pValue
                ________________    ____________________    ________________    ____________________

    (Intercept)     1.00025461492583    0.00013448068082622      7437.9056439965    2.54708157154619e-270
    x1               45.7425113159989     0.501622809052083     91.1890577751807     8.99651984788726e-93
    x2              -2808.60375876096      39.2655426673252    -71.5284589991961     4.21208329640699e-83
    x3               39058.1400453254      540.642957953032     72.2438708777531     1.69656136879227e-83
    x4              -191020.594276462      2577.43668348836    -74.1126234061083     1.64254371614105e-84
    x5               405466.66678231       5340.04971962418     75.9293804498221     1.79106047855058e-85
```

 

| | | | | |
|---|---|---|---|---|
| x6 | -385387.962331859 | 4971.84201354067 | -77.5141207790323 | 2.70179487631246e-86 |
| x7 | 134655.610401773 | 1707.35013735067 | 78.8681872897617 | 5.52799270581174e-87 |

Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.000149
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 4.13e+09, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 645.004627345209 | 100 | 6.45004627345209 | | |
| Model | 645.004625267824 | 7 | 92.1435178954035 | 4125066262.00115 | 0 |
| Residual | 2.07738412427716e-06 | 93 | 2.23374637019049e-08 | | |

A = 1.624058, B = 2.000000
List of powers: [3.624058, 2.624058, 2.290725, 2.124058, 2.024058, 1.957392, 1.909773]
Fitting 10^x in range (0.000000, 1.000000)
MSS of errors squared = 1.427043e-05
Corrected MSS of errors squared = 2.018143e-05
Particle swarm AICc =  -1.467461e+03
AIC =  -1.485026e+03
AICc =  -1.467461e+03

## Using Power A+B*sqrt(i)

Fitting 10^x in range (0.000000, 1.000000)
Shammas polynomial power is A+B*sqrt(i)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 1.00023979716084 | 0.000124928993171979 | 8006.46648759823 | 2.69746912898663e-273 |
| x1 | 70.989142642152 | 0.666816532207643 | 106.459781983999 | 5.74689314968483e-99 |
| x2 | -873.516680419914 | 11.1627737263629 | -78.2526549254463 | 1.13338463023205e-86 |
| x3 | 4333.36243264121 | 59.9427914422039 | 72.2916355475268 | 1.59711698902239e-83 |
| x4 | -10549.5983494673 | 148.695781403919 | -70.9475295792708 | 8.8715573053588e-83 |
| x5 | 13546.4065911842 | 189.025855777587 | 71.6643050521261 | 3.54171681549539e-83 |
| x6 | -8828.31754897161 | 119.948302642589 | -73.6010210605265 | 3.09494197047163e-84 |
| x7 | 2309.6738229145 | 30.1715786770461 | 76.5513083566839 | 8.48725996342939e-86 |

Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.00014
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 4.72e+09, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 645.004627338154 | 100 | 6.45004627338154 | | |
| Model | 645.00462552358 | 7 | 92.14351793194 | 4722509958.32032 | 0 |
| Residual | 1.81457471626345e-06 | 93 | 1.95115560888543e-08 | | |

A = 0.479193, B = 1.000000
List of powers: [1.479193, 1.893407, 2.211244, 2.479193, 2.715261, 2.928683, 3.124945]
Fitting 10^x in range (0.000000, 1.000000)
MSS of errors squared = 1.333724e-05
Corrected MSS of errors squared = 1.886171e-05

 

```
Particle swarm AICc =  -1.481122e+03
AIC =  -1.498687e+03
AICc =  -1.481122e+03
```

## Using Power A+B*sqrt(i+C)

```
Fitting 10^x in range (0.000000, 1.000000)
Shammas polynomial power is A+B*sqrt(i+C)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat               pValue
                ________________    ____________________    ________________    ____________________

    (Intercept)     1.00023979335299    0.00012492898897511     8006.46672608768    2.69746165647724e-273
    x1               70.9889507583213     0.666815201102822     106.459706738711     5.74726787454171e-99
    x2              -873.514456480099      11.1627526115241    -78.2526037151723     1.13345259042569e-86
    x3               4333.35275138876      59.9426828950057      72.29160494833     1.59717877230663e-83
    x4              -10549.5777193578      148.695521785197    -70.9475147112875     8.87172710447038e-83
    x5               13546.383418183      189.025535967217     71.6643037083223     3.54172288312268e-83
    x6              -8828.30435812136      119.948105256797    -73.6010322065597     3.09489911021719e-84
    x7               2309.67082415437       30.171530246341     76.551331844843     8.48702152567514e-86


Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.00014
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 4.72e+09, p-value = 0
                    SumSq            DF            MeanSq                F            pValue
                ____________________    ___    ____________________    ________________    ______

    Total           645.004627337327    100        6.45004627337327
    Model           645.004625522753      7        92.1435179318218    4722512325.31514       0
    Residual    1.81457380676875e-06     93    1.95115463093414e-08

A = 0.479193, B = 1.000000, C = 0.000000
List of powers: [1.479193, 1.893406, 2.211243, 2.479193, 2.715261, 2.928682, 3.124944]
Fitting 10^x in range (0.000000, 1.000000)
MSS of errors squared = 1.333724e-05
Corrected MSS of errors squared = 2.310078e-05
Particle swarm AICc =  -1.481122e+03
AIC =  -1.498687e+03
AICc =  -1.481122e+03
```

## Using Power A+B*i+C*(i–1)

```
Fitting 10^x in range (0.000000, 1.000000)
Shammas polynomial power is A+B*i+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat               pValue
                ________________    ____________________    ________________    ____________________

    (Intercept)     0.99957590338363    0.000220610408152252     4530.95532416486    2.66224610013822e-250
    x1               0.60041536024524     0.0187028253558537     32.1029228911299     4.12984868092494e-52
    x2               2.79851940587066      0.201413273575976     13.8944139886342     2.03850391985499e-24
    x3               4.15012529307046      0.925358071673569     4.48488581891839     2.08151570364401e-05
    x4              -19.3172516505318       2.19892140021529    -8.78487591627445     7.59996881722953e-14
```

```
    x5              44.9101441058108        2.82740513635976     15.8838730001148     3.10076582511592e-28
    x6             -41.8125331762327        1.86759528502419    -22.3884336780659     3.14635557455373e-39
    x7              17.6703934089773       0.496777858230699      35.570010048176     6.06541766672694e-56


Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.000228
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.78e+09, p-value = 0
                       SumSq            DF            MeanSq                 F           pValue
                ____________________    ___    ____________________    _____________    ______

    Total           645.004627337436    100        6.45004627337436
    Model           645.004622515875      7        92.1435175022679    1777297215.4549       0
    Residual    4.82156110592769e-06     93    5.18447430744912e-08

A = 0.500000, B = 0.270000, C = 0.270000
List of powers: [0.770000, 1.310000, 1.850000, 2.390000, 2.930000, 3.470000, 4.010000]
Fitting 10^x in range (0.000000, 1.000000)
MSS of errors squared = 2.174064e-05
Corrected MSS of errors squared = 3.765590e-05
Particle swarm AICc =  -1.382420e+03
AIC =  -1.399985e+03
AICc =  -1.382420e+03
```

## Using Power A+B*i–C*(i–1)

```
Fitting 10^x in range (0.000000, 1.000000)
Shammas polynomial power is A+B*i-C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                       Estimate                SE                tStat                pValue
                   ________________    ____________________    _____________    ____________________

    (Intercept)    0.999114635376011    0.00119535088961409    835.833765680777    4.91107580002351e-182
    x1              66.1702723954412       4.67606615433406     14.150841799813     6.38864974322026e-25
    x2             -625.993531773452       39.9674598830297    -15.6625798488448     8.04540028035634e-28
    x3              2508.22197089915       140.988607874015     17.7902456710578      1.0867822907892e-31
    x4             -5273.7874750037        261.918730545604    -20.1352055426424     1.13851255798602e-35
    x5              6159.99875841742       270.066860417644     22.8091619567514     7.25239563698674e-40
    x6             -3800.11959150627       146.578609343178    -25.9254717215199     2.42554022274219e-44
    x7              974.507005544767       32.7347807775365     29.7697733846899     2.48797321353041e-49


Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.0012
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 6.39e+07, p-value = 9.45e-308
                       SumSq            DF            MeanSq                 F                  pValue
                ____________________    ___    ____________________    _____________    ____________________

    Total           645.004627337556    100        6.45004627337556
    Model           645.004493187339      7        92.1434990267628    63878729.8068441    9.44913904400657e-
308
    Residual    0.000134150216126727     93    1.442475442222287e-06

A = 0.750000, B = 0.250000, C = 0.000000
List of powers: [1.000000, 1.250000, 1.500000, 1.750000, 2.000000, 2.250000, 2.500000]
Fitting 10^x in range (0.000000, 1.000000)
```

 

```
MSS of errors squared = 1.146765e-04
Corrected MSS of errors squared = 1.986255e-04
Particle swarm AICc =  -1.046508e+03
AIC =  -1.064073e+03
AICc =  -1.046508e+03
```

## Using Power A+B*i+C*sqrt(i–1)

```
Fitting 10^x in range (0.000000, 1.000000)
Shammas polynomial power is A+B*i+C*sqrt(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
```

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 0.999400151282682 | 0.000249584805259911 | 4004.2507805791 | 2.60852108356399e-245 |
| x1 | 0.530408377437921 | 0.0123062766652169 | 43.1006381432244 | 2.93678454856638e-63 |
| x2 | 6.75444486472513 | 0.293740616756791 | 22.9945893737862 | 3.82119293288529e-40 |
| x3 | -8.27236698916749 | 1.7112494152212 | -4.83410946153528 | 5.26740989226483e-06 |
| x4 | -2.85209862549303 | 4.48323112315872 | -0.636170330536862 | 0.526227989761666 |
| x5 | 38.7938086009361 | 6.01325764474972 | 6.45137975001081 | 4.92679513829041e-09 |
| x6 | -47.781438037037 | 4.03346875781765 | -11.846239776725 | 2.80214436223674e-20 |
| x7 | 21.8272151130589 | 1.07398563780345 | 20.3235633185008 | 5.61276360628317e-36 |

```
Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.00026
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.36e+09, p-value = 0
```

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 645.004627337435 | 100 | 6.45004627337435 | | |
| Model | 645.00462104058 | 7 | 92.1435172915114 | 1360893152.10948 | 0 |
| Residual | 6.29685519015766e-06 | 93 | 6.77081203242759e-08 | | |

```
A = 0.340000, B = 0.390000, C = 0.360000
List of powers: [0.730000, 1.480000, 2.019117, 2.523538, 3.010000, 3.484984, 3.951816]
Fitting 10^x in range (0.000000, 1.000000)
MSS of errors squared = 2.484508e-05
Corrected MSS of errors squared = 4.303295e-05
Particle swarm AICc =  -1.355458e+03
AIC =  -1.373023e+03
```

## Using Power A+B*sqrt(i)+C*(i–1)

```
Fitting 10^x in range (0.000000, 1.000000)
Shammas polynomial power is A+B*sqrt(i)+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
```

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 0.999619995229897 | 0.00014303770421184 | 6988.50698658762 | 8.37916354227936e-268 |
| x1 | 1.89080433920688 | 0.0234121321879991 | 80.7617317390725 | 6.28005416377478e-88 |
| x2 | -0.71352133753564 | 0.285041978343565 | -2.5032149358563 | 0.0140513571046596 |
| x3 | 16.3039838146016 | 1.35850405878241 | 12.0014244412449 | 1.34136062436542e-20 |

 

| | | | | |
|---|---|---|---|---|
| x4 | -48.985167792865 | 3.2368309503569 | -15.1336812283829 | 8.05526883217011e-27 |
| x5 | 86.3247262236463 | 4.10997235682007 | 21.0037242903591 | 4.5183934775795e-37 |
| x6 | -72.6177498349313 | 2.66173259297417 | -27.2821357136367 | 3.64463568713833e-46 |
| x7 | 26.7968816945804 | 0.691925925298763 | 38.7279630879706 | 3.65234539445541e-59 |

Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.000153
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 3.92e+09, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 645.004627337435 | 100 | 6.45004627337435 | | |
| Model | 645.004625151733 | 7 | 92.1435178788191 | 3920639632.7033 | 0 |
| Residual | 2.18570130527951e-06 | 93 | 2.35021645728979e-08 | | |

A = 0.550000, B = 0.400000, C = 0.400000
List of powers: [0.950000, 1.515685, 2.042820, 2.550000, 3.044427, 3.529796, 4.008301]
Fitting 10^x in range (0.000000, 1.000000)
MSS of errors squared = 1.463774e-05
Corrected MSS of errors squared = 2.535331e-05
Particle swarm AICc =  -1.462327e+03
AIC =  -1.479893e+03
AICc =  -1.462327e+03

## Using Power A+B*sqrt(i)+C*sqrt(i–1)

Fitting 10^x in range (0.000000, 1.000000)
Shammas polynomial power is A+B*sqrt(i)+C*sqrt(i-1)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 1.00007294950223 | 4.19788720573513e-05 | 23823.2448965264 | 2.45414263864847e-317 |
| x1 | 2.92387358144242 | 0.00488638047566073 | 598.372066196311 | 1.53980888772098e-168 |
| x2 | -16.3706763586712 | 0.31820097688009 | -51.4475993102947 | 4.01782065160063e-70 |
| x3 | 146.808665331349 | 2.54536418557721 | 57.6768802528182 | 1.33925433867057e-74 |
| x4 | -473.265529561328 | 7.92423539048894 | -59.7238100889033 | 5.68793653487605e-76 |
| x5 | 754.497870751332 | 11.7718952324536 | 64.0931520246016 | 9.3362520065239e-79 |
| x6 | -585.530765237322 | 8.39752769874265 | -69.7265655134418 | 4.32837710160272e-82 |
| x7 | 179.936351878896 | 2.31788243650855 | 77.629628252387 | 2.35731025883896e-86 |

Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 4.95e-05
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 3.76e+10, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 645.004627337442 | 100 | 6.45004627337442 | | |
| Model | 645.004627109591 | 7 | 92.143518158513 | 37609482810.947 | 0 |
| Residual | 2.27850705414312e-07 | 93 | 2.45000758510013e-09 | | |

A = 0.345852, B = 0.700000, C = 0.700000
List of powers: [1.045852, 2.035802, 2.548237, 2.958288, 3.311100, 3.625742, 3.912521]
Fitting 10^x in range (0.000000, 1.000000)

 

```
MSS of errors squared = 4.726107e-06
Corrected MSS of errors squared = 8.185857e-06
Particle swarm AICc =  -1.690689e+03
AIC =  -1.708254e+03
AICc =  -1.690689e+03
```

## Comments

The following table shows the summary results for the seven models that fit the common exponent function.

| *Power* | *A* | *B* | *C* | *F* | *AICc* | *MSSE* | *Pwr Rng* |
|---|---|---|---|---|---|---|---|
| A+B*i | 0.500 | 0.500 | | 4528199721.3 | -1.476878e+03 | 1.362039e-05 | 1, 4 |
| A+B/i | 1.624 | 2.000 | | 4125066262.0 | -1.467461e+03 | 1.427043e-05 | 1.90, 3.62 |
| A+B*√(i) | 0.479 | 1.000 | | 4722509958.3 | -1.481122e+03 | 1.333724e-05 | 1.47, 3.12 |
| A+B*√(i+C) | 0.479 | 1.000 | 0.000 | 4722509958.3 | -1.481122e+03 | 1.333724e-05 | 1.47, 3.12 |
| A+B*i+C*(i-1) | 0.500 | 0.270 | 0.270 | 1777297215.4 | -1.382420e+03 | 2.174064e-05 | 0.77, 4.01 |
| A+B*i-C*(i-1) | 0.750 | 0.250 | 0.000 | 63878729.8 | -1.046508e+03 | 1.146765e-04 | 1, 2.5 |
| A+B*i+C*√(i-1) | 0.340 | 0.390 | 0.360 | 1360893152.1 | -1.355458e+03 | 2.484508e-05 | 0.73, 3.95 |
| A+B*√(i)+C*(i-1) | 0.550 | 0.400 | 0.400 | 3920639632.7 | -1.462327e+03 | 1.463774e-05 | 0.95, 4.008 |
| A+B*√(i)+C*√(i-1) | 0.345 | 0.700 | 0.700 | 37609482810.9 | -1.690689e+03 | 4.726107e-06 | 1.04, 3.91 |

The above table shows that the top two Shammas polynomials use the following power equations:

$0.345+0.7*\sqrt{(i)}+0.7*\sqrt{(i-1)}$

And,

$0.479+\sqrt{(i)}$

 

# RESULTS FOR THE SINE FUNCTION

## Using Power A+B* i

```
Fitting sin(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
```

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | -4.01676721334551e-07 | 1.39386041234946e-07 | -2.8817571528378 | 0.00491054659707566 |
| x1 | 1.0004757945029 | 3.39157574456657e-05 | 29498.8486135301 | 0 |
| x2 | -0.00672867296313523 | 0.000337401113593459 | -19.9426519120584 | 2.35625747866364e-35 |
| x3 | 0.0387685786085 | 0.00141538219371442 | 27.3908904468826 | 2.6216058023079e-46 |
| x4 | -0.117111806482408 | 0.00310639046348845 | -37.7002852213536 | 3.84100417655154e-58 |
| x5 | 0.0326734001173057 | 0.0037342238278584 | 8.7497165739109 | 9.01607438358424e-14 |
| x6 | -0.187099771270348 | 0.00232963108085064 | -80.3130473353878 | 1.0467219649295e-87 |
| x7 | 0.0804935369043733 | 0.000590104356094303 | 136.405596862786 | 6.48585941108969e-109 |

```
Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 1.49e-07
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 4.04e+13, p-value = 0
```

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 6.31499836761147 | 100 | 0.0631499836761147 | | |
| Model | 6.3149983676094 | 7 | 0.9021426239442 | 40437574294499.5 | 0 |
| Residual | 2.07478478841949e-12 | 93 | 2.2309513853973e-14 | | |

```
A = 0.500000, B = 0.500000
List of powers: [1.000000, 1.500000, 2.000000, 2.500000, 3.000000, 3.500000, 4.000000]
Fitting sin(x) in range (0.000000, 1.000000)
MSS of errors squared = 1.427189e-08
Corrected MSS of errors squared = 2.018349e-08
Particle swarm AICc =  -2.862807e+03
AIC =  -2.880372e+03
AICc =  -2.862807e+03
```

## Using Power A+B/i

```
Fitting sin(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B/i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
```

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 4.59086480683109e-07 | 1.43090162936758e-06 | 0.320837205899342 | 0.749053608709888 |
| x1 | 0.152977810960072 | 0.00161441130964635 | 94.7576432635271 | 2.63004168596415e-94 |
| x2 | -16.7769004991993 | 0.102715175600382 | -163.334194788028 | 3.67518930393641e-116 |
| x3 | 238.362228600371 | 1.2969391285398 | 183.788293031716 | 6.52361557953636e-121 |
| x4 | -1227.63799868604 | 5.89813345574469 | -208.14008497728 | 6.32493314135752e-126 |
| x5 | 2753.52857535623 | 11.8643750220641 | 232.083744001307 | 2.57963106558251e-130 |

 

| | | | | |
|---|---|---|---|---|
| x6 | -2758.09448870773 | 10.8255751789225 | -254.775791874575 | 4.46345052372868e-134 |
| x7 | 1011.30707319253 | 3.66349716895924 | 276.049639607019 | 2.59901288473017e-137 |

Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 1.44e-06
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 4.32e+11, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 6.31499836763245 | 100 | 0.0631499836763245 | | |
| Model | 6.31499836743843 | 7 | 0.902142623919775 | 432416153431.387 | 0 |
| Residual | 1.94024352140332e-10 | 93 | 2.08628335634765e-12 | | |

A = 1.105748, B = 2.000000
List of powers: [3.105748, 2.105748, 1.772415, 1.605748, 1.505748, 1.439082, 1.391463]
Fitting sin(x) in range (0.000000, 1.000000)
MSS of errors squared = 1.379122e-07
Corrected MSS of errors squared = 1.950372e-07
Particle swarm AICc =  -2.404605e+03
AIC =  -2.422170e+03
AICc =  -2.404605e+03

## Using Power A+B*sqrt(i)

Fitting sin(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*sqrt(i)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 3.55952304572048e-07 | 9.42382471091777e-07 | 0.377715328426756 | 0.70650275494108 |
| x1 | 1.57917915320751 | 0.00120984441515663 | 1305.27457367571 | 4.89353703633998e-200 |
| x2 | -4.57287543387088 | 0.0223956317234772 | -204.186043525496 | 3.7500911014037e-125 |
| x3 | 19.8191660184687 | 0.129424264680513 | 153.133309796217 | 1.44683259453798e-113 |
| x4 | -46.7427824292905 | 0.339856972812875 | -137.536629136713 | 3.02050863209774e-109 |
| x5 | 59.5339332425509 | 0.45248591339539 | 131.570799178734 | 1.82847916630683e-107 |
| x6 | -38.4788183581012 | 0.298499969232948 | -128.907277468001 | 1.21245466715503e-106 |
| x7 | 9.70366608142305 | 0.0776338547009229 | 124.992712506747 | 2.09950511681842e-105 |

Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 9.59e-07
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 9.81e+11, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 6.3149983676111 | 100 | 0.063149983676111 | | |
| Model | 6.31499836752558 | 7 | 0.902142623932225 | 980997108282.068 | 0 |
| Residual | 8.55244763897645e-11 | 93 | 9.19618025696392e-13 | | |

A = 0.052116, B = 1.000000
List of powers: [1.052116, 1.466330, 1.784167, 2.052116, 2.288184, 2.501606, 2.697867]
Fitting sin(x) in range (0.000000, 1.000000)
MSS of errors squared = 9.156305e-08
Corrected MSS of errors squared = 1.294897e-07

 

```
Particle swarm AICc =  -2.487342e+03
AIC =  -2.504907e+03
AICc =  -2.487342e+03
```

## Using Power A+B*sqrt(i+C)

```
Fitting sin(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*sqrt(i+C)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                        Estimate                SE                tStat              pValue
                    ____________________    ____________________    _______________    ____________________

    (Intercept)    3.55963770964791e-07    9.42382459439627e-07    0.377727500548407     0.706493742371618
    x1                  1.57918071675391      0.0012098449250129     1305.27531595595    4.89327825271932e-200
    x2                 -4.57288701256804      0.0223956402679972    -204.186482629951    3.74934281944021e-125
    x3                  19.8192105294483       0.129424310541194     153.133599449541    1.44657909872935e-113
    x4                 -46.7428756571678       0.339857086365466    -137.536857498104     3.0200445165143e-109
    x5                  59.5340400066686       0.452486057341953     131.570993273009     1.8282296529208e-107
    x6                 -38.4788816179317       0.298500060261187    -128.907450083135    1.21230451745237e-106
    x7                  9.70368130918404      0.0776338775091953     124.992871933193    2.09925754607147e-105


Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 9.59e-07
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 9.81e+11, p-value = 0
                        SumSq            DF            MeanSq                F            pValue
                    ____________________    ___    ____________________    _______________    ______

    Total           6.31499836761141    100      0.0631499836761141
    Model           6.31499836752588      7       0.902142623932269    980997108282.116       0
    Residual    8.55244763897645e-11     93    9.19618025696392e-13

A = 0.052116, B = 1.000000, C = 0.000000
List of powers: [1.052116, 1.466330, 1.784167, 2.052116, 2.288184, 2.501606, 2.697868]
Fitting sin(x) in range (0.000000, 1.000000)
MSS of errors squared = 9.156305e-08
Corrected MSS of errors squared = 1.585919e-07
Particle swarm AICc =  -2.487342e+03
AIC =  -2.504907e+03
AICc =  -2.487342e+03
```

## Using Power A+B*i+C*(i–1)

```
Fitting sin(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*i+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                        Estimate                SE                tStat              pValue
                    ____________________    ____________________    _______________    ____________________

    (Intercept)    4.14208110923306e-08    4.00302034411589e-07    0.103473896037565     0.917809674838167
    x1                -0.000895124634171511    1.01816649381931e-05    -87.9153497591292     2.59400941219196e-91
    x2                  0.952930194143688    0.000127954607680464     7447.4081974711     2.2619084144389e-270
    x3                  0.144526333681065    0.000671510719369739     215.225653905739     2.82986285633676e-127
    x4                 -0.266363946739223     0.00176382197914334    -151.015210088601     5.25706839638756e-113
```

| | | | | |
|---|---|---|---|---|
| x5 | 0.328037494630371 | 0.0024526006694799 | 133.750878694792 | 3.99751028752544e-108 |
| x6 | -0.445629294047673 | 0.00172575194096872 | -258.223261100624 | 1.28105937114085e-134 |
| x7 | 0.128864282764922 | 0.000483720186129137 | 266.402532828182 | 7.07545178747116e-136 |

Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 4.02e-07
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 5.59e+12, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 6.31499836761147 | 100 | 0.0631499836761147 | | |
| Model | 6.31499836759646 | 7 | 0.902142623942352 | 5592124884664.77 | 0 |
| Residual | 1.50031098655745e-11 | 93 | 1.61323761995425e-13 | | |

A = 0.179462, B = 0.270000, C = 0.270000
List of powers: [0.449462, 0.989462, 1.529462, 2.069462, 2.609462, 3.149462, 3.689462]
Fitting sin(x) in range (0.000000, 1.000000)
MSS of errors squared = 3.835004e-08
Corrected MSS of errors squared = 6.642423e-08
Particle swarm AICc =  -2.663137e+03
AIC =  -2.680702e+03
AICc =  -2.663137e+03

## Using Power A+B*i–C*(i–1)

Fitting sin(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*i-C*(i-1)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 8.17340424980174e-08 | 3.05207292361329e-06 | 0.0267798458764393 | 0.978692709175599 |
| x1 | -188.868974477425 | 2.94840178154834 | -64.058085861772 | 9.81277207636916e-79 |
| x2 | 1451.36176558065 | 20.5199534995922 | 70.72929115602 | 1.17541427353395e-82 |
| x3 | -4620.96886005737 | 59.3310598763729 | -77.8844819169925 | 1.74591110222559e-86 |
| x4 | 7846.61301087254 | 91.2310318711808 | 86.0081580788436 | 1.94604104469046e-90 |
| x5 | -7497.37145846643 | 78.6890731447337 | -95.2784314116451 | 1.58790170088328e-94 |
| x6 | 3823.5867012294 | 36.1001193727589 | 105.916178884291 | 9.21559664221425e-99 |
| x7 | -813.510723040488 | 6.88257217304445 | -118.198647625752 | 3.67663977626872e-103 |

Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 3.05e-06
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 9.68e+10, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 6.3149983676165 | 100 | 0.063149983676165 | | |
| Model | 6.31499836674991 | 7 | 0.902142623821416 | 96815963237.3199 | 0 |
| Residual | 8.66585025960376e-10 | 93 | 9.31811855871372e-12 | | |

A = 0.750000, B = 0.250000, C = 0.160083
List of powers: [1.000000, 1.089917, 1.179835, 1.269752, 1.359669, 1.449586, 1.539504]
Fitting sin(x) in range (0.000000, 1.000000)
MSS of errors squared = 2.914640e-07

 

```
Corrected MSS of errors squared = 5.048305e-07
Particle swarm AICc =  -2.253449e+03
AIC =  -2.271014e+03
AICc =  -2.253449e+03
```

## Using Power A+B*i+C*sqrt(i–1)

```
Fitting sin(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*i+C*sqrt(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                        Estimate                SE                tStat              pValue
                    ____________________    ____________________    _______________    ____________________

    (Intercept)    -3.38414288425646e-07    1.51690490319884e-06    -0.223095256473889      0.823950654340787
    x1               -0.0015744777893669    3.92234748996849e-05      -40.141211185232     1.56970831220534e-60
    x2                  0.76188122368261    0.000793954380379253      959.603275088271    1.30148799544439e-187
    x3                 0.724943835534918     0.00458473279758445      158.121283734762    7.42503944527506e-115
    x4                 -1.35107444060145      0.0122073695758942     -110.676950689639    1.59416897978157e-100
    x5                  1.77856304835363      0.0167643190181751      106.092173885822     7.90698660953927e-99
    x6                 -1.3956318650792      0.0115378521265397      -120.96115028801    4.34822686452769e-104
    x7                 0.32435939347709     0.00315247121790473      102.890517012451     1.33446775332467e-97


Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 1.52e-06
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 3.91e+11, p-value = 0
                        SumSq             DF            MeanSq                 F             pValue
                    ____________________    ___    ____________________    _______________    ______

    Total             6.31499836761147    100      0.0631499836761147
    Model             6.31499836739703      7       0.902142623913861    391238438514.828       0
    Residual    2.14445350366077e-10     93    2.30586398243094e-12

A = 0.000000, B = 0.390000, C = 0.174095
List of powers: [0.390000, 0.954095, 1.416207, 1.861541, 2.298190, 2.729288, 3.156444]
Fitting sin(x) in range (0.000000, 1.000000)
MSS of errors squared = 1.449897e-07
Corrected MSS of errors squared = 2.511296e-07
Particle swarm AICc =  -2.394496e+03
AIC =  -2.412061e+03
AICc =  -2.394496e+03
```

## Using Power A+B*sqrt(i)+C*(i–1)

```
Fitting sin(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*sqrt(i)+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                        Estimate                SE                tStat              pValue
                    ____________________    ____________________    _______________    ____________________

    (Intercept)    4.40876087167138e-08    6.30426815877553e-07    0.0699329527335284      0.9443971805791
    x1               -0.00124153424669722    1.42834771545883e-05      -86.9210090274413     7.37769893885947e-91
    x2                  0.905448730282711     0.00022163678103497      4085.28190156239    4.04788455698762e-246
    x3                  0.301538485987495     0.00128897698969111      233.936283113755    1.23306121235383e-130
```

| | | | | |
|---|---|---|---|---|
| x4 | -0.587350590415936 | 0.00356771038002142 | -164.629560096862 | 1.76730926096536e-116 |
| x5 | 0.798709878535768 | 0.00509679126487712 | 156.708375334069 | 1.70562266634916e-114 |
| x6 | -0.788811795034484 | 0.0036332016672985 | -217.11203155453 | 1.25892998739153e-127 |
| x7 | 0.213176145347145 | 0.00102322204446572 | 208.338108527026 | 5.79067585100414e-126 |

Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 6.32e-07
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 2.26e+12, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 6.31499836761147 | 100 | 0.0631499836761147 | | |
| Model | 6.31499836757434 | 7 | 0.902142623939192 | 2259860611278.14 | 0 |
| Residual | 3.71258579434652e-11 | 93 | 3.99202773585648e-13 | | |

A = 0.014078, B = 0.400000, C = 0.400000
List of powers: [0.414078, 0.979764, 1.506899, 2.014078, 2.508506, 2.993874, 3.472379]
Fitting sin(x) in range (0.000000, 1.000000)
MSS of errors squared = 6.032837e-08
Corrected MSS of errors squared = 1.044918e-07
Particle swarm AICc =  -2.571622e+03
AIC =  -2.589187e+03
AICc =  -2.571622e+03

## Using Power A+B*sqrt(i)+C*sqrt(i–1)

Fitting sin(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*sqrt(i)+C*sqrt(i-1)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 2.90377953696882e-08 | 2.00070071886799e-08 | 1.45138126336646 | 0.150038718105442 |
| x1 | 1.00050114815166 | 2.01284365501674e-06 | 497058.549807404 | 0 |
| x2 | -0.0180035456136966 | 0.000132820058550137 | -135.548393896398 | 1.16231375707924e-108 |
| x3 | 0.159905147240535 | 0.00107137731324283 | 149.251944449465 | 1.56019664026007e-112 |
| x4 | -0.690580485760389 | 0.00335589518593714 | -205.781303496683 | 1.82138488245777e-125 |
| x5 | 0.966136382106075 | 0.00500942429884935 | 192.863755287807 | 7.4607581785608e-123 |
| x6 | -0.910417231110021 | 0.00358764720854504 | -253.76442503644 | 6.45791887056213e-134 |
| x7 | 0.33392950244853 | 0.000993578304088446 | 336.087755815977 | 2.98155463736066e-145 |

Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 2.31e-08
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.69e+15, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 6.31499836761147 | 100 | 0.0631499836761147 | | |
| Model | 6.31499836761142 | 7 | 0.902142623944488 | 1.68682452771395e+15 | 0 |
| Residual | 4.9737991503207e-14 | 93 | 5.3481711293771e-16 | | |

A = 0.300091, B = 0.700000, C = 0.700000
List of powers: [1.000091, 1.990041, 2.502476, 2.912527, 3.265339, 3.579982, 3.866760]
Fitting sin(x) in range (0.000000, 1.000000)

 

```
MSS of errors squared = 2.225106e-09
Corrected MSS of errors squared = 3.853997e-09
Particle swarm AICc =  -3.238209e+03
AIC =  -3.255774e+03
AICc =  -3.238209e+03
```

## Comments

The following table shows the summary results for the seven models that fit the sine function.

| *Power* | *A* | *B* | *C* | *F* | *AICc* | *MSSE* | *Pwr Rng* |
|---|---|---|---|---|---|---|---|
| A+B*i | 0.500 | 0.500 | | 40437574294499.5 | -2.862807e+03 | 1.427189e-08 | 1, 4 |
| A+B/i | 1.105 | 2.000 | | 432416153431.3 | -2.404605e+03 | 1.379122e-07 | 1.39, 3.10 |
| A+B*√(i) | 0.052 | 1.000 | | 980997108282.0 | -2.487342e+03 | 9.156305e-08 | 1.05, 2.69 |
| A+B*√(i+C) | 0.052 | 1.000 | 0.000 | 980997108282.0 | -2.487342e+03 | 9.156305e-08 | 1.05, 2.69 |
| A+B*i+C*(i-1) | 0.179 | 0.270 | 0.270 | 5592124884664.7 | -2.663137e+03 | 3.835004e-08 | 0.44, 3.68 |
| A+B*i-C*(i-1) | 0.750 | 0.250 | 0.160 | 96815963237.3 | -2.253449e+03 | 2.914640e-07 | 1, 1.53 |
| A+B*i+C*√(i-1) | 0.000 | 0.390 | 0.174 | 391238438514.8 | -2.394496e+03 | 1.449897e-07 | 0.39, 3.15 |
| A+B*√(i)+C*(i-1) | 0.014 | 0.400 | 0.400 | 2259860611278.1 | -2.571622e+03 | 6.032837e-08 | 0.41, 3.47 |
| A+B*√(i)+C*√(i-1) | 0.300 | 0.700 | 0.700 | 1.68682452771395e+15 | -3.238209e+03 | 2.225106e-09 | 1, 3.86 |

The above table shows that the top two Shammas polynomials use the following power equations:

$0.3+0.7*\sqrt{(i)}+0.7*\sqrt{(i-1)}$

And,

$0.5+0.5*i$

 

# RESULTS FOR THE HYPERBOLIC SINE FUNCTION

## Using Power A+B* i

```
Fitting sinh(x) in range (0.000000, 5.000000)
Shammas polynomial power is A+B*i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat               pValue
                ________________    ________________    ________________    ____________________

    (Intercept)    -0.156680255738382    0.0165413706454538    -9.47202375768323     1.14267753395508e-19
    x1               16.0914798649265     0.724916366636451     22.1977052878384     3.38484439880601e-76
    x2              -80.5592453744439      3.07845927511384    -26.1686896512428      2.7543586511749e-95
    x3               178.211255050104      5.59182854763746     31.8699426371715    7.98772830174513e-122
    x4              -205.063072457692      5.36369466072272    -38.2316827166409    1.43131811943907e-149
    x5               129.545321525008      2.83525460466027     45.6908953827555     2.4907894050286e-179
    x6              -42.691954861169      0.78104069810565    -54.6603460801912    2.25228285267534e-211
    x7               5.82745776110099    0.0876174090394353     66.5102726157782    2.33970041108701e-248


Number of observations: 501, Error degrees of freedom: 493
Root Mean Squared Error: 0.0276
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 3.19e+07, p-value = 0
                    SumSq          DF           MeanSq                F            pValue
                ________________    ___    ____________________    ________________    ______

    Total       169774.232792727    500        339.548465585454
    Model       169773.858062127      7        24253.4082945896    31908070.1670428       0
    Residual    0.37473060033517    493    0.000760102637596693

A = 0.500000, B = 0.500000
List of powers: [1.000000, 1.500000, 2.000000, 2.500000, 3.000000, 3.500000, 4.000000]
Fitting sinh(x) in range (0.000000, 5.000000)
MSS of errors squared = 1.221861e-03
Corrected MSS of errors squared = 1.727973e-03
Particle swarm AICc =  -2.152206e+03
AIC =  -2.168499e+03
AICc =  -2.152206e+03
```

## Using Power A+B/i

```
Fitting sinh(x) in range (0.000000, 5.000000)
Shammas polynomial power is A+B/i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat               pValue
                ________________    ________________    ________________    ____________________

    (Intercept)    -0.0626939730694216    0.00717677055465285    -8.73568028850745     3.79373215061768e-17
    x1               6.57824270627823     0.0939231728737016     70.0385485818711     2.28805053538434e-258
    x2              -2059.46519152516      41.1005874411383    -50.1079259383967     1.36383988591069e-195
    x3               45935.3127840602      1010.83296702973     45.4430299390002     2.17756975585481e-178
    x4              -279690.460913241      6447.87255620865    -43.3771695198794     2.04300482577515e-170
    x5               671937.356294706       15914.17406591     42.2225717471617     7.30875767759142e-166
```

 

| | | | | |
|---|---|---|---|---|
| x6 | -690896.363101549 | 16652.3827373999 | -41.4893396336521 | 6.20232661426249e-163 |
| x7 | 254768.293577246 | 6216.26154187642 | 40.9841657821145 | 6.71670067064738e-161 |

Number of observations: 501, Error degrees of freedom: 493
Root Mean Squared Error: 0.0179
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 7.54e+07, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 169774.232723508 | 500 | 339.548465447017 | | |
| Model | 169774.074114373 | 7 | 24253.4391591962 | 75386234.8729913 | 0 |
| Residual | 0.158609135018196 | 493 | 0.000321722383404049 | | |

A = 2.000000, B = 2.000000
List of powers: [4.000000, 3.000000, 2.666667, 2.500000, 2.400000, 2.333333, 2.285714]
Fitting sinh(x) in range (0.000000, 5.000000)
MSS of errors squared = 7.949254e-04
Corrected MSS of errors squared = 1.124194e-03
Particle swarm AICc =  -2.582948e+03
AIC =  -2.599241e+03
AICc =  -2.582948e+03

## Using Power A+B*sqrt(i)

Fitting sinh(x) in range (0.000000, 5.000000)
Shammas polynomial power is A+B*sqrt(i)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 0.0201696943937905 | 0.0014380027259233 | 14.0261864808637 | 7.53846648564724e-38 |
| x1 | 179.383185745367 | 1.8558418343154 | 96.658660467981 | 1.38338380835549e-322 |
| x2 | -1360.56952498957 | 14.3821020138359 | -94.6015765762663 | 3.12742565686217e-318 |
| x3 | 4095.60591813573 | 42.7570258909688 | 95.7879046260047 | 9.20938363848084e-321 |
| x4 | -6337.28134751632 | 64.5786741327408 | -98.132726207573 | 0 |
| x5 | 5373.95991457484 | 53.1488728078657 | 101.111456004002 | 0 |
| x6 | -2384.59449732566 | 22.8121179641785 | -104.531920318409 | 0 |
| x7 | 434.647239473806 | 4.01231077309653 | 108.328408254968 | 0 |

Number of observations: 501, Error degrees of freedom: 493
Root Mean Squared Error: 0.005
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 9.69e+08, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 169774.2327933 | 500 | 339.548465586601 | | |
| Model | 169774.220452246 | 7 | 24253.4600646065 | 968876351.915868 | 0 |
| Residual | 0.0123410544474609 | 493 | 2.5032564802152e-05 | | |

A = 1.400000, B = 1.000000
List of powers: [2.400000, 2.814214, 3.132051, 3.400000, 3.636068, 3.849490, 4.045751]
Fitting sinh(x) in range (0.000000, 5.000000)
MSS of errors squared = 2.217371e-04
Corrected MSS of errors squared = 3.135836e-04

 

```
Particle swarm AICc =  -3.862257e+03
AIC =  -3.878550e+03
AICc =  -3.862257e+03
```

## Using Power A+B*sqrt(i+C)

```
Fitting sinh(x) in range (0.000000, 5.000000)
Shammas polynomial power is A+B*sqrt(i+C)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                       Estimate                SE                tStat                pValue
                   ________________    ________________    ________________    ____________________

    (Intercept)    0.0313523386764381    0.00120516287009652      26.015022080731    1.48745663342452e-94
    x1               1098.27639898006       9.92213583638106     110.689514545151                       0
    x2              -6920.97059891605       63.375556955139    -109.205676942849                       0
    x3               18133.9560429423       165.860493100619     109.332582485097                       0
    x4              -25228.3987488295       228.545157707071    -110.386931851625                       0
    x5               19652.2041832143       175.365521941661     112.064241394908                       0
    x6              -8130.45684421823       71.1885200345068    -114.210224349055                       0
    x7               1396.52888270737       11.9620418183785     116.746697922569                       0


Number of observations: 501, Error degrees of freedom: 493
Root Mean Squared Error: 0.00445
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.23e+09, p-value = 0
                       SumSq           DF           MeanSq                F            pValue
                   ________________    ___    ____________________    ________________    ______

    Total          169774.232794454    500       339.548465588908
    Model          169774.223047678      7       24253.4604353825    1226760041.13425       0
    Residual    0.00974677654448897    493    1.9770337818436e-05

A = 1.000000, B = 1.000000, C = 2.000000
List of powers: [2.732051, 3.000000, 3.236068, 3.449490, 3.645751, 3.828427, 4.000000]
Fitting sinh(x) in range (0.000000, 5.000000)
MSS of errors squared = 1.970574e-04
Corrected MSS of errors squared = 3.413135e-04
Particle swarm AICc =  -3.980491e+03
AIC =  -3.996783e+03
AICc =  -3.980491e+03
```

## Using Power A+B*i+C*(i–1)

```
Fitting sinh(x) in range (0.000000, 5.000000)
Shammas polynomial power is A+B*i+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                       Estimate                SE                tStat                pValue
                   ________________    ________________    ________________    ____________________

    (Intercept)    -0.204782722428205    0.0239400244969107    -8.55398967760582    1.51044325883036e-16
    x1               9.41116491926592     0.508046155579999     18.5242321310785    1.55424436031384e-58
    x2              -48.7257687743476       2.096632059166    -23.2400189443491    3.13447259351325e-81
    x3               113.634246836405      3.82506601916502     29.707787072708    6.35784929636675e-112
    x4              -134.663030975679      3.68090512070782    -36.5842168052907    1.35984384382477e-142
```

 

| | | | | |
|---|---|---|---|---|
| x5 | 86.5258028553657 | 1.93838852376347 | 44.6380082189984 | 2.61895461062416e-175 |
| x6 | -28.7033884681962 | 0.528017553656691 | -54.3606709084121 | 2.30424222976716e-210 |
| x7 | 3.93500850351567 | 0.0581833866169444 | 67.6311354892791 | 1.39226077803365e-251 |

Number of observations: 501, Error degrees of freedom: 493
Root Mean Squared Error: 0.0337
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 2.13e+07, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 169774.232792761 | 500 | 339.548465585521 | | |
| Model | 169773.671689518 | 7 | 24253.3816699312 | 21309656.1690886 | 0 |
| Residual | 0.561103242042009 | 493 | 0.00113814045038947 | | |

A = 0.500000, B = 0.270000, C = 0.270000
List of powers: [0.770000, 1.310000, 1.850000, 2.390000, 2.930000, 3.470000, 4.010000]
Fitting sinh(x) in range (0.000000, 5.000000)
MSS of errors squared = 1.495146e-03
Corrected MSS of errors squared = 2.589669e-03
Particle swarm AICc =  -1.949954e+03
AIC =  -1.966246e+03
AICc =  -1.949954e+03

## Using Power A+B*i–C*(i–1)

Fitting sinh(x) in range (0.000000, 5.000000)
Shammas polynomial power is A+B*i-C*(i-1)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | -0.711780289189312 | 0.109407264632388 | -6.50578635322726 | 1.90160040588906e-10 |
| x1 | 1161.44604208876 | 54.5830002354454 | 21.278530624532 | 9.31678896482425e-72 |
| x2 | -7259.12634907983 | 296.386243530323 | -24.4921163094979 | 2.91793313774782e-87 |
| x3 | 18740.8051593287 | 674.724069311975 | 27.7755100369236 | 6.72335157532059e-103 |
| x4 | -25483.0770785712 | 817.137189155204 | -31.1858001529912 | 1.02613125310065e-118 |
| x5 | 19241.7678513246 | 553.001646916217 | 34.7951366123867 | 7.54019319085552e-135 |
| x6 | -7656.13728578288 | 197.9183841879 | -38.6833053291012 | 1.85656991530611e-151 |
| x7 | 1256.18648544143 | 29.2437690160886 | 42.9556971521125 | 9.21062224547892e-169 |

Number of observations: 501, Error degrees of freedom: 493
Root Mean Squared Error: 0.125
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.55e+06, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 169774.232792704 | 500 | 339.548465585408 | | |
| Model | 169766.506895943 | 7 | 24252.3581279919 | 1547576.01958412 | 0 |
| Residual | 7.72589676099597 | 493 | 0.015671190184576 | | |

A = 0.750000, B = 0.250000, C = 0.000000
List of powers: [1.000000, 1.250000, 1.500000, 1.750000, 2.000000, 2.250000, 2.500000]
Fitting sinh(x) in range (0.000000, 5.000000)
MSS of errors squared = 5.548003e-03

 

```
Corrected MSS of errors squared = 9.609424e-03
Particle swarm AICc =  -6.361170e+02
AIC =  -6.524097e+02
AICc =  -6.361170e+02
```

## Using Power A+B*i+C*sqrt(i–1)

```
Fitting sinh(x) in range (0.000000, 5.000000)
Shammas polynomial power is A+B*i+C*sqrt(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                       Estimate                SE                tStat                pValue
                   ________________    ________________    ________________    ____________________

    (Intercept)    -0.181445271060561    0.0203683768754449    -8.90818508367759     1.00261037696331e-17
    x1               5.18257405712497     0.276368691611925     18.7523920560521     1.27354740444006e-59
    x2              -43.7634819986441      1.77353318191702    -24.6758743759956     3.82140921309361e-88
    x3               130.023665667277      4.15309469640483     31.3076573428084     2.8563343902645e-119
    x4              -178.308505401378      4.7002145514145     -37.9362481118479     2.48981931009185e-148
    x5               128.224954131646      2.82720801170672     45.3539158069376     4.7565049020751e-178
    x6              -47.0043697146283     0.870795005267943    -53.9786854888598     4.5248243351579e-209
    x7               7.03359099530136     0.108243844551305     64.9791313719242    7.01735871425136e-244


Number of observations: 501, Error degrees of freedom: 493
Root Mean Squared Error: 0.0301
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 2.68e+07, p-value = 0
                       SumSq          DF            MeanSq                F            pValue
                   ________________    ___    ____________________    ________________    ______

    Total          169774.232792737    500        339.548465585473
    Model          169773.786266683      7        24253.3980380975    26777665.3420656       0
    Residual     0.446526053710841    493    0.000905732360468237

A = 0.340000, B = 0.390000, C = 0.360000
List of powers: [0.730000, 1.480000, 2.019117, 2.523538, 3.010000, 3.484984, 3.951816]
Fitting sinh(x) in range (0.000000, 5.000000)
MSS of errors squared = 1.333785e-03
Corrected MSS of errors squared = 2.310183e-03
Particle swarm AICc =  -2.064386e+03
AIC =  -2.080678e+03
AICc =  -2.064386e+03
```

## Using Power A+B*sqrt(i)+C*(i–1)

```
Fitting sinh(x) in range (0.000000, 5.000000)
Shammas polynomial power is A+B*sqrt(i)+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                       Estimate                SE                tStat                pValue
                   ________________    ________________    ________________    ____________________

    (Intercept)    -0.151230202879885    0.0159727991655236    -9.46798374616181     1.18068719845543e-19
    x1               11.1723440255857     0.512464664256233     21.8011988042156     2.78667275318063e-74
    x2              -62.0052318885213      2.37885518174979    -26.0651561995937     8.57854417936926e-95
    x3               150.949690237579      4.69358140607809     32.160876136526    3.86714709798633e-123
```

| | | | | |
|---|---|---|---|---|
| x4 | -186.930653639948 | 4.83269674135644 | -38.6804021945479 | 1.90900059614051e-151 |
| x5 | 125.702028823403 | 2.72425961192735 | 46.1417216894653 | 4.91746566821065e-181 |
| x6 | -43.8390264418141 | 0.798094169460913 | -54.9296412871002 | 2.80854027209393e-212 |
| x7 | 6.30558844384639 | 0.0951623234794628 | 66.2613964570465 | 1.23378674800498e-247 |

Number of observations: 501, Error degrees of freedom: 493
Root Mean Squared Error: 0.0266
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 3.44e+07, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 169774.232792697 | 500 | 339.548465585394 | | |
| Model | 169773.884755295 | 7 | 24253.4121078992 | 34355308.0659802 | 0 |
| Residual | 0.348037402145565 | 493 | 0.0007059582193622 | | |

A = 0.550000, B = 0.400000, C = 0.400000
List of powers: [0.950000, 1.515685, 2.042820, 2.550000, 3.044427, 3.529796, 4.008301]
Fitting sinh(x) in range (0.000000, 5.000000)
MSS of errors squared = 1.177539e-03
Corrected MSS of errors squared = 2.039557e-03
Particle swarm AICc =  -2.189229e+03
AIC =  -2.205521e+03
AICc =  -2.189229e+03

## Using Power A+B*sqrt(i)+C*sqrt(i–1)

Fitting sinh(x) in range (0.000000, 5.000000)
Shammas polynomial power is A+B*sqrt(i)+C*sqrt(i-1)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | -0.0480411025025841 | 0.0052318745510732 | -9.18238807785053 | 1.16500705453909e-18 |
| x1 | 4.82813548363392 | 0.134792509476933 | 35.8190191900846 | 2.65775417941841e-139 |
| x2 | -61.3113527839338 | 1.66857086197064 | -36.7448300706408 | 2.79725834908386e-143 |
| x3 | 240.938761550224 | 5.656929121849 | 42.5917943040219 | 2.51169848208929e-167 |
| x4 | -428.411594370926 | 8.88853156351672 | -48.1982418928854 | 1.11310792859e-188 |
| x5 | 395.836422184148 | 7.34757878770527 | 53.8730422117422 | 1.03363842253951e-208 |
| x6 | -185.984530968523 | 3.11225742973285 | -59.7587234242661 | 5.75663482414243e-228 |
| x7 | 35.3343937715942 | 0.534783587789454 | 66.0723226710268 | 4.37784996166438e-247 |

Number of observations: 501, Error degrees of freedom: 493
Root Mean Squared Error: 0.0125
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.55e+08, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 169774.232792736 | 500 | 339.548465585471 | | |
| Model | 169774.155798703 | 7 | 24253.4508283862 | 155297117.309084 | 0 |
| Residual | 0.076994032249786 | 493 | 0.000156174507606057 | | |

A = 0.500000, B = 0.700000, C = 0.700000
List of powers: [1.200000, 2.189949, 2.702385, 3.112436, 3.465248, 3.779890, 4.066669]
Fitting sinh(x) in range (0.000000, 5.000000)

 

```
MSS of errors squared = 5.538483e-04
Corrected MSS of errors squared = 9.592933e-04
Particle swarm AICc =  -2.945028e+03
AIC =  -2.961321e+03
AICc =  -2.945028e+03
```

## Comments

The following table shows the summary results for the seven models that fit the hyperbolic sine function.

| *Power* | *A* | *B* | *C* | *F* | *AICc* | *MSSE* | *Pwr Rng* |
|---|---|---|---|---|---|---|---|
| A+B*i | 0.500 | 0.500 | | 31908070.1 | -2.152206e+03 | 1.221861e-03 | 1, 4 |
| A+B/i | 2.000 | 2.000 | | 75386234.8 | -2.582948e+03 | 7.949254e-04 | 2.28, 4 |
| A+B*√(i) | 1.000 | 1.000 | | 968876351.9 | -3.862257e+03 | 2.217371e-04 | 2.40, 4.04 |
| A+B*√(i+C) | 1.000 | 1.000 | 2.000 | 1226760041.1 | -3.980491e+03 | 1.970574e-04 | 2.73, 4 |
| A+B*i+C*(i-1) | 0.500 | 0.270 | 0.270 | 21309656.1 | -1.949954e+03 | 1.495146e-03 | 0.77, 4.01 |
| A+B*i-C*(i-1) | 0.750 | 0.250 | 0.000 | 1547576.0 | -6.361170e+02 | 5.548003e-03 | 1, 2.5 |
| A+B*i+C*√(i-1) | 0.340 | 0.390 | 0.360 | 26777665.3 | -2.064386e+03 | 1.333785e-03 | 0.73, 3.95 |
| A+B*√(i)+C*(i-1) | 0.550 | 0.400 | 0.400 | 34355308.0 | -2.189229e+03 | 1.177539e-03 | 0.95, 4.008 |
| A+B*√(i)+C*√(i-1) | 0.500 | 0.700 | 0.700 | 155297117.3 | -2.945028e+03 | 5.538483e-04 | 1.2, 4.06 |

The above table shows that the top two Shammas polynomials use the following power equations:

$1+\sqrt{(i+2)}$

And,

$1+\sqrt{(i)}$

 

# RESULTS FOR THE TANGENT

## Using Power A+B* i

```
Fitting tan(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
```

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | -0.000380334336706991 | 0.000185803487314083 | -2.04697092721447 | 0.0434824363035331 |
| x1 | 1.34694988308588 | 0.0452101656125044 | 29.7930756244196 | 2.32922285888958e-49 |
| x2 | -4.42850992305786 | 0.449760269922943 | -9.84637865816069 | 4.32412877054787e-16 |
| x3 | 22.6712221295936 | 1.88672370019548 | 12.0161855852262 | 1.25072271751779e-20 |
| x4 | -59.3445650880063 | 4.14086070571792 | -14.3314565027653 | 2.83486325160753e-25 |
| x5 | 84.6014682658305 | 4.97777111309108 | 16.9958534339469 | 2.82907346719701e-30 |
| x6 | -62.1005046235862 | 3.10542989199125 | -19.9973938499595 | 1.91524421981447e-35 |
| x7 | 18.8110224119627 | 0.786617126579702 | 23.9138225908646 | 1.68144090047499e-41 |

```
Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.000199
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 6.66e+07, p-value = 1.36e-308
```

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 18.4802838293815 | 100 | 0.184802838293815 | | |
| Model | 18.4802801426441 | 7 | 2.64004002037772 | 66596476.0629308 | 1.36136936625351e-308 |
| Residual | 3.68673744333137e-06 | 93 | 3.96423381003373e-08 | | |

```
A = 0.500000, B = 0.500000
List of powers: [1.000000, 1.500000, 2.000000, 2.500000, 3.000000, 3.500000, 4.000000]
Fitting tan(x) in range (0.000000, 1.000000)
MSS of errors squared = 1.901077e-05
Corrected MSS of errors squared = 2.688529e-05
Particle swarm AICc =  -1.409524e+03
AIC =  -1.427089e+03
AICc =  -1.409524e+03
```

## Using Power A+B/i

```
Fitting tan(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B/i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
```

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 0.000327907961813827 | 0.000131855651524407 | 2.48687074101734 | 0.0146702515237926 |
| x1 | 32.5183368404979 | 0.706690132542153 | 46.0149864036177 | 8.75826988374427e-66 |
| x2 | -2515.47290620253 | 57.9740867973439 | -43.3896081019041 | 1.6241822854636e-63 |
| x3 | 36356.7202536332 | 812.896875833338 | 44.7248861872698 | 1.10111214474887e-64 |
| x4 | -180290.744757568 | 3912.44584437517 | -46.0813393792447 | 7.70262792586395e-66 |
| x5 | 384934.478744367 | 8153.28047685029 | 47.2122208768993 | 8.85390236462425e-67 |
| x6 | -366822.938952166 | 7620.83609409246 | -48.1342118401577 | 1.57172517275867e-67 |

 

```
    x7                128306.995882392         2624.3721819136     48.8905486678473     3.8936564732993e-68


Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.000155
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.1e+08, p-value = 1.21e-318
                      SumSq            DF           MeanSq                F                  pValue
                ____________________    ___    ____________________    ________________    ____________________

    Total           18.4802838278662    100      0.184802838278662
    Model           18.4802815867161      7       2.64004022667372    109552564.622737    1.20784722504455e-318
    Residual    2.24115009928028e-06     93    2.4098388164304e-08

A = 1.778018, B = 2.000000
List of powers: [3.778018, 2.778018, 2.444684, 2.278018, 2.178018, 2.111351, 2.063732]
Fitting tan(x) in range (0.000000, 1.000000)
MSS of errors squared = 1.482225e-05
Corrected MSS of errors squared = 2.096182e-05
Particle swarm AICc =  -1.459797e+03
AIC =  -1.477362e+03
AICc =  -1.459797e+03
```

## Using Power A+B*sqrt(i)

```
Fitting tan(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*sqrt(i)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                        Estimate                  SE                tStat              pValue
                    ____________________    ____________________    ________________    ____________________

    (Intercept)    0.000313082962605466    0.000125757096641013     2.48958485022277     0.0145658166822452
    x1                   65.4635889471944         1.05385529386131      62.118195285936    1.60600747224531e-77
    x2                  -841.341138156137         17.2502948083376    -48.7725657737449    4.83413389643152e-68
    x3                   4088.01047082602         91.0335673148963     44.9066272080175    7.67737829276652e-65
    x4                  -9664.70917590877         222.663543942297    -43.4049912473026    1.57392659301552e-63
    x5                   11999.7609732573         279.736606656814     42.8966416539785    4.47104970657214e-63
    x6                  -7542.05263081527         175.719441770924    -42.9209912961563    4.25187052023895e-63
    x7                   1896.42454506753         43.8094420736883     43.2880323350777    1.99930360068109e-63


Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.000149
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.19e+08, p-value = 2.33e-320
                      SumSq            DF           MeanSq                F                  pValue
                ____________________    ___    ____________________    ________________    ____________________

    Total           18.4802838294856    100      0.184802838294856
    Model           18.4802817708574      7       2.64004025297963    119265707.977254    2.32556699497475e-320
    Residual    2.05862814794955e-06     93    2.21357865370919e-08

A = 0.625852, B = 1.000000
List of powers: [1.625852, 2.040066, 2.357903, 2.625852, 2.861920, 3.075342, 3.271603]
Fitting tan(x) in range (0.000000, 1.000000)
MSS of errors squared = 1.420586e-05
Corrected MSS of errors squared = 2.009012e-05
Particle swarm AICc =  -1.468377e+03
AIC =  -1.485942e+03
```

 

```
AICc =  -1.468377e+03
```

## Using Power A+B*sqrt(i+C)

```
Fitting tan(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*sqrt(i+C)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
```

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 0.000313077576308128 | 0.000125757163777599 | 2.48954069019723 | 0.0145675105494273 |
| x1 | 65.4632553866049 | 1.05385181834306 | 62.1180836310845 | 1.60626970816994e-77 |
| x2 | -841.337011681297 | 17.2502410032203 | -48.7724786873549 | 4.8349068159643e-68 |
| x3 | 4087.99194782995 | 91.0332960441731 | 44.9065575506163 | 7.67843757882598e-65 |
| x4 | -9664.66872195291 | 222.662905620892 | -43.4049339965413 | 1.57411068471532e-63 |
| x5 | 11999.71441134 | 279.735831353996 | 42.8965940947151 | 4.4714887868337e-63 |
| x6 | -7542.02541153024 | 175.718969187496 | -42.9209518266793 | 4.25221686929932e-63 |
| x7 | 1896.41816382885 | 43.8093274128523 | 43.2879999721817 | 1.99943610986843e-63 |

```
Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.000149
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.19e+08, p-value = 2.33e-320
```

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 18.4802838293233 | 100 | 0.184802838293233 | | |
| Model | 18.4802817706951 | 7 | 2.64004025295644 | 119265707.564556 | 2.32556699497475e-320 |
| Residual | 2.05862815505498e-06 | 93 | 2.21357866134944e-08 | | |

```
A = 0.625851, B = 1.000000, C = 0.000000
List of powers: [1.625851, 2.040064, 2.357902, 2.625851, 2.861919, 3.075341, 3.271602]
Fitting tan(x) in range (0.000000, 1.000000)
MSS of errors squared = 1.420586e-05
Corrected MSS of errors squared = 2.460527e-05
Particle swarm AICc =  -1.468377e+03
AIC =  -1.485942e+03
AICc =  -1.468377e+03
```

## Using Power A+B*i+C*(i–1)

```
Fitting tan(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*i+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
```

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | -0.000381394607145454 | 0.000242222582064591 | -1.57456255273404 | 0.118753357494063 |
| x1 | 0.334164775876061 | 0.0205350540237044 | 16.2728948991477 | 5.88951856625935e-29 |
| x2 | 0.18047739932053 | 0.221144793649015 | 0.816105124351113 | 0.416525444912458 |
| x3 | 6.99114187435533 | 1.01601109092003 | 6.88097003746745 | 6.81274756315773e-10 |
| x4 | -25.3821475343546 | 2.41433948551351 | -10.5130813983088 | 1.6946732774232e-17 |
| x5 | 42.9690509412276 | 3.10439284532349 | 13.8413703040055 | 2.59398727376375e-24 |
| x6 | -35.3513100557693 | 2.05055489439108 | -17.2398749979658 | 1.0307192755765e-30 |
| x7 | 11.8155495686989 | 0.545444870625106 | 21.662225102892 | 4.14531420724408e-38 |

 

```
Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.00025
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 4.22e+07, p-value = 2.13e-299
                    SumSq           DF           MeanSq                F                 pValue
              ___________________    ___    ___________________    _____________    ____________________

    Total          18.4802838293817    100       0.184802838293817
    Model          18.4802780168553      7        2.64003971669362    42240443.6929916    2.12854431772875e-299
    Residual    5.81252638909291e-06     93    6.25002837536872e-08

A = 0.500000, B = 0.270000, C = 0.270000
List of powers: [0.770000, 1.310000, 1.850000, 2.390000, 2.930000, 3.470000, 4.010000]
Fitting tan(x) in range (0.000000, 1.000000)
MSS of errors squared = 2.387048e-05
Corrected MSS of errors squared = 4.134488e-05
Particle swarm AICc =  -1.363541e+03
AIC =  -1.381107e+03
AICc =  -1.363541e+03
```

## Using Power A+B*i–C*(i–1)

```
Fitting tan(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*i-C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                        Estimate                SE                tStat               pValue
                   ____________________    ____________________    _____________    ____________________

    (Intercept)    -0.000443503200444142    0.000679651060581877    -0.652545440103397      0.515659120972157
    x1                  28.0332641571744         2.6587116375261      10.5439280294643    1.45941728417703e-17
    x2                 -259.794007523514        22.7246465738896      -11.432257336931    2.01993461787644e-19
    x3                  1018.23793914074        80.1631200546245      12.7020747002724    4.95712118887476e-22
    x4                 -2088.04809397415         148.92141257288     -14.0211408010403    1.14778179354285e-24
    x5                  2368.54025717287        153.554265701955      15.4247766829881    2.25691807866719e-27
    x6                 -1412.81733838047        83.3414758497145     -16.9521516624944    3.39251882032519e-30
    x7                  347.403436447988        18.6122992559537      18.6652616998332    3.28713327595876e-33


Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.000683
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 5.66e+06, p-value = 8.2e-259
                    SumSq           DF           MeanSq                F                 pValue
              ___________________    ___    ___________________    _____________    ____________________

    Total          18.4802838293805    100       0.184802838293805
    Model          18.4802404610648      7        2.64003435158068    5661349.54435602    8.19662015806338e-259
    Residual    4.33683157652354e-05     93    4.66325975970273e-07

A = 0.750000, B = 0.250000, C = 0.000000
List of powers: [1.000000, 1.250000, 1.500000, 1.750000, 2.000000, 2.250000, 2.500000]
Fitting tan(x) in range (0.000000, 1.000000)
MSS of errors squared = 6.520260e-05
Corrected MSS of errors squared = 1.129342e-04
Particle swarm AICc =  -1.160560e+03
AIC =  -1.178126e+03
AICc =  -1.160560e+03
```

 

## Using Power A+B*i+C*sqrt(i–1)

```
Fitting tan(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*i+C*sqrt(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                        Estimate                SE                tStat              pValue
                    ____________________    ____________________    _____________    _____________

    (Intercept)    -0.000467031878709295    0.000240003298671393    -1.94593941539422    0.0546810624750585
    x1                 0.269065919996396     0.0118338413708282     22.7369889087473    9.31590456404649e-40
    x2                  1.57789179786784      0.28246397813354     5.58616998986635    2.29466063498553e-07
    x3                  4.57791917247804      1.64555492100122     2.78199111682803     0.00654177654261831
    x4                 -29.1866751705734      4.31112085186371    -6.77008976864007    1.13954951040413e-09
    x5                  57.9781493091025      5.78240998684992     10.026641044297    1.79851396313378e-16
    x6                 -50.8877054978129      3.87862476626389    -13.120038303378     7.0974326451403e-23
    x7                  17.2284035905251      1.03275556190251     16.6819760900512    1.04835014838455e-29


Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.00025
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 4.22e+07, p-value = 2.31e-299
                    SumSq            DF           MeanSq                F                 pValue
                ____________________    ___    ____________________    _____________    ____________________

    Total           18.4802838293814    100       0.184802838293814
    Model           18.4802780067159      7        2.64003971524512    42166889.4289576    2.30822864324086e-299
    Residual    5.82266552839883e-06     93    6.26093067569767e-08

A = 0.340000, B = 0.390000, C = 0.360000
List of powers: [0.730000, 1.480000, 2.019117, 2.523538, 3.010000, 3.484984, 3.951816]
Fitting tan(x) in range (0.000000, 1.000000)
MSS of errors squared = 2.389129e-05
Corrected MSS of errors squared = 4.138092e-05
Particle swarm AICc =  -1.363365e+03
AIC =  -1.380931e+03
AICc =  -1.363365e+03
```

## Using Power A+B*sqrt(i)+C*(i–1)

```
Fitting tan(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*sqrt(i)+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                        Estimate                SE                tStat              pValue
                    ____________________    ____________________    _____________    _____________

    (Intercept)    -0.000401020325542179    0.000186483321343771    -2.15043534538364    0.0341141367463968
    x1                 0.954811018930887     0.0305232259858661     31.2814582368526    3.75902625734777e-51
    x2                  -2.4264326356533      0.371619323288238    -6.52935001921656    3.45089120042418e-09
    x3                  17.7234938408071       1.77112985933793     10.0068855749699    1.97993002005659e-16
    x4                 -53.6231503710865       4.21997115779285    -12.7069945186859    4.84447700882519e-22
    x5                  83.0070589179934       5.35831653586796     15.4912570697072    1.69032290965954e-27
    x6                 -64.0697701999472       3.47019505942059    -18.4628728653209    7.32073019272107e-33
    x7                  19.9911009739273      0.902088336670146     22.160912807852    7.01326823232789e-39
```

```
Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 0.0002
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 6.61e+07, p-value = 1.94e-308
                     SumSq            DF           MeanSq                F                pValue
                ____________________    ___    ____________________    _____________    ____________________

    Total           18.4802838293816    100       0.184802838293816
    Model           18.4802801142881      7        2.64004001632687    66088168.6655603    1.944044326649e-308
    Residual    3.71509343466414e-06     93    3.99472412329477e-08

A = 0.550000, B = 0.400000, C = 0.400000
List of powers: [0.950000, 1.515685, 2.042820, 2.550000, 3.044427, 3.529796, 4.008301]
Fitting tan(x) in range (0.000000, 1.000000)
MSS of errors squared = 1.908374e-05
Corrected MSS of errors squared = 3.305401e-05
Particle swarm AICc =  -1.408750e+03
AIC =  -1.426315e+03
AICc =  -1.408750e+03
```

## Using Power A+B*sqrt(i)+C*sqrt(i–1)

```
Fitting tan(x) in range (0.000000, 1.000000)
Shammas polynomial power is A+B*sqrt(i)+C*sqrt(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                       Estimate                 SE               tStat              pValue
                  ____________________    ____________________    _____________    ____________________

    (Intercept)    0.000167080005545374    7.42883590882885e-05      2.249073846776     0.0268680498139929
    x1                  1.94809961852379     0.0113845186256807     171.118312734747    4.9096519879686e-118
    x2                 -30.3563493559086      0.724057236927629    -41.9253448590872    3.39187674043407e-62
    x3                  219.666286073117       5.70616177010586     38.496329918989    6.17717811962077e-59
    x4                 -668.815703940724       17.5697795882415    -38.0662546494508    1.65104576200334e-58
    x5                  1001.04725697084       25.8743473830113     38.6887924998684    3.99098438269394e-59
    x6                 -729.899550706026       18.3252826995487    -39.8301932184654    3.11093214118067e-60
    x7                  207.966900057836       5.02734153859496     41.3671715878605    1.10793496250663e-61


Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 9.13e-05
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 3.17e+08, p-value = 0
                     SumSq            DF           MeanSq                F            pValue
                ____________________    ___    ____________________    _____________    ______

    Total           18.4802838293835    100       0.184802838293835
    Model           18.4802830549418      7        2.64004043642026    317033264.434833       0
    Residual    7.74441637929613e-07     93    8.32732944010336e-09

A = 0.434588, B = 0.700000, C = 0.700000
List of powers: [1.134588, 2.124538, 2.636973, 3.047024, 3.399836, 3.714479, 4.001257]
Fitting tan(x) in range (0.000000, 1.000000)
MSS of errors squared = 8.713105e-06
Corrected MSS of errors squared = 1.509154e-05
Particle swarm AICc =  -1.567120e+03
AIC =  -1.584685e+03
AICc =  -1.567120e+03
```

 

## Comments

The following table shows the summary results for the seven models that fit the tangent function.

| Power | A | B | C | F | AICc | MSSE | Pwr Rng |
|---|---|---|---|---|---|---|---|
| A+B*i | 0.500 | 0.500 | | 66596476.0 | -1.409524e+03 | 1.901077e-05 | 1, 4 |
| A+B/i | 1.778 | 2.000 | | 109552564.6 | -1.459797e+03 | 1.482225e-05 | 2.06, 3.77 |
| A+B*√(i) | 0.625 | 1.000 | | 119265707.9 | -1.468377e+03 | 1.420586e-05 | 1.62, 3.27 |
| A+B*√(i+C) | 0.625 | 1.000 | 0.000 | 119265707.5 | -1.468377e+03 | 1.420586e-05 | 1.62, 3.27 |
| A+B*i+C*(i-1) | 0.500 | 0.270 | 0.270 | 42240443.6 | -1.363541e+03 | 2.387048e-05 | 0.77, 4.01 |
| A+B*i-C*(i-1) | 0.750 | 0.250 | 0.000 | 5661349.5 | -1.160560e+03 | 6.520260e-05 | 1, 2.5 |
| A+B*i+C*√(i-1) | 0.340 | 0.390 | 0.360 | 42166889.4 | -1.363365e+03 | 2.389129e-05 | 0.73, 3.95 |
| A+B*√(i)+C*(i-1) | 0.550 | 0.400 | 0.400 | 66088168.6 | -1.408750e+03 | 1.908374e-05 | 0.95, 4.00 |
| A+B*√(i)+C*√(i-1) | 0.434 | 0.700 | 0.700 | -1.567120e+03 | -1.567120e+03 | 8.713105e-06 | 1.13, 4.00 |

The above table shows that the top two Shammas polynomials use the following power equations:

$0.434+0.7*\sqrt{(i)} + 0.7*\sqrt{(i-1)}$

And,

$0.625+ \sqrt{(i)}$

 

# RESULTS FOR THE HYPERBOLIC TANGENT FUNCTION

## Using Power A+B* i

```
Fitting tanh(x) in range (0.000000, 3.000000)
Shammas polynomial power is A+B*i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat               pValue
                ____________________    ____________________    ________________    ____________________

    (Intercept)    -0.00173552887328545    0.000181598085307549     -9.556977819156     5.14675091961746e-19
    x1                1.33828123410121      0.0134948692252479     99.1696334187061     1.8355376013959e-227
    x2               -2.41423813894805      0.0745862626321958    -32.3684020856935     9.17611469117307e-99
    x3                6.88956163506566      0.175957385904541      39.154716919944    1.94479834921235e-118
    x4               -9.66567908560466      0.218873769127053     -44.1609751783178    1.55771806816332e-131
    x5                6.3720090150864       0.149878843456582      42.5143994184361    2.38306372534742e-127
    x6               -1.99967451993254      0.0534457648153704    -37.4150230021118    1.29216873451932e-113
    x7               0.242772293465347      0.00775677539233889    31.2980950441242    1.92629141381761e-95


Number of observations: 301, Error degrees of freedom: 293
Root Mean Squared Error: 0.000252
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 5.2e+07, p-value = 0
                    SumSq             DF            MeanSq                 F             pValue
                ____________________    ___    ____________________    ________________    ______

    Total           23.0506117213921    300      0.0768353724046403
    Model           23.0505931684065      7        3.29294188120093    52004135.3192267       0
    Residual    1.85529855514233e-05    293    6.33207698000794e-08

A = 0.500000, B = 0.500000
List of powers: [1.000000, 1.500000, 2.000000, 2.500000, 3.000000, 3.500000, 4.000000]
Fitting tanh(x) in range (0.000000, 3.000000)
MSS of errors squared = 1.431003e-05
Corrected MSS of errors squared = 2.023743e-05
Particle swarm AICc =  -4.110505e+03
AIC =  -4.126998e+03
AICc =  -4.110505e+03
```

## Using Power A+B/i

```
Fitting tanh(x) in range (0.000000, 3.000000)
Shammas polynomial power is A+B/i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat               pValue
                ____________________    ____________________    ________________    ____________________

    (Intercept)    0.00133765252364434    0.000101409118206813     13.1906533386508     1.66790428366356e-31
    x1               0.466911188709886      0.00636013515720889     73.4121488252755    1.08988453748989e-190
    x2               -160.127478712212        1.52833638852744    -104.772404762603    2.89480524436798e-234
    x3               3609.61513331987         30.6681636905484     117.699095705307    1.00133327929122e-248
    x4               -22170.8873645215        176.623791596402    -125.526052657637    9.26734880459301e-257
    x5                53736.0960619538        409.981679032189     131.069505810124    3.66362583337629e-262
```

| | | | | |
|---|---|---|---|---|
| x6 | -55711.4503226869 | 411.802991023264 | -135.286657788116 | 3.97635276966939e-266 |
| x7 | 20697.0471811922 | 149.299017308056 | 138.628154118971 | 3.48594320893235e-269 |

Number of observations: 301, Error degrees of freedom: 293
Root Mean Squared Error: 0.000158
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.32e+08, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 23.050611725123 | 300 | 0.0768353724170765 | | |
| Model | 23.0506043946977 | 7 | 3.29294348495681 | 131620254.326388 | 0 |
| Residual | 7.33042529077466e-06 | 293 | 2.50185163507667e-08 | | |

A = 1.644421, B = 2.000000
List of powers: [3.644421, 2.644421, 2.311088, 2.144421, 2.044421, 1.977754, 1.930135]
Fitting tanh(x) in range (0.000000, 3.000000)
MSS of errors squared = 8.994937e-06
Corrected MSS of errors squared = 1.272076e-05
Particle swarm AICc =  -4.390013e+03
AIC =  -4.406506e+03
AICc =  -4.390013e+03

## Using Power A+B*sqrt(i)

Fitting tanh(x) in range (0.000000, 3.000000)
Shammas polynomial power is A+B*sqrt(i)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 0.00115714820796833 | 8.82698382601607e-05 | 13.1092140959614 | 3.28331564740031e-31 |
| x1 | 18.4874049094318 | 0.0884928489836432 | 208.914111385983 | 7.74694932679075e-321 |
| x2 | -138.539224792646 | 0.922981277657081 | -150.099712904598 | 3.67820654893135e-279 |
| x3 | 456.443850403956 | 3.45597255812412 | 132.073922094946 | 4.06077358533767e-263 |
| x4 | -771.178872304203 | 6.33464871316897 | -121.73980077238 | 6.1823162351426e-253 |
| x5 | 702.499715322558 | 6.17380905672579 | 113.787081665127 | 1.62209290409911e-244 |
| x6 | -329.700416705225 | 3.08296530426673 | -106.942629632883 | 8.29897961590008e-237 |
| x7 | 62.7478595493856 | 0.622409294425139 | 100.81446423023 | 1.69797041690064e-229 |

Number of observations: 301, Error degrees of freedom: 293
Root Mean Squared Error: 0.000139
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.72e+08, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 23.0506117215468 | 300 | 0.0768353724051559 | | |
| Model | 23.0506060963301 | 7 | 3.29294372804716 | 171519174.412679 | 0 |
| Residual | 5.62521663027837e-06 | 293 | 1.91986915709159e-08 | | |

A = 0.494120, B = 1.000000
List of powers: [1.494120, 1.908333, 2.226171, 2.494120, 2.730188, 2.943610, 3.139871]
Fitting tanh(x) in range (0.000000, 3.000000)
MSS of errors squared = 7.879581e-06
Corrected MSS of errors squared = 1.114341e-05

 

```
Particle swarm AICc =  -4.469710e+03
AIC =  -4.486203e+03
AICc =  -4.469710e+03
```

## Using Power A+B*sqrt(i+C)

```
Fitting tanh(x) in range (0.000000, 3.000000)
Shammas polynomial power is A+B*sqrt(i+C)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                       Estimate                SE                tStat               pValue
                   ____________________    ____________________    _______________    ____________________

    (Intercept)    0.00115714245237057    8.8269864248022e-05     13.1091450318674    3.28520057365108e-31
    x1                18.4873778529422     0.0884927940272108     208.913935379387    7.74694932679075e-321
    x2               -138.539010210646      0.922980751502706    -150.099565982379    3.67924810013247e-279
    x3                456.443153674756       3.45597073595485     132.073790129663    4.06194300984613e-263
    x4               -771.17769754768       6.33464559960884    -121.739675159617    6.18414944238413e-253
    x5                702.498630672476       6.17380621044974     113.786958437962    1.62259633433434e-244
    x6               -329.699895348574       3.08296396470104    -106.942506991173    8.30169920067847e-237
    x7                62.7477572951716      0.622409038567632     100.814341384847    1.69855981508498e-229


Number of observations: 301, Error degrees of freedom: 293
Root Mean Squared Error: 0.000139
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.72e+08, p-value = 0
                       SumSq             DF            MeanSq                 F             pValue
                   ____________________    ___    ____________________    _______________    ______

    Total             23.0506117213316    300      0.0768353724044387
    Model             23.0506060961151      7        3.29294372801645    171519178.744128       0
    Residual    5.62521648816983e-06    293    1.91986910859038e-08

A = 0.494119, B = 1.000000, C = 0.000000
List of powers: [1.494119, 1.908333, 2.226170, 2.494119, 2.730187, 2.943609, 3.139871]
Fitting tanh(x) in range (0.000000, 3.000000)
MSS of errors squared = 7.879581e-06
Corrected MSS of errors squared = 1.364784e-05
Particle swarm AICc =  -4.469710e+03
AIC =  -4.486203e+03
AICc =  -4.469710e+03
```

## Using Power A+B*i+C*(i–1)

```
Fitting tanh(x) in range (0.000000, 3.000000)
Shammas polynomial power is A+B*i+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                       Estimate                SE                tStat               pValue
                   ____________________    ____________________    _______________    ____________________

    (Intercept)    0.000391376623690417    0.000289300716541454     1.35283668968838     0.177150793987653
    x1                -1.00514572351686      0.0479936871197904    -20.9432903333318    3.60084075261492e-60
    x2                 11.2170492147002       0.308715296216004     36.3346078156482    1.5169452730393e-110
    x3                -36.4497898656799       0.837794282473147    -43.5068496267139    6.93428907939283e-130
    x4                 65.3994012868346         1.1999845869063     54.5002010862837    2.15934663859398e-155
```

| | | | | |
|---|---|---|---|---|
| x5 | -62.3118279989862 | 0.950186715012339 | -65.578508954608 | 3.74829166474043e-177 |
| x6 | 29.2546464817645 | 0.39372751684497 | 74.3017575103432 | 3.82405977689083e-192 |
| x7 | -5.34350307145124 | 0.0667171462315578 | -80.0919010070566 | 3.02494168193543e-201 |

Number of observations: 301, Error degrees of freedom: 293
Root Mean Squared Error: 0.000299
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 3.67e+07, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 23.0506117213923 | 300 | 0.076835372404641 | | |
| Model | 23.0505854591244 | 7 | 3.29294077987491 | 36738321.7172835 | 0 |
| Residual | 2.62622679372271e-05 | 293 | 8.9632313778932e-08 | | |

A = 0.500000, B = 0.270000, C = 0.048491
List of powers: [0.770000, 1.088491, 1.406981, 1.725472, 2.043962, 2.362453, 2.680943]
Fitting tanh(x) in range (0.000000, 3.000000)
MSS of errors squared = 1.702549e-05
Corrected MSS of errors squared = 2.948901e-05
Particle swarm AICc =  -4.005907e+03
AIC =  -4.022400e+03
AICc =  -4.005907e+03

## Using Power A+B*i–C*(i–1)

Fitting tanh(x) in range (0.000000, 3.000000)
Shammas polynomial power is A+B*i-C*(i-1)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 0.0004637043183193 | 0.000267841751863162 | 1.7312622662213 | 0.0844576517140317 |
| x1 | -5.72767891698914 | 0.258961998053572 | -22.1178356671632 | 1.9693851634579e-64 |
| x2 | 53.9161957838799 | 1.62407756864999 | 33.1980422762058 | 2.69094387444239e-101 |
| x3 | -174.386099219913 | 4.2551696226044 | -40.982173376482 | 2.38598769582765e-123 |
| x4 | 290.528478740437 | 5.91708680162495 | 49.0999183349232 | 2.01673956276197e-143 |
| x5 | -261.860599002596 | 4.59052367376092 | -57.0437313065981 | 1.06756796722353e-160 |
| x6 | 120.417150545494 | 1.88128269274083 | 64.0080042250634 | 2.86897810467693e-174 |
| x7 | -22.1266714073458 | 0.318040522698532 | -69.5718621627329 | 3.19521965661711e-184 |

Number of observations: 301, Error degrees of freedom: 293
Root Mean Squared Error: 0.000283
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 4.1e+07, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 23.0506117213744 | 300 | 0.0768353724045813 | | |
| Model | 23.0505881804445 | 7 | 3.29294116863492 | 40985286.7006103 | 0 |
| Residual | 2.35409299307321e-05 | 293 | 8.03444707533518e-08 | | |

A = 0.750000, B = 0.250000, C = 0.004592
List of powers: [1.000000, 1.245408, 1.490816, 1.736224, 1.981633, 2.227041, 2.472449]
Fitting tanh(x) in range (0.000000, 3.000000)
MSS of errors squared = 1.611927e-05

 

```
Corrected MSS of errors squared = 2.791939e-05
Particle swarm AICc =  -4.038834e+03
AIC =  -4.055327e+03
AICc =  -4.038834e+03
```

## Using Power A+B*i+C*sqrt(i–1)

```
Fitting tanh(x) in range (0.000000, 3.000000)
Shammas polynomial power is A+B*i+C*sqrt(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                        Estimate                SE                tStat              pValue
                    ____________________    ____________________    _____________    _____________________

    (Intercept)     0.000305257053399266    0.000296612405853011     1.02914459198493       0.304260538952076
    x1                -0.130485169925351      0.0083257828658512    -15.6724204831891     1.25766480622687e-40
    x2                  6.61024312677922       0.141183718393244     46.8201517994269     4.70683222097584e-138
    x3                 -28.7744159954431       0.575941497430295    -49.9606576776067     2.13023651695469e-145
    x4                  63.7908794109912        1.05319676484372     60.5688144327491     9.84156434867318e-168
    x5                 -70.180339720336       0.990907067365883    -70.8243406789858     2.29653911769995e-186
    x6                  36.8812900596433       0.469422520786518     78.5673639983201     6.57321983025244e-199
    x7                 -7.4362602364387       0.0888927654734002    -83.6542793649938     1.49431280168468e-206


Number of observations: 301, Error degrees of freedom: 293
Root Mean Squared Error: 0.000303
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 3.58e+07, p-value = 0
                        SumSq             DF            MeanSq                 F             pValue
                    ____________________    ___    ____________________    _____________    ______

    Total              23.050611721396     300      0.0768353724046532
    Model              23.0505847692083      7        3.29294068131547    35797896.2599789       0
    Residual     2.69521877100942e-05     293    9.19869887716526e-08

A = 0.340000, B = 0.205476, C = 0.360000
List of powers: [0.545476, 1.110953, 1.465546, 1.785444, 2.087382, 2.377843, 2.660151]
Fitting tanh(x) in range (0.000000, 3.000000)
MSS of errors squared = 1.724767e-05
Corrected MSS of errors squared = 2.987385e-05
Particle swarm AICc =  -3.998101e+03
AIC =  -4.014594e+03
AICc =  -3.998101e+03
```

## Using Power A+B*sqrt(i)+C*(i–1)

```
Fitting tanh(x) in range (0.000000, 3.000000)
Shammas polynomial power is A+B*sqrt(i)+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                        Estimate                SE                tStat              pValue
                    ____________________    ____________________    _____________    _____________________

    (Intercept)    -0.00195187321616303    0.000189740944670021    -10.2870427864557     2.15783358637562e-21
    x1                 1.01309616504276      0.0100549899499222     100.755562172452     2.00548869028332e-229
    x2                -1.33466686485747      0.0629017301631498    -21.2182854334167      3.57941852198582e-61
    x3                 5.28697184901668       0.163484270543142     32.3393304533325     1.12739827383547e-98
```

| | | | | |
|---|---|---|---|---|
| x4 | -8.52995041065996 | 0.219094208134211 | -38.9327973719632 | 7.86917691928237e-118 |
| x5 | 6.12598858735799 | 0.159527871402987 | 38.4007417229495 | 2.29582570000817e-116 |
| x6 | -2.06732565202328 | 0.0600407169074281 | -34.4320614161007 | 5.37198690298428e-105 |
| x7 | 0.269131569613389 | 0.00916012297428105 | 29.380781280888 | 2.46153169750145e-89 |

Number of observations: 301, Error degrees of freedom: 293
Root Mean Squared Error: 0.000263
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 4.78e+07, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 23.0506117213916 | 300 | 0.0768353724046386 | | |
| Model | 23.0505915169553 | 7 | 3.29294164527933 | 47753468.0298985 | 0 |
| Residual | 2.02044362822562e-05 | 293 | 6.89571204172567e-08 | | |

A = 0.550000, B = 0.400000, C = 0.400000
List of powers: [0.950000, 1.515685, 2.042820, 2.550000, 3.044427, 3.529796, 4.008301]
Fitting tanh(x) in range (0.000000, 3.000000)
MSS of errors squared = 1.493334e-05
Corrected MSS of errors squared = 2.586530e-05
Particle swarm AICc =  -4.084838e+03
AIC =  -4.101331e+03
AICc =  -4.084838e+03

## Using Power A+B*sqrt(i)+C*sqrt(i–1)

Fitting tanh(x) in range (0.000000, 3.000000)
Shammas polynomial power is A+B*sqrt(i)+C*sqrt(i-1)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 0.000194420520885205 | 3.67497946948728e-05 | 5.29038386471123 | 2.39394248026949e-07 |
| x1 | 1.33970448593861 | 0.0014873938736267 | 900.705932499251 | 0 |
| x2 | -5.59023749672771 | 0.0332005900396462 | -168.37765503722 | 1.29107199152208e-293 |
| x3 | 26.3483884230964 | 0.152195328285718 | 173.122189227992 | 4.07397538145012e-297 |
| x4 | -55.2121947712262 | 0.30378942374045 | -181.74495376244 | 3.032438843078e-303 |
| x5 | 56.8849319679448 | 0.308185802315308 | 184.579988891718 | 3.38195970471435e-305 |
| x6 | -28.7272443841622 | 0.15658369491963 | -183.462552719216 | 1.97359173416149e-304 |
| x7 | 5.71798695676854 | 0.031743191392365 | 180.13270581684 | 4.03289009141291e-302 |

Number of observations: 301, Error degrees of freedom: 293
Root Mean Squared Error: 6.16e-05
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 8.67e+08, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 23.0506117214049 | 300 | 0.076835372404683 | | |
| Model | 23.05061060818 | 7 | 3.29294437259714 | 866700591.232665 | 0 |
| Residual | 1.1132249255752e-06 | 293 | 3.79940247636586e-09 | | |

A = 0.432612, B = 0.633150, C = 0.700000
List of powers: [1.065762, 2.028021, 2.519209, 2.911347, 3.248378, 3.548754, 3.822412]
Fitting tanh(x) in range (0.000000, 3.000000)

 

```
MSS of errors squared = 3.505298e-06
Corrected MSS of errors squared = 6.071355e-06
Particle swarm AICc =  -4.957329e+03
AIC =  -4.973822e+03
AICc =  -4.957329e+03
```

## Comments

The following table shows the summary results for the seven models that fit the hyperbolic tangent function.

| *Power* | *A* | *B* | *C* | *F* | *AICc* | *MSSE* | *Pwr Rng* |
|---|---|---|---|---|---|---|---|
| A+B*i | 0.500 | 0.500 | | 52004135.3 | -4.110505e+03 | 1.431003e-05 | 1, 4 |
| A+B/i | 1.644 | 2.000 | | 131620254.3 | -4.390013e+03 | 8.994937e-06 | 1.93, 3.64 |
| A+B*√(i) | 0.494 | 1.000 | | 171519174.4 | -4.469710e+03 | 7.879581e-06 | 1.49, 3.13 |
| A+B*√(i+C) | 0.494 | 1.000 | 0.000 | 171519178.7 | -4.469710e+03 | 7.879581e-06 | 1.49, 3.13 |
| A+B*i+C*(i-1) | 0.500 | 0.270 | 0.048 | 36738321.7 | -4.005907e+03 | 1.702549e-05 | 0.77, 2.68 |
| A+B*i-C*(i-1) | 0.750 | 0.250 | 0.004 | 40985286.7 | -4.038834e+03 | 1.611927e-05 | 1, 2.47 |
| A+B*i+C*√(i-1) | 0.340 | 0.205 | 0.360 | 35797896.2 | -3.998101e+03 | 1.724767e-05 | 0.54, 2.66 |
| A+B*√(i)+C*(i-1) | 0.550 | 0.400 | 0.400 | 47753468.0 | -4.084838e+03 | 1.493334e-05 | 0.95, 4.008 |
| A+B*√(i)+C*√(i-1) | 0.432 | 0.633 | 0.700 | 866700591.2 | -4.957329e+03 | 3.505298e-06 | 1.06, 3.82 |

The above table shows that the top two Shammas polynomials use the following power equations:

$0.432+0.633*\sqrt{(i)}+0.7*\sqrt{(i-1)}$

And,

$0.494+\sqrt{(i)}$

 

# RESULTS FOR THE TWO-SIDED T-INVERSE DISTRIBUTION

## Using Power A+B* i

```
Fitting tinv(0.975,x) in range (2.000000, 100.000000)
Shammas polynomial power is A+B*i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat                pValue
                ________________    ________________    ________________    ____________________

    (Intercept)     52268.4738261619    1580.11816716967     33.0788386034355    1.64774584882112e-52
    x1             -265442.802188092    8392.67263118641    -31.6279228146861    7.06596492986515e-51
    x2              577379.412600186    19048.2166675138     30.3114681378488    2.42794139047334e-49
    x3             -697116.385711081     23948.118943139    -29.1094422641826    6.86209094478175e-48
    x4              504494.136276393    18013.2458206958     28.0068423702279    1.62512049623866e-46
    x5             -218806.319208925    8106.50806529501    -26.9914391556166    3.27489924733966e-45
    x6              52656.4133862782    2021.11923487857     26.0530959666226    5.68904540407223e-44
    x7             -5423.68149837244    215.368223555784    -25.183294957938     8.61535770872412e-43


Number of observations: 99, Error degrees of freedom: 91
Root Mean Squared Error: 0.00132
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 6.34e+05, p-value = 2.33e-210
                    SumSq             DF          MeanSq                F                  pValue
                ________________    ____    ____________________    ________________    ____________________

    Total           7.69602628704596    98      0.0785308804800608
    Model            7.6958683685875     7        1.09940976694107    633531.315908234    2.33275544090282e-210
    Residual    0.000157918458455697    91    1.73536767533733e-06

A = 0.000000, B = 0.100000
List of powers: [0.100000, 0.200000, 0.300000, 0.400000, 0.500000, 0.600000, 0.700000]
Fitting tinv(0.975,x) in range (2.000000, 100.000000)
MSS of errors squared = 1.269350e-04
Corrected MSS of errors squared = 1.795131e-04
Particle swarm AICc =  -1.006957e+03
AIC =  -1.024557e+03
AICc =  -1.006957e+03
```

## Using Power A+B/i

```
Fitting tinv(0.975,x) in range (2.000000, 100.000000)
Shammas polynomial power is A+B/i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat                pValue
                ________________    ________________    ________________    ____________________

    (Intercept)     78503801.4203727    2485625.00650571     31.5831234457741    7.95359344286021e-51
    x1             -33856.2314819102    1236.68704400643    -27.3765554883053    1.03760304404207e-45
    x2              23374267.0671907    797231.979635936     29.3192792866447    3.79881276858011e-48
    x3             -808670556.531707    26924873.2941812    -30.0343309955881    5.19522503407491e-49
    x4              6676549189.66432    219579496.353692     30.4060684195666    1.87515040711126e-49
    x5             -20247882441.4277    660963502.024544    -30.6338888295769    1.00925569898742e-49
```

```
    x6              25141436990.0848     816603437.50545     30.7878167484656    6.65517473954862e-50
    x7              -10863277384.326     351576178.812015    -30.8987867751258    4.93457526617325e-50


Number of observations: 99, Error degrees of freedom: 91
Root Mean Squared Error: 0.000724
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 2.1e+06, p-value = 5.18e-234
                        SumSq            DF          MeanSq                F                 pValue
                    _____________________    __    _____________________    _______________    ____________________

    Total           7.69600371601967     98      0.078530650163466
    Model           7.69595601098688      7       1.09942228728384    2097209.08472623    5.17507506186397e-234
    Residual    4.77050327845063e-05     91    5.2423112950007e-07

A = 0.000000, B = 0.365639
List of powers: [0.365639, 0.182820, 0.121880, 0.091410, 0.073128, 0.060940, 0.052234]
Fitting tinv(0.975,x) in range (2.000000, 100.000000)
MSS of errors squared = 6.979472e-05
Corrected MSS of errors squared = 9.870464e-05
Particle swarm AICc =  -1.125384e+03
AIC =  -1.142984e+03
AICc =  -1.125384e+03
```

## Using Power A+B*sqrt(i)

```
Fitting tinv(0.975,x) in range (2.000000, 100.000000)
Shammas polynomial power is A+B*sqrt(i)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                        Estimate                SE                tStat                pValue
                    _____________________    _____________    _____________________    _____________________

    (Intercept)      2228107.9127341     69838.5340221266     31.9037039355405    3.42090272215252e-51
    x1              -128599525.338367     4204657.30501092     -30.585019422417    1.15231758404037e-49
    x2               881358153.017835     29307659.9788843      30.072621070834    4.67532242247049e-49
    x3              -2550211805.51592     85889414.8768491    -29.6918055522033    1.34086082328194e-48
    x4               3882013618.04334     132136875.208635     29.3787302894358    3.21484238680957e-48
    x5              -3276024841.14222     112544419.786288    -29.1087274461329    6.87596812152111e-48
    x6               1456063685.60172     50436622.4790615     28.8691750960565     1.3562846207503e-47
    x7              -266827382.965166      9312504.203862    -28.6525919477717    2.51641787957512e-47


Number of observations: 99, Error degrees of freedom: 91
Root Mean Squared Error: 0.000833
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.58e+06, p-value = 1.78e-228
                        SumSq            DF          MeanSq                F                 pValue
                    _____________________    __    _____________________    _______________    ____________________

    Total            7.6961443845579     98      0.0785320855567133
    Model           7.69608124981046      7       1.09944017854435    1584690.8444111    1.78277554833465e-228
    Residual    6.31347474495669e-05     91    6.93788433511724e-07

A = 0.000000, B = 0.100000
List of powers: [0.100000, 0.141421, 0.173205, 0.200000, 0.223607, 0.244949, 0.264575]
Fitting tinv(0.975,x) in range (2.000000, 100.000000)
MSS of errors squared = 8.024455e-05
Corrected MSS of errors squared = 1.134829e-04
```

 

```
Particle swarm AICc =  -1.097759e+03
AIC =  -1.115359e+03
AICc =  -1.097759e+03
```

## Using Power A+B*sqrt(i+C)

```
Fitting tinv(0.975,x) in range (2.000000, 100.000000)
Shammas polynomial power is A+B*sqrt(i+C)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat                pValue
                ________________    ________________    ________________    ____________________

    (Intercept)      2227937.7550233    69812.9610725757     31.9129531364125    3.33898695627928e-51
    x1              -128598696.090189    4203412.58831415    -30.5938789943449    1.12494029585667e-49
    x2               881359386.337327     29299203.426713     30.0813429464695    4.56443858888897e-49
    x3              -2550224533.83727    85864916.9767045    -29.7004250819826    1.30910346182362e-48
    x4               3882041567.53923    132099447.791182     29.3872656733268    3.13879044562911e-48
    x5               -3276053516.2025    112512693.953589    -29.1171902572509    6.71347711174151e-48
    x6               1456078149.52217    50422454.9435594     28.8775735166412    1.32426396030298e-47
    x7              -266830285.409808    9309895.57533301    -28.6609321501723    2.45706034541435e-47


Number of observations: 99, Error degrees of freedom: 91
Root Mean Squared Error: 0.000833
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.59e+06, p-value = 1.74e-228
                    SumSq            DF           MeanSq                F                pValue
                ________________    __    ________________    ________________    ____________________

    Total           7.69615907319741    98      0.0785322354407899
    Model           7.69609597072099     7       1.09944228153157    1585504.2987129    1.74163017725636e-228
    Residual    6.31024764174981e-05    91    6.93433806785693e-07

A = 0.000003, B = 0.100000, C = 0.000000
List of powers: [0.100003, 0.141424, 0.173208, 0.200003, 0.223609, 0.244952, 0.264578]
Fitting tinv(0.975,x) in range (2.000000, 100.000000)
MSS of errors squared = 8.020880e-05
Corrected MSS of errors squared = 1.389257e-04
Particle swarm AICc =  -1.097847e+03
AIC =  -1.115447e+03
AICc =  -1.097847e+03
```

## Using Power A+B*i+C*(i–1)

```
Fitting tinv(0.975,x) in range (2.000000, 100.000000)
Shammas polynomial power is A+B*i+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat                pValue
                ________________    ________________    ________________    ____________________

    (Intercept)      52268.4738261619    1580.11816716967     33.0788386034355    1.64774584882112e-52
    x1              -265442.802188092    8392.67263118641    -31.6279228146861    7.06596492986515e-51
    x2               577379.412600186    19048.2166675138     30.3114681378488    2.42794139047334e-49
    x3              -697116.385711081     23948.118943139    -29.1094422641826    6.86209094478175e-48
    x4               504494.136276393    18013.2458206958     28.0068423702279    1.62512049623866e-46
```

| | | | | |
|---|---|---|---|---|
| x5 | -218806.319208925 | 8106.50806529501 | -26.9914391556166 | 3.27489924733966e-45 |
| x6 | 52656.4133862782 | 2021.11923487857 | 26.0530959666226 | 5.68904540407223e-44 |
| x7 | -5423.68149837244 | 215.368223555784 | -25.183294957938 | 8.61535770872412e-43 |

Number of observations: 99, Error degrees of freedom: 91
Root Mean Squared Error: 0.00132
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 6.34e+05, p-value = 2.33e-210

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 7.69602628704596 | 98 | 0.0785308804800608 | | |
| Model | 7.6958683685875 | 7 | 1.09940976694107 | 633531.315908234 | 2.33275544090282e-210 |
| Residual | 0.000157918458455697 | 91 | 1.73536767533733e-06 | | |

A = 0.000000, B = 0.100000, C = 0.000000
List of powers: [0.100000, 0.200000, 0.300000, 0.400000, 0.500000, 0.600000, 0.700000]
Fitting tinv(0.975,x) in range (2.000000, 100.000000)
MSS of errors squared = 1.269350e-04
Corrected MSS of errors squared = 2.198578e-04
Particle swarm AICc =  -1.006957e+03
AIC =  -1.024557e+03
AICc =  -1.006957e+03

## Using Power A+B*i–C*(i–1)

Fitting tinv(0.975,x) in range (2.000000, 100.000000)
Shammas polynomial power is A+B*i+C*(i-1)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 52268.4738261619 | 1580.11816716967 | 33.0788386034355 | 1.64774584882112e-52 |
| x1 | -265442.802188092 | 8392.67263118641 | -31.6279228146861 | 7.06596492986515e-51 |
| x2 | 577379.412600186 | 19048.2166675138 | 30.3114681378488 | 2.42794139047334e-49 |
| x3 | -697116.385711081 | 23948.118943139 | -29.1094422641826 | 6.86209094478175e-48 |
| x4 | 504494.136276393 | 18013.2458206958 | 28.0068423702279 | 1.62512049623866e-46 |
| x5 | -218806.319208925 | 8106.50806529501 | -26.9914391556166 | 3.27489924733966e-45 |
| x6 | 52656.4133862782 | 2021.11923487857 | 26.0530959666226 | 5.68904540407223e-44 |
| x7 | -5423.68149837244 | 215.368223555784 | -25.183294957938 | 8.61535770872412e-43 |

Number of observations: 99, Error degrees of freedom: 91
Root Mean Squared Error: 0.00132
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 6.34e+05, p-value = 2.33e-210

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 7.69602628704596 | 98 | 0.0785308804800608 | | |
| Model | 7.6958683685875 | 7 | 1.09940976694107 | 633531.315908234 | 2.33275544090282e-210 |
| Residual | 0.000157918458455697 | 91 | 1.73536767533733e-06 | | |

A = 0.000000, B = 0.100000, C = 0.000000
List of powers: [0.100000, 0.200000, 0.300000, 0.400000, 0.500000, 0.600000, 0.700000]
Fitting tinv(0.975,x) in range (2.000000, 100.000000)
MSS of errors squared = 1.269350e-04

 

```
Corrected MSS of errors squared = 2.198578e-04
Particle swarm AICc =  -1.006957e+03
AIC =  -1.024557e+03
AICc =  -1.006957e+03
```

## Using Power A+B*i+C*sqrt(i–1)

```
Fitting tinv(0.975,x) in range (2.000000, 100.000000)
Shammas polynomial power is A+B*i+C*sqrt(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat                pValue
                ________________    ________________    ________________    ____________________

    (Intercept)     52268.4738261619    1580.11816716967     33.0788386034355    1.64774584882112e-52
    x1             -265442.802188092    8392.67263118641    -31.6279228146861    7.06596492986515e-51
    x2              577379.412600186    19048.2166675138     30.3114681378488    2.42794139047334e-49
    x3             -697116.385711081     23948.118943139    -29.1094422641826    6.86209094478175e-48
    x4              504494.136276393    18013.2458206958     28.0068423702279    1.62512049623866e-46
    x5             -218806.319208925    8106.50806529501    -26.9914391556166    3.27489924733966e-45
    x6              52656.4133862782    2021.11923487857     26.0530959666226    5.68904540407223e-44
    x7             -5423.68149837244    215.368223555784     -25.183294957938    8.61535770872412e-43


Number of observations: 99, Error degrees of freedom: 91
Root Mean Squared Error: 0.00132
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 6.34e+05, p-value = 2.33e-210
                    SumSq            DF           MeanSq                F                  pValue
                ____________________    __    ____________________    ________________    ____________________

    Total           7.69602628704596    98      0.0785308804800608
    Model            7.6958683685875     7       1.09940976694107    633531.315908234    2.33275544090282e-210
    Residual    0.000157918458455697    91    1.73536767533733e-06

A = 0.000000, B = 0.100000, C = 0.000000
List of powers: [0.100000, 0.200000, 0.300000, 0.400000, 0.500000, 0.600000, 0.700000]
Fitting tinv(0.975,x) in range (2.000000, 100.000000)
MSS of errors squared = 1.269350e-04
Corrected MSS of errors squared = 2.198578e-04
Particle swarm AICc =  -1.006957e+03
AIC =  -1.024557e+03
AICc =  -1.006957e+03
```

## Using Power A+B*sqrt(i)+C*(i–1)

```
Fitting tinv(0.975,x) in range (2.000000, 100.000000)
Shammas polynomial power is A+B*sqrt(i)+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat                pValue
                ________________    ________________    ________________    ____________________

    (Intercept)      2228107.9127341    69838.5340221266     31.9037039355405    3.42090272215252e-51
    x1              -128599525.338367    4204657.30501092    -30.585019422417    1.15231758404037e-49
    x2               881358153.017835    29307659.9788843     30.072621070834    4.67532242247049e-49
    x3              -2550211805.51592    85889414.8768491    -29.6918055522033    1.34086082328194e-48
```

| | | | | |
|---|---|---|---|---|
| x4 | 3882013618.04334 | 132136875.208635 | 29.3787302894358 | 3.21484238680957e-48 |
| x5 | -3276024841.14222 | 112544419.786288 | -29.1087274461329 | 6.87596812152111e-48 |
| x6 | 1456063685.60172 | 50436622.4790615 | 28.8691750960565 | 1.3562846207503e-47 |
| x7 | -266827382.965166 | 9312504.203862 | -28.6525919477717 | 2.51641787957512e-47 |

Number of observations: 99, Error degrees of freedom: 91
Root Mean Squared Error: 0.000833
R-squared: 1, Adjusted R-Squared: 1
F-statistic vs. constant model: 1.58e+06, p-value = 1.78e-228

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 7.6961443845579 | 98 | 0.0785320855567133 | | |
| Model | 7.69608124981046 | 7 | 1.09944017854435 | 1584690.8444111 | 1.78277554833465e-228 |
| Residual | 6.31347474495669e-05 | 91 | 6.93788433511724e-07 | | |

A = 0.000000, B = 0.100000, C = 0.000000
List of powers: [0.100000, 0.141421, 0.173205, 0.200000, 0.223607, 0.244949, 0.264575]
Fitting tinv(0.975,x) in range (2.000000, 100.000000)
MSS of errors squared = 8.024455e-05
Corrected MSS of errors squared = 1.389876e-04
Particle swarm AICc = -1.097759e+03
AIC = -1.115359e+03
AICc = -1.097759e+03

## Using Power A+B*sqrt(i)+C*sqrt(i–1)

Fitting tinv(0.975,x) in range (2.000000, 100.000000)
Shammas polynomial power is A+B*sqrt(i)+C*sqrt(i-1)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 2228107.9127341 | 69838.5340221266 | 31.9037039355405 | 3.42090272215252e-51 |
| x1 | -128599525.338367 | 4204657.30501092 | -30.585019422417 | 1.15231758404037e-49 |
| x2 | 881358153.017835 | 29307659.9788843 | 30.072621070834 | 4.67532242247049e-49 |
| x3 | -2550211805.51592 | 85889414.8768491 | -29.6918055522033 | 1.34086082328194e-48 |
| x4 | 3882013618.04334 | 132136875.208635 | 29.3787302894358 | 3.21484238680957e-48 |
| x5 | -3276024841.14222 | 112544419.786288 | -29.1087274461329 | 6.87596812152111e-48 |
| x6 | 1456063685.60172 | 50436622.4790615 | 28.8691750960565 | 1.3562846207503e-47 |
| x7 | -266827382.965166 | 9312504.203862 | -28.6525919477717 | 2.51641787957512e-47 |

Number of observations: 99, Error degrees of freedom: 91
Root Mean Squared Error: 0.000833
R-squared: 1, Adjusted R-Squared: 1
F-statistic vs. constant model: 1.58e+06, p-value = 1.78e-228

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 7.6961443845579 | 98 | 0.0785320855567133 | | |
| Model | 7.69608124981046 | 7 | 1.09944017854435 | 1584690.8444111 | 1.78277554833465e-228 |
| Residual | 6.31347474495669e-05 | 91 | 6.93788433511724e-07 | | |

A = 0.000000, B = 0.100000, C = 0.000000
List of powers: [0.100000, 0.141421, 0.173205, 0.200000, 0.223607, 0.244949, 0.264575]
Fitting tinv(0.975,x) in range (2.000000, 100.000000)

 

```
MSS of errors squared = 8.024455e-05
Corrected MSS of errors squared = 1.389876e-04
Particle swarm AICc =  -1.097759e+03
AIC =  -1.115359e+03
AICc =  -1.097759e+03
```

## Comments

The following table shows the summary results for the seven models that fit the two-sided t-inverse function.

| *Power* | *A* | *B* | *C* | *F* | *AICc* | *MSSE* | *Pwr Rng* |
|---|---|---|---|---|---|---|---|
| A+B*i | 0.000 | 0.100 | | 633531.3 | -1.006957e+03 | 1.269350e-04 | 0.1, 0.7 |
| A+B/i | 0.000 | 0.365 | | 2097209.0 | -1.125384e+03 | 9.870464e-05 | 0.05, 0.36 |
| A+B*√(i) | 0.000 | 0.100 | | 1584690.8 | -1.097759e+03 | 8.024455e-05 | 0.1, 0.26 |
| A+B*√(i+C) | 0.000 | 0.100 | 0.000 | 1585504.2 | -1.097847e+03 | 8.020880e-05 | 0.1, 0.26 |
| A+B*i+C*(i-1) | 0.000 | 0.100 | 0.000 | 633531.3 | -1.006957e+03 | 1.269350e-04 | 0.1, 0.7 |
| A+B*i-C*(i-1) | 0.000 | 0.100 | 0.250 | 25181844.6 | -1.371529e+03 | 2.013381e-05 | 0.1, -0.8 |
| A+B*i+C*√(i-1) | 0.000 | 0.100 | 0.000 | 633531.3 | -1.006957e+03 | 1.269350e-04 | 0.1, 0.7 |
| A+B*√(i)+C*(i-1) | 0.000 | 0.100 | 0.000 | 1584690.8 | -1.097759e+03 | 8.024455e-05 | 0.1, 0.26 |
| A+B*√(i)+C*√(i-1) | 0.000 | 0.100 | 0.000 | 1584690.8 | -1.097759e+03 | 8.024455e-05 | 0.1, 0.26 |

The above table shows that the top two Shammas polynomials use the following power equations:

0.1*i–0.25*(i–1)

And,

0.1*√ (i)

 

# RESULTS FOR THE ONE-SIDED T-INVERSE DISTRIBUTION

## Using Power A+B* i

```
Fitting tinv(0.95,x) in range (2.000000, 100.000000)
Shammas polynomial power is A+B*i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE              tStat              pValue
                ________________    ________________    ________________    ____________________

    (Intercept)     21220.6947608789    590.900256187231     35.9124819098995     1.5757079088011e-55
    x1             -107344.416525911    3138.51996066007    -34.2022411427759    9.86773285639566e-54
    x2              232687.792994425    7123.26226139465     32.6659028484047    4.73368597403346e-52
    x3             -280080.926258397    8955.62744149096    -31.2742940780225    1.80510548578034e-50
    x4              202132.254755644    6736.22504402847     30.0067550348292    5.60548636667372e-49
    x5             -87449.0687137593    3031.50599245805    -28.8467411680267    1.44570591335465e-47
    x6              20997.1633101914    755.816810722388     27.7807572050732    3.14796102773923e-46
    x7              -2158.254071226    80.5390008910875    -26.7976265827359    5.86834290763214e-45


Number of observations: 99, Error degrees of freedom: 91
Root Mean Squared Error: 0.000493
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.42e+06, p-value = 2.76e-226
                    SumSq            DF          MeanSq                F                  pValue
                ________________    __    ____________________    ________________    ____________________

    Total           2.40972607471063    98      0.0245890415786798
    Model           2.40970399048783     7       0.344243427212548    1418485.59361335    2.75752422711502e-226
    Residual    2.20842227918183e-05    91    2.42683766943058e-07

A = 0.000000, B = 0.100000
List of powers: [0.100000, 0.200000, 0.300000, 0.400000, 0.500000, 0.600000, 0.700000]
Fitting tinv(0.95,x) in range (2.000000, 100.000000)
MSS of errors squared = 4.746854e-05
Corrected MSS of errors squared = 6.713065e-05
Particle swarm AICc =  -1.201711e+03
AIC =  -1.219311e+03
AICc =  -1.201711e+03
```

## Using Power A+B/i

```
Fitting tinv(0.95,x) in range (2.000000, 100.000000)
Shammas polynomial power is A+B/i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE              tStat              pValue
                ________________    ________________    ________________    ____________________

    (Intercept)     30114676.8542194    886318.589726245     33.9772596482726    1.72307857575145e-53
    x1             -12854.2971603216    440.816477408146    -29.1602011701118    5.94563508796017e-48
    x2              8916213.28004906    284224.986554376     31.3702654651838    1.39823128510326e-50
    x3             -309012768.478023    9599687.60381923    -32.189877549253    1.62047121022222e-51
    x4              2553622028.69185    78290315.2436295     32.6173425250021    5.36311462230454e-52
    x5             -7748727805.74271    235668421.232814    -32.8797883280588    2.73655315163486e-52
```

 

| | | | | |
|---|---|---|---|---|
| x6 | 9625156649.29341 | 291165695.327049 | 33.0573168603603 | 1.74043075249166e-52 |
| x7 | -4160056134.17769 | 125358004.484021 | -33.1854048834031 | 1.25719237006733e-52 |

Number of observations: 99, Error degrees of freedom: 91
Root Mean Squared Error: 0.000259
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 5.14e+06, p-value = 9.93e-252

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 2.409712142666 | 98 | 0.0245888994149591 | | |
| Model | 2.4097060490146 | 7 | 0.3442437212878 | 5140789.42419512 | 9.92739333120449e-252 |
| Residual | 6.09365139325746e-06 | 91 | 6.69632021237083e-08 | | |

A = 0.000000, B = 0.365760
List of powers: [0.365760, 0.182880, 0.121920, 0.091440, 0.073152, 0.060960, 0.052251]
Fitting tinv(0.95,x) in range (2.000000, 100.000000)
MSS of errors squared = 2.493087e-05
Corrected MSS of errors squared = 3.525758e-05
Particle swarm AICc =  -1.329215e+03
AIC =  -1.346815e+03
AICc =  -1.329215e+03

## Using Power A+B*sqrt(i)

Fitting tinv(0.95,x) in range (2.000000, 100.000000)
Shammas polynomial power is A+B*sqrt(i)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 867899.982221184 | 25232.0609918669 | 34.3967138673665 | 6.11020307097923e-54 |
| x1 | -49927637.53697 | 1519106.47975975 | -32.8664502470335 | 2.83143861088212e-52 |
| x2 | 341750021.719269 | 10588605.1944019 | 32.2752634029597 | 1.29804421933482e-51 |
| x3 | -987939995.973539 | 31031106.0372745 | -31.8370861414616 | 4.07408733516926e-51 |
| x4 | 1502738027.58669 | 47739915.2376634 | 31.4776015019218 | 1.05160940904019e-50 |
| x5 | -1267336765.28761 | 40661329.7959861 | -31.1681091505453 | 2.39641121870404e-50 |
| x6 | 562959011.708941 | 18222317.413081 | 30.8939307195263 | 4.99949972311359e-50 |
| x7 | -103110556.815618 | 3364527.58278714 | -30.6463698925013 | 9.75685719528346e-50 |

Number of observations: 99, Error degrees of freedom: 91
Root Mean Squared Error: 0.000301
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 3.8e+06, p-value = 9.16e-246

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 2.40974740529676 | 98 | 0.024589259237722 | | |
| Model | 2.40973916422549 | 7 | 0.344248452032213 | 3801278.75440043 | 9.15948511377701e-246 |
| Residual | 8.24107127073148e-06 | 91 | 9.0561222755291e-08 | | |

A = 0.000000, B = 0.100000
List of powers: [0.100000, 0.141421, 0.173205, 0.200000, 0.223607, 0.244949, 0.264575]
Fitting tinv(0.95,x) in range (2.000000, 100.000000)
MSS of errors squared = 2.899379e-05
Corrected MSS of errors squared = 4.100341e-05

 

```
Particle swarm AICc =  -1.299322e+03
AIC =  -1.316922e+03
AICc =  -1.299322e+03
```

## Using Power A+B*sqrt(i+C)

```
Fitting tinv(0.95,x) in range (2.000000, 100.000000)
Shammas polynomial power is A+B*sqrt(i+C)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat                pValue
                ________________    ________________    ________________    ____________________

    (Intercept)     866397.802350986     25171.554639619     34.4197176040653    5.77430241369517e-54
    x1             -49980614.7535907    1519757.34356161    -32.8872335872114    2.68499190942516e-52
    x2              342093625.803794    10592490.5344929      32.29586325235    1.23048445605702e-51
    x3             -988837689.251713    31039439.6682523    -31.8574594071398    3.86195695111347e-51
    x4              1503984907.80571    47748949.6266432     31.4977589992159    9.96915763960609e-51
    x5             -1268308447.63394    40666465.5611634     -31.188066878504    2.27198559778818e-50
    x6              563361736.415658    18223689.4853124     30.9137036641074     4.7404156567605e-50
    x7             -103179910.805023    3364638.61778993    -30.6659711564499    9.25225462165476e-50


Number of observations: 99, Error degrees of freedom: 91
Root Mean Squared Error: 0.000301
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 3.81e+06, p-value = 8.76e-246
                    SumSq            DF           MeanSq                F                    pValue
                ____________    ____    ________________    ________________    ____________________

    Total           2.40973409816984    98      0.0245891234507127
    Model           2.40972586523849     7       0.344246552176928    3805016.08921142    8.75896275998237e-246
    Residual    8.23293134999403e-06    91    9.04717730768575e-08

A = 0.000000, B = 0.100000, C = 0.001305
List of powers: [0.100065, 0.141467, 0.173243, 0.200033, 0.223636, 0.244976, 0.264600]
Fitting tinv(0.95,x) in range (2.000000, 100.000000)
MSS of errors squared = 2.897786e-05
Corrected MSS of errors squared = 5.019113e-05
Particle swarm AICc =  -1.299431e+03
AIC =  -1.317031e+03
AICc =  -1.299431e+03
```

## Using Power A+B*i+C*(i–1)

```
Fitting tinv(0.95,x) in range (2.000000, 100.000000)
Shammas polynomial power is A+B*i+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat                pValue
                ________________    ________________    ________________    ____________________

    (Intercept)     21220.6947608789    590.900256187231     35.9124819098995     1.5757079088011e-55
    x1             -107344.416525911    3138.51996066007    -34.2022411427759    9.86773285639566e-54
    x2              232687.792994425    7123.26226139465     32.6659028484047    4.73368597403346e-52
    x3             -280080.926258397    8955.62744149096    -31.2742940780225    1.80510548578034e-50
    x4              202132.254755644    6736.22504402847     30.0067550348292    5.60548636667372e-49
```

 

| | | | | |
|---|---|---|---|---|
| x5 | -87449.0687137593 | 3031.50599245805 | -28.8467411680267 | 1.44570591335465e-47 |
| x6 | 20997.1633101914 | 755.816810722388 | 27.7807572050732 | 3.14796102773923e-46 |
| x7 | -2158.254071226 | 80.5390008910875 | -26.7976265827359 | 5.86834290763214e-45 |

Number of observations: 99, Error degrees of freedom: 91
Root Mean Squared Error: 0.000493
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.42e+06, p-value = 2.76e-226

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 2.40972607471063 | 98 | 0.0245890415786798 | | |
| Model | 2.40970399048783 | 7 | 0.344243427212548 | 1418485.59361335 | 2.75752422711502e-226 |
| Residual | 2.20842227918183e-05 | 91 | 2.42683766943058e-07 | | |

A = 0.000000, B = 0.100000, C = 0.000000
List of powers: [0.100000, 0.200000, 0.300000, 0.400000, 0.500000, 0.600000, 0.700000]
Fitting tinv(0.95,x) in range (2.000000, 100.000000)
MSS of errors squared = 4.746854e-05
Corrected MSS of errors squared = 8.221792e-05
Particle swarm AICc =  -1.201711e+03
AIC =  -1.219311e+03
AICc =  -1.201711e+03

## Using Power A+B*i–C*(i–1)

Fitting tinv(0.95,x) in range (2.000000, 100.000000)
Shammas polynomial power is A+B*i-C*(i-1)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 13850.6209295534 | 460.688391669129 | 30.0650530380654 | 4.77373958092222e-49 |
| x1 | -1119.02047525044 | 38.6750854941865 | -28.9338849792238 | 1.12840316313362e-47 |
| x2 | -21598.7703658351 | 704.465904887137 | -30.6597810000406 | 9.4086962743552e-50 |
| x3 | 21655.8730050737 | 664.079513354108 | 32.6103615148358 | 5.46030603128409e-52 |
| x4 | -26411.0249333907 | 758.230844420563 | -34.832432797658 | 2.10529696998229e-54 |
| x5 | 22159.5572037053 | 592.676261921705 | 37.3889737575362 | 5.07165438830923e-57 |
| x6 | -10889.6488291781 | 269.7432193768 | -40.3704265647046 | 6.97820177762675e-60 |
| x7 | 2358.14008870749 | 53.6772473242737 | 43.9318371611262 | 4.61386792656579e-63 |

Number of observations: 99, Error degrees of freedom: 91
Root Mean Squared Error: 6.77e-05
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 7.5e+07, p-value = 1.07e-304

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 2.40972607710542 | 98 | 0.0245890416031165 | | |
| Model | 2.40972565954799 | 7 | 0.34424652279257 | 75023053.8188932 | 1.06507517682541e-304 |
| Residual | 4.17557430409943e-07 | 91 | 4.58854319131805e-09 | | |

A = 0.000000, B = 0.100000, C = 0.250000
List of powers: [0.100000, -0.050000, -0.200000, -0.350000, -0.500000, -0.650000, -0.800000]
Fitting tinv(0.95,x) in range (2.000000, 100.000000)
MSS of errors squared = 6.527140e-06

 

```
Corrected MSS of errors squared = 1.130534e-05
Particle swarm AICc =  -1.594563e+03
AIC =  -1.612163e+03
AICc =  -1.594563e+03
```

## Using Power A+B*i+C*sqrt(i–1)

```
Fitting tinv(0.95,x) in range (2.000000, 100.000000)
Shammas polynomial power is A+B*i+C*sqrt(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat                pValue
                ________________    ________________    ________________    ____________________

    (Intercept)     21220.6947608789    590.900256187231     35.9124819098995     1.5757079088011e-55
    x1             -107344.416525911    3138.51996066007    -34.2022411427759    9.86773285639566e-54
    x2              232687.792994425    7123.26226139465     32.6659028484047    4.73368597403346e-52
    x3             -280080.926258397    8955.62744149096    -31.2742940780225    1.80510548578034e-50
    x4              202132.254755644    6736.22504402847     30.0067550348292    5.60548636667372e-49
    x5             -87449.0687137593    3031.50599245805    -28.8467411680267    1.44570591335465e-47
    x6              20997.1633101914    755.816810722388     27.7807572050732    3.14796102773923e-46
    x7              -2158.254071226    80.5390008910875    -26.7976265827359    5.86834290763214e-45


Number of observations: 99, Error degrees of freedom: 91
Root Mean Squared Error: 0.000493
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.42e+06, p-value = 2.76e-226
                    SumSq            DF           MeanSq                F                    pValue
                ____________________    __    ____________________    ________________    ____________________

    Total           2.40972607471063    98      0.0245890415786798
    Model           2.40970399048783     7       0.344243427212548    1418485.59361335    2.75752422711502e-226
    Residual    2.20842227918183e-05    91    2.42683766943058e-07

A = 0.000000, B = 0.100000, C = 0.000000
List of powers: [0.100000, 0.200000, 0.300000, 0.400000, 0.500000, 0.600000, 0.700000]
Fitting tinv(0.95,x) in range (2.000000, 100.000000)
MSS of errors squared = 4.746854e-05
Corrected MSS of errors squared = 8.221792e-05
Particle swarm AICc =  -1.201711e+03
AIC =  -1.219311e+03
AICc =  -1.201711e+03
```

## Using Power A+B*sqrt(i)+C*(i–1)

```
Fitting tinv(0.95,x) in range (2.000000, 100.000000)
Shammas polynomial power is A+B*sqrt(i)+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat                pValue
                ________________    ________________    ________________    ____________________

    (Intercept)     867899.982221184    25232.0609918669     34.3967138673665    6.11020307097923e-54
    x1               -49927637.53697    1519106.47975975    -32.8664502470335    2.83143861088212e-52
    x2              341750021.719269    10588605.1944019     32.2752634029597    1.29804421933482e-51
    x3             -987939995.973539    31031106.0372745    -31.8370861414616    4.07408733516926e-51
```

| | | | | |
|---|---|---|---|---|
| x4 | 1502738027.58669 | 47739915.2376634 | 31.4776015019218 | 1.05160940904019e-50 |
| x5 | -1267336765.28761 | 40661329.7959861 | -31.1681091505453 | 2.39641121870404e-50 |
| x6 | 562959011.708941 | 18222317.413081 | 30.8939307195263 | 4.99949972311359e-50 |
| x7 | -103110556.815618 | 3364527.58278714 | -30.6463698925013 | 9.75685719528346e-50 |

Number of observations: 99, Error degrees of freedom: 91
Root Mean Squared Error: 0.000301
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 3.8e+06, p-value = 9.16e-246

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 2.40974740529676 | 98 | 0.024589259237722 | | |
| Model | 2.40973916422549 | 7 | 0.344248452032213 | 3801278.75440043 | 9.15948511377701e-246 |
| Residual | 8.24107127073148e-06 | 91 | 9.0561222755291e-08 | | |

A = 0.000000, B = 0.100000, C = 0.000000
List of powers: [0.100000, 0.141421, 0.173205, 0.200000, 0.223607, 0.244949, 0.264575]
Fitting tinv(0.95,x) in range (2.000000, 100.000000)
MSS of errors squared = 2.899379e-05
Corrected MSS of errors squared = 5.021872e-05
Particle swarm AICc =  -1.299322e+03
AIC =  -1.316922e+03
AICc =  -1.299322e+03

## Using Power A+B*sqrt(i)+C*sqrt(i–1)

Fitting tinv(0.95,x) in range (2.000000, 100.000000)
Shammas polynomial power is A+B*sqrt(i)+C*sqrt(i-1)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 867899.982221184 | 25232.0609918669 | 34.3967138673665 | 6.11020307097923e-54 |
| x1 | -49927637.53697 | 1519106.47975975 | -32.8664502470335 | 2.83143861088212e-52 |
| x2 | 341750021.719269 | 10588605.1944019 | 32.2752634029597 | 1.29804421933482e-51 |
| x3 | -987939995.973539 | 31031106.0372745 | -31.8370861414616 | 4.07408733516926e-51 |
| x4 | 1502738027.58669 | 47739915.2376634 | 31.4776015019218 | 1.05160940904019e-50 |
| x5 | -1267336765.28761 | 40661329.7959861 | -31.1681091505453 | 2.39641121870404e-50 |
| x6 | 562959011.708941 | 18222317.413081 | 30.8939307195263 | 4.99949972311359e-50 |
| x7 | -103110556.815618 | 3364527.58278714 | -30.6463698925013 | 9.75685719528346e-50 |

Number of observations: 99, Error degrees of freedom: 91
Root Mean Squared Error: 0.000301
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 3.8e+06, p-value = 9.16e-246

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 2.40974740529676 | 98 | 0.024589259237722 | | |
| Model | 2.40973916422549 | 7 | 0.344248452032213 | 3801278.75440043 | 9.15948511377701e-246 |
| Residual | 8.24107127073148e-06 | 91 | 9.0561222755291e-08 | | |

A = 0.000000, B = 0.100000, C = 0.000000
List of powers: [0.100000, 0.141421, 0.173205, 0.200000, 0.223607, 0.244949, 0.264575]
Fitting tinv(0.95,x) in range (2.000000, 100.000000)

 

```
MSS of errors squared = 2.899379e-05
Corrected MSS of errors squared = 5.021872e-05
Particle swarm AICc =  -1.299322e+03
AIC =  -1.316922e+03
AICc =  -1.299322e+03
```

## Comments

The following table shows the summary results for the seven models that fit the one-sided t-inverse function.

| *Power* | *A* | *B* | *C* | *F* | *AICc* | *MSSE* | *Pwr Rng* |
|---|---|---|---|---|---|---|---|
| A+B*i | 0.000 | 0.100 | | 1418485.5 | -1.201711e+03 | 4.746854e-05 | 0.1, 0.7 |
| A+B/i | 0.000 | 0.365 | | 5140789.4 | -1.329215e+03 | 2.493087e-05 | 0.05, 0.36 |
| A+B*√(i) | 0.000 | 0.100 | | 3801278.7 | -1.299322e+03 | 2.899379e-05 | 0.1, 0.26 |
| A+B*√(i+C) | 0.000 | 0.100 | 0.001 | 3805016.0 | -1.299431e+03 | 2.897786e-05 | 0.1, 0.26 |
| A+B*i+C*(i-1) | 0.000 | 0.100 | 0.000 | 1418485.5 | -1.201711e+03 | 4.746854e-05 | 0.1, 0.7 |
| A+B*i-C*(i-1) | 0.000 | 0.100 | 0.250 | 75023053.8 | -1.594563e+03 | 6.527140e-06 | 0.1, -0.8 |
| A+B*i+C*√(i-1) | 0.000 | 0.100 | 0.000 | 1418485.5 | -1.201711e+03 | 4.746854e-05 | 0.1, 0.7 |
| A+B*√(i)+C*(i-1) | 0.000 | 0.100 | 0.000 | 3801278.7 | -1.299322e+03 | 2.899379e-05 | 0.1, 0.26 |
| A+B*√(i)+C*√(i-1) | 0.000 | 0.100 | 0.000 | 3801278.7 | -1.299322e+03 | 2.899379e-05 | 0.1, 0.26 |

The above table shows that the top two Shammas polynomials use the following power equations:

0.365/i

And,

0.1*√ (i+0.001)

 

# RESULTS FOR THE COMMON LOGARITHM OF THE GAMMA FUNCTION

## Using Power A+B* i

```
Fitting log10Gamma(x) in range (2.000000, 100.000000)
Shammas polynomial power is A+B*i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                       Estimate          SE    tStat    pValue
                    ________________    __    _____    ______

    (Intercept)      6.53527078084081    0      Inf       0
    x1              -37.6558634046871    0     -Inf       0
    x2               81.646242861782     0      Inf       0
    x3              -92.9964355628003    0     -Inf       0
    x4               65.1737019732104    0      Inf       0
    x5              -29.4835916112019    0     -Inf       0
    x6               5.17404747870673    0      Inf       0
    x7               1.60614165681286    0      Inf       0


Number of observations: 99, Error degrees of freedom: 91
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: Inf, p-value = 0
                       SumSq          DF         MeanSq          F     pValue
                    ______________    __    ______________    ___    ______

    Total       220712.747500411     98    2252.17089286134
    Model       220712.747500411      7    31530.3925000587    Inf       0
    Residual                   0     91                   0

A = 0.086023, B = 0.141652
List of powers: [0.227675, 0.369328, 0.510980, 0.652632, 0.794284, 0.935936, 1.077588]
Fitting log10Gamma(x) in range (2.000000, 100.000000)
MSS of errors squared = 1.016754e-07
Corrected MSS of errors squared = 1.437907e-07
Particle swarm AICc =  -1.000000e+99
AIC =  -Inf
AICc =  -Inf
```

## Using Power A+B/i

```
Fitting log10Gamma(x) in range (2.000000, 100.000000)
Shammas polynomial power is A+B/i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                         Estimate          SE    tStat    pValue
                    ____________________    __    _____    ______

    (Intercept)          4.58479667173045    0      Inf       0
    x1             0.000875431026385695    0      Inf       0
    x2                   15.3706984903855    0      Inf       0
    x3                  -349.79954764452     0     -Inf       0
```

 

```
    x4                2620.79392530365    0      Inf       0
    x5               -7530.56838185866    0     -Inf       0
    x6                8908.10612653357    0      Inf       0
    x7               -3668.4888881453     0     -Inf       0


Number of observations: 99, Error degrees of freedom: 91
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: Inf, p-value = 0
                     SumSq          DF         MeanSq          F     pValue
                ________________    __    ________________    ___    ______

    Total       220712.747500404    98    2252.17089286127
    Model       220712.747500404     7    31530.3925000578    Inf       0
    Residual                   0    91                   0

A = 0.281791, B = 1.327267
List of powers: [1.609058, 0.945425, 0.724213, 0.613608, 0.547244, 0.503002, 0.471401]
Fitting log10Gamma(x) in range (2.000000, 100.000000)
MSS of errors squared = 4.980147e-08
Corrected MSS of errors squared = 7.042991e-08
Particle swarm AICc =  -1.000000e+99
AIC =  -Inf
AICc =  -Inf
```

## Using Power A+B*sqrt(i)

```
Fitting log10Gamma(x) in range (2.000000, 100.000000)
Shammas polynomial power is A+B*sqrt(i)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                     Estimate          SE    tStat    pValue
                ________________    __    _____    ______

    (Intercept)    3.65644346460212    0      Inf       0
    x1            -41.9744000688039    0     -Inf       0
    x2             154.287257960655    0      Inf       0
    x3            -256.282287113784    0     -Inf       0
    x4             196.188157991014    0      Inf       0
    x5            -25.2780778612935    0     -Inf       0
    x6             -61.586323551546    0     -Inf       0
    x7             30.9884062913761    0      Inf       0


Number of observations: 99, Error degrees of freedom: 91
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: Inf, p-value = 0
                     SumSq          DF         MeanSq          F     pValue
                ________________    __    ________________    ___    ______

    Total       220712.747500409    98    2252.17089286132
    Model       220712.747500409     7    31530.3925000584    Inf       0
    Residual                   0    91                   0

A = 0.000000, B = 0.360264
List of powers: [0.360264, 0.509491, 0.623996, 0.720529, 0.805576, 0.882464, 0.953170]
Fitting log10Gamma(x) in range (2.000000, 100.000000)
MSS of errors squared = 1.955465e-07
Corrected MSS of errors squared = 2.765446e-07
```

 

```
Particle swarm AICc =  -1.000000e+99
AIC =  -Inf
AICc =  -Inf
```

## Using Power A+B*sqrt(i+C)

```
Fitting log10Gamma(x) in range (2.000000, 100.000000)
Shammas polynomial power is A+B*sqrt(i+C)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                      Estimate          SE    tStat    pValue
                  __________________    __    _____    ______

    (Intercept)     2.80903618191575    0      Inf       0
    x1             -4098.34733739572    0     -Inf       0
    x2              24088.9076360418    0      Inf       0
    x3             -52465.1638822328    0     -Inf       0
    x4              50238.2627138938    0      Inf       0
    x5             -15089.9918319481    0     -Inf       0
    x6             -6557.04956214478    0     -Inf       0
    x7              3880.57217657583    0      Inf       0


Number of observations: 99, Error degrees of freedom: 91
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: Inf, p-value = 0
                     SumSq           DF          MeanSq           F     pValue
                  _________________    __    _________________    ___    ______

    Total       220712.747500638     98    2252.17089286366
    Model       220712.747500638      7    31530.3925000912     Inf       0
    Residual                   0     91                   0

A = 0.430644, B = 0.141447, C = 0.524561
List of powers: [0.605293, 0.655388, 0.696195, 0.731517, 0.763107, 0.791946, 0.818647]
Fitting log10Gamma(x) in range (2.000000, 100.000000)
MSS of errors squared = 2.621128e-07
Corrected MSS of errors squared = 4.539926e-07
Particle swarm AICc =  -1.000000e+99
AIC =  -Inf
AICc =  -Inf
```

## Using Power A+B*i+C*(i–1)

```
Fitting log10Gamma(x) in range (2.000000, 100.000000)
Shammas polynomial power is A+B*i+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                      Estimate          SE    tStat    pValue
                  __________________    __    _____    ______

    (Intercept)     7.95017901402682    0      Inf       0
    x1             -28.2349455359749    0     -Inf       0
    x2              43.3015698545906    0      Inf       0
    x3             -32.6952197387337    0     -Inf       0
    x4              8.59074907836052    0      Inf       0
    x5              6.23999699350626    0      Inf       0
```

 

```
    x6             -10.0934854576234    0     -Inf       0
    x7              4.94066462579192    0      Inf       0


Number of observations: 99, Error degrees of freedom: 91
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: Inf, p-value = 0
                     SumSq          DF         MeanSq          F     pValue
                 _______________    __    _______________    ___    ______

    Total        220712.74750041    98    2252.17089286133
    Model        220712.74750041     7    31530.3925000586    Inf       0
    Residual                   0    91                   0

A = 0.045952, B = 0.125850, C = 0.016988
List of powers: [0.171802, 0.314641, 0.457479, 0.600318, 0.743156, 0.885994, 1.028833]
Fitting log10Gamma(x) in range (2.000000, 100.000000)
MSS of errors squared = 1.091339e-07
Corrected MSS of errors squared = 1.890254e-07
Particle swarm AICc =  -1.000000e+99
AIC =  -Inf
AICc =  -Inf
```

## Using Power A+B*i–C*(i–1)

```
Fitting log10Gamma(x) in range (2.000000, 100.000000)
Shammas polynomial power is A+B*i-C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                     Estimate          SE    tStat    pValue
                 _______________    __    _____    ______

    (Intercept)     8.0510431139232    0      Inf       0
    x1             -36.3998946435387    0     -Inf       0
    x2              71.8105621391838    0      Inf       0
    x3             -79.3560619930403    0     -Inf       0
    x4              55.4516101123207    0      Inf       0
    x5             -25.6485000685374    0     -Inf       0
    x6              4.61288175153273    0      Inf       0
    x7              1.47795081205442    0      Inf       0


Number of observations: 99, Error degrees of freedom: 91
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: Inf, p-value = 0
                     SumSq          DF         MeanSq          F     pValue
                 _______________    __    _______________    ___    ______

    Total        220712.747500411    98    2252.17089286134
    Model        220712.747500411     7    31530.3925000588    Inf       0
    Residual                   0    91                   0

A = 0.044607, B = 0.148534, C = 0.000000
List of powers: [0.193140, 0.341674, 0.490208, 0.638741, 0.787275, 0.935809, 1.084342]
Fitting log10Gamma(x) in range (2.000000, 100.000000)
MSS of errors squared = 8.205692e-08
Corrected MSS of errors squared = 1.421268e-07
Particle swarm AICc =  -1.000000e+99
AIC =  -Inf
```

 

```
AICc =  -Inf
```

## Using Power A+B*i+C*sqrt(i–1)

```
Fitting log10Gamma(x) in range (2.000000, 100.000000)
Shammas polynomial power is A+B*i+C*sqrt(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                       Estimate          SE    tStat    pValue
                    ____________________    __    _____    ______

    (Intercept)       8.2240236273305     0      Inf       0
    x1               -25.6290418632253    0     -Inf       0
    x2                41.5267082420757    0      Inf       0
    x3               -42.9031327275285    0     -Inf       0
    x4                30.0515349726949    0      Inf       0
    x5               -16.8034211878306    0     -Inf       0
    x6                5.40211539793737    0      Inf       0
    x7               0.130768772561619    0      Inf       0


Number of observations: 99, Error degrees of freedom: 91
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: Inf, p-value = 0
                      SumSq          DF          MeanSq           F     pValue
                  _____________    ___    _________________    ___    ______

    Total        220712.74750041    98    2252.17089286133
    Model        220712.74750041     7    31530.3925000586    Inf       0
    Residual                   0    91                   0

A = 0.000000, B = 0.165916, C = 0.003779
List of powers: [0.165916, 0.335611, 0.503092, 0.670209, 0.837138, 1.003946, 1.170668]
Fitting log10Gamma(x) in range (2.000000, 100.000000)
MSS of errors squared = 9.134889e-08
Corrected MSS of errors squared = 1.582209e-07
Particle swarm AICc =  -1.000000e+99
AIC =  -Inf
AICc =  -Inf
```

## Using Power A+B*sqrt(i)+C*(i–1)

```
Fitting log10Gamma(x) in range (2.000000, 100.000000)
Shammas polynomial power is A+B*sqrt(i)+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                         Estimate          SE    tStat    pValue
                    ____________________    __    _____    ______

    (Intercept)       6.70345403085121     0      Inf       0
    x1               -8.47488591514746     0     -Inf       0
    x2                4.18567274075492     0      Inf       0
    x3               -4.26541851477234     0     -Inf       0
    x4                1.70694018941489     0      Inf       0
    x5               0.152497428677308     0      Inf       0
    x6              -0.00929996906794242   0     -Inf       0
    x7               0.000302163819806944  0      Inf       0
```

 

```
Number of observations: 99, Error degrees of freedom: 91
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: Inf, p-value = 0
                    SumSq          DF          MeanSq           F      pValue
                ________________    __    ________________    ___    ______

    Total       220712.74750041     98    2252.17089286133
    Model       220712.74750041      7    31530.3925000586    Inf       0
    Residual                  0     91                   0

A = 0.000000, B = 0.100000, C = 0.278119
List of powers: [0.100000, 0.419541, 0.729444, 1.034358, 1.336084, 1.635546, 1.933292]
Fitting log10Gamma(x) in range (2.000000, 100.000000)
MSS of errors squared = 1.359618e-07
Corrected MSS of errors squared = 2.354928e-07
Particle swarm AICc =  -1.000000e+99
AIC =  -Inf
AICc =  -Inf
```

## Using Power A+B*sqrt(i)+C*sqrt(i–1)

```
Fitting log10Gamma(x) in range (2.000000, 100.000000)
Shammas polynomial power is A+B*sqrt(i)+C*sqrt(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                      Estimate          SE    tStat    pValue
                  _________________    __    _____    ______

    (Intercept)    4.53047316562135    0      Inf       0
    x1              -12.822200723033    0     -Inf       0
    x2              44.8963868616164    0      Inf       0
    x3             -72.0241207741881    0     -Inf       0
    x4              13.4093116602654    0      Inf       0
    x5              85.7884278516069    0      Inf       0
    x6             -101.884600639884    0     -Inf       0
    x7              38.1055831360504    0      Inf       0


Number of observations: 99, Error degrees of freedom: 91
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: Inf, p-value = 0
                    SumSq          DF          MeanSq           F      pValue
                ________________    __    ________________    ___    ______

    Total       220712.74750041     98    2252.17089286133
    Model       220712.74750041      7    31530.3925000586    Inf       0
    Residual                  0     91                   0

A = 0.117838, B = 0.109893, C = 0.221760
List of powers: [0.227731, 0.495010, 0.621794, 0.721723, 0.807086, 0.882890, 0.951786]
Fitting log10Gamma(x) in range (2.000000, 100.000000)
MSS of errors squared = 1.745450e-07
Corrected MSS of errors squared = 3.023207e-07
Particle swarm AICc =  -1.000000e+99
AIC =  -Inf
AICc =  -Inf
```

## Comments

 

The following table shows the summary results for the seven models that fit the common logarithm of the gamma function.

| Power | A | B | C | F | AICc | MSSE | Pwr Rng |
|---|---|---|---|---|---|---|---|
| A+B*i | 0.086 | 0.141 | | Infinity | Infinity | 1.016754e-07 | 0.22, 1.07 |
| A+B/i | 0.281 | 1.327 | | Infinity | Infinity | 4.980147e-08 | 0.47, 1.69 |
| A+B*√(i) | 0.000 | 0.360 | | Infinity | Infinity | 1.955465e-07 | 0.36, 0.95 |
| A+B*√(i+C) | 0.430 | 0.141 | 0.524 | Infinity | Infinity | 2.621128e-07 | 0.60, 0.81 |
| A+B*i+C*(i-1) | 0.045 | 0.125 | 0.016 | Infinity | Infinity | 1.091339e-07 | 0.17, 1.02 |
| A+B*i-C*(i-1) | 0.044 | 0.148 | 0.000 | Infinity | Infinity | 8.205692e-08 | 0.19, 1.08 |
| A+B*i+C*√(i-1) | 0.000 | 0.165 | 0.003 | Infinity | Infinity | 9.134889e-08 | 0.165, 1.17 |
| A+B*√(i)+C*(i-1) | 0.000 | 0.100 | 0.278 | Infinity | Infinity | 1.359618e-07 | 0.10, 1.93 |
| A+B*√(i)+C*√(i-1) | 0.117 | 0.109 | 0.221 | Infinity | Infinity | 1.745450e-07 | 0.22, 0.95 |

The above table shows that the top two Shammas polynomials use the following power equations:

0.281+1.327/i

And,

0.044+0.148*i

 

# RESULTS FOR THE DIGAMMA FUNCTION

## Using Power A+B* i

```
Fitting digamma(x) in range (2.000000, 100.000000)
Shammas polynomial power is A+B*i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat                pValue
                ________________    ________________    ________________    ____________________

    (Intercept)    -1154.496927854    17.3217914949193    -66.6499725615928    4.95921800665753e-79
    x1              5594.34694825975    92.0033250822792     60.8059213431329    1.75082451216613e-75
    x2             -11735.7651184572    208.813014317612    -56.2022686028883    1.88280033936396e-72
    x3              13754.9238937353    262.527405638034     52.3942399853684    9.23237820544277e-70
    x4             -9699.13460073305    197.467312721121    -49.1176715127073    2.69896765264032e-67
    x5              4110.44306798617    88.8662920131109     46.254243030414    5.18267169338207e-65
    x6             -968.80304618935    22.1561948342424    -43.7260573594552    6.93930422162447e-63
    x7              97.922043980918    2.36093954273476     41.4758794998586    6.76947075967666e-61


Number of observations: 99, Error degrees of freedom: 91
Root Mean Squared Error: 1.44e-05
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 5.28e+10, p-value = 0
                    SumSq             DF           MeanSq                 F             pValue
                ________________    ____    ____________________    ________________    ______

    Total           77.1021496434692    98       0.786756629014992
    Model           77.1021496244917     7        11.0145928034988    52816602631.1941       0
    Residual    1.89775164471939e-08    91    2.08544136782351e-10

A = 0.000000, B = 0.100000
List of powers: [0.100000, 0.200000, 0.300000, 0.400000, 0.500000, 0.600000, 0.700000]
Fitting digamma(x) in range (2.000000, 100.000000)
MSS of errors squared = 1.391496e-06
Corrected MSS of errors squared = 1.967873e-06
Particle swarm AICc =  -1.900589e+03
AIC =  -1.918189e+03
AICc =  -1.900589e+03
```

## Using Power A+B/i

```
Fitting digamma(x) in range (2.000000, 100.000000)
Shammas polynomial power is A+B/i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat                pValue
                ________________    ________________    ________________    ____________________

    (Intercept)   -1012803.18512092    16235.2760234582    -62.3828744061715    1.79813017505528e-76
    x1              392.328907121984     8.072308526771     48.6018226162769    6.81231825802248e-67
    x2             -284159.875777892    5205.5743606327    -54.5876124500034    2.47746075643825e-71
    x3              10012673.247167    175826.852868545     56.9462120478994    5.87919459889829e-73
    x4             -83470552.8248907    1433992.54007133    -58.2084986444481    8.42869753978322e-74
    x5              254659924.0083    4316648.11082272     58.9948305885336    2.56446000000428e-74
```

```
    x6             -317496076.295767    5333222.50425052    -59.5317514022199    1.14784765927971e-74
    x7              137590602.026427    2296172.85852572     59.921709080199     6.42984559253445e-75


Number of observations: 99, Error degrees of freedom: 91
Root Mean Squared Error: 4.75e-06
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 4.88e+11, p-value = 0
                      SumSq            DF           MeanSq                F           pValue
                  _____________________    __    ____________________    _______________    ______

    Total           77.1021536186853     98       0.786756669578422
    Model           77.1021536166324      7        11.0145933738046    488236255427.96       0
    Residual    2.05295691557694e-09     91    2.25599661052411e-11

A = 0.000000, B = 0.365864
List of powers: [0.365864, 0.182932, 0.121955, 0.091466, 0.073173, 0.060977, 0.052266]
Fitting digamma(x) in range (2.000000, 100.000000)
MSS of errors squared = 4.575766e-07
Corrected MSS of errors squared = 6.471111e-07
Particle swarm AICc =  -2.120803e+03
AIC =  -2.138403e+03
AICc =  -2.120803e+03
```

## Using Power A+B*sqrt(i)

```
Fitting digamma(x) in range (2.000000, 100.000000)
Shammas polynomial power is A+B*sqrt(i)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                      Estimate                SE                tStat                pValue
                  _____________________    ________________    _________________    ____________________

    (Intercept)    -33075.1214995678     523.790472161324    -63.1457104652733    6.09831264439729e-77
    x1              1842383.56427166     31535.0181086343     58.4234186238571    6.07944430867739e-74
    x2             -12461085.6224837     219808.065464544    -56.6907569845011    8.75354783817037e-73
    x3              35709366.896732      644172.415728992     55.4344862102185    6.35564478272536e-72
    x4             -53932428.788949      991029.339669774    -54.4206176649826    3.24738715224008e-71
    x5              45208767.357697      844085.512452269     53.5594636926711    1.32749501040969e-70
    x6             -19975002.6346251     378275.728040915     -52.805403978932    4.63571168815731e-70
    x7              3641073.77945367     69843.9771430453     52.1315355796034    1.43758256271111e-69


Number of observations: 99, Error degrees of freedom: 91
Root Mean Squared Error: 6.25e-06
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 2.82e+11, p-value = 0
                      SumSq            DF           MeanSq                F           pValue
                  _____________________    __    ____________________    _______________    ______

    Total           77.1021513112867     98      0.786756646033537
    Model           77.1021513077353      7       11.0145930439622    282238620798.308       0
    Residual    3.55134943674784e-09     91    3.902581798624e-11

A = 0.000000, B = 0.100000
List of powers: [0.100000, 0.141421, 0.173205, 0.200000, 0.223607, 0.244949, 0.264575]
Fitting digamma(x) in range (2.000000, 100.000000)
MSS of errors squared = 6.017223e-07
Corrected MSS of errors squared = 8.509638e-07
```

 

```
Particle swarm AICc =  -2.066580e+03
AIC =  -2.084180e+03
AICc =  -2.066580e+03
```

## Using Power A+B*sqrt(i+C)

```
Fitting digamma(x) in range (2.000000, 100.000000)
Shammas polynomial power is A+B*sqrt(i+C)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat                pValue
                ________________    ________________    ________________    ____________________

    (Intercept)    -33075.1214995678    523.790472161324    -63.1457104652733    6.09831264439729e-77
    x1              1842383.56427166    31535.0181086343     58.4234186238571    6.07944430867739e-74
    x2             -12461085.6224837    219808.065464544    -56.6907569845011    8.75354783817037e-73
    x3              35709366.896732     644172.415728992     55.4344862102185    6.35564478272536e-72
    x4             -53932428.788949     991029.339669774    -54.4206176649826    3.24738715224008e-71
    x5              45208767.357697     844085.512452269     53.5594636926711    1.32749501040969e-70
    x6             -19975002.6346251    378275.728040915    -52.805403978932     4.63571168815731e-70
    x7              3641073.77945367    69843.9771430453     52.1315355796034    1.43758256271111e-69


Number of observations: 99, Error degrees of freedom: 91
Root Mean Squared Error: 6.25e-06
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 2.82e+11, p-value = 0
                    SumSq            DF          MeanSq                F            pValue
                ____________    ____    ________________    ________________    ______

    Total           77.1021513112867    98     0.786756646033537
    Model           77.1021513077353     7      11.0145930439622    282238620798.308       0
    Residual    3.55134943674784e-09    91    3.902581798624e-11

A = 0.000000, B = 0.100000, C = 0.000000
List of powers: [0.100000, 0.141421, 0.173205, 0.200000, 0.223607, 0.244949, 0.264575]
Fitting digamma(x) in range (2.000000, 100.000000)
MSS of errors squared = 6.017223e-07
Corrected MSS of errors squared = 1.042214e-06
Particle swarm AICc =  -2.066580e+03
AIC =  -2.084180e+03
AICc =  -2.066580e+03
```

## Using Power A+B*i+C*(i–1)

```
Fitting digamma(x) in range (2.000000, 100.000000)
Shammas polynomial power is A+B*i+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat                pValue
                ________________    ________________    ________________    ____________________

    (Intercept)     -1154.496927854    17.3217914949193    -66.6499725615928    4.95921800665753e-79
    x1               5594.34694825975    92.0033250822792     60.8059213431329    1.75082451216613e-75
    x2             -11735.7651184572    208.813014317612    -56.2022686028883    1.88280033936396e-72
    x3              13754.9238937353    262.527405638034     52.3942399853684    9.23237820544277e-70
    x4             -9699.13460073305    197.467312721121    -49.1176715127073    2.69896765264032e-67
```

| | | | | |
|---|---|---|---|---|
| x5 | 4110.44306798617 | 88.8662920131109 | 46.254243030414 | 5.18267169338207e-65 |
| x6 | -968.80304618935 | 22.1561948342424 | -43.7260573594552 | 6.93930422162447e-63 |
| x7 | 97.922043980918 | 2.36093954273476 | 41.4758794998586 | 6.76947075967666e-61 |

Number of observations: 99, Error degrees of freedom: 91
Root Mean Squared Error: 1.44e-05
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 5.28e+10, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 77.1021496434692 | 98 | 0.786756629014992 | | |
| Model | 77.1021496244917 | 7 | 11.0145928034988 | 52816602631.1941 | 0 |
| Residual | 1.89775164471939e-08 | 91 | 2.08544136782351e-10 | | |

A = 0.000000, B = 0.100000, C = 0.000000
List of powers: [0.100000, 0.200000, 0.300000, 0.400000, 0.500000, 0.600000, 0.700000]
Fitting digamma(x) in range (2.000000, 100.000000)
MSS of errors squared = 1.391496e-06
Corrected MSS of errors squared = 2.410142e-06
Particle swarm AICc =  -1.900589e+03
AIC =  -1.918189e+03
AICc =  -1.900589e+03

## Using Power A+B*i–C*(i–1)

Fitting digamma(x) in range (2.000000, 100.000000)
Shammas polynomial power is A+B*i-C*(i-1)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | -94.0573212717299 | 1.89277982821906 | -49.6926900157371 | 9.71789965814101e-68 |
| x1 | 10.2526944659401 | 0.158900078668835 | 64.5229036500847 | 8.9323277510262e-78 |
| x2 | 158.279632314538 | 2.89436173897805 | 54.6855046427003 | 2.11480168689399e-71 |
| x3 | -190.720624789117 | 2.72843060502584 | -69.9012188317358 | 7.05969225408192e-81 |
| x4 | 249.412367151966 | 3.11525984462724 | 80.0614971435263 | 3.71948809436464e-86 |
| x5 | -224.135855545085 | 2.43506390331487 | -92.0451636772108 | 1.33253494047691e-91 |
| x6 | 119.523359228789 | 1.10826435757463 | 107.847336614126 | 8.31010194816912e-98 |
| x7 | -29.1311243121635 | 0.220537814294831 | -132.091289674336 | 9.04425055703089e-106 |

Number of observations: 99, Error degrees of freedom: 91
Root Mean Squared Error: 2.78e-07
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.42e+14, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 77.1021496437131 | 98 | 0.786756629017481 | | |
| Model | 77.1021496437061 | 7 | 11.0145928062437 | 142202739455115 | 0 |
| Residual | 7.04858393874019e-12 | 91 | 7.74569663597824e-14 | | |

A = 0.000000, B = 0.100000, C = 0.250000
List of powers: [0.100000, -0.050000, -0.200000, -0.350000, -0.500000, -0.650000, -0.800000]
Fitting digamma(x) in range (2.000000, 100.000000)
MSS of errors squared = 2.632725e-08

 

```
Corrected MSS of errors squared = 4.560013e-08
Particle swarm AICc =  -2.686094e+03
AIC =  -2.703694e+03
AICc =  -2.686094e+03
```

## Using Power A+B*i+C*sqrt(i–1)

```
Fitting digamma(x) in range (2.000000, 100.000000)
Shammas polynomial power is A+B*i+C*sqrt(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat                pValue
                ________________    ________________    ________________    ____________________

    (Intercept)     -1154.496927854    17.3217914949193    -66.6499725615928    4.95921800665753e-79
    x1               5594.34694825975    92.0033250822792     60.8059213431329    1.75082451216613e-75
    x2              -11735.7651184572    208.813014317612    -56.2022686028883    1.88280033936396e-72
    x3               13754.9238937353    262.527405638034     52.3942399853684    9.23237820544277e-70
    x4              -9699.13460073305    197.467312721121    -49.1176715127073    2.69896765264032e-67
    x5               4110.44306798617    88.8662920131109     46.254243030414    5.18267169338207e-65
    x6               -968.80304618935    22.1561948342424    -43.7260573594552    6.93930422162447e-63
    x7               97.922043980918    2.36093954273476     41.4758794998586    6.76947075967666e-61


Number of observations: 99, Error degrees of freedom: 91
Root Mean Squared Error: 1.44e-05
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 5.28e+10, p-value = 0
                    SumSq            DF           MeanSq                F            pValue
                ____________________    __    ____________________    ________________    ______

    Total           77.1021496434692    98       0.786756629014992
    Model           77.1021496244917     7        11.0145928034988    52816602631.1941       0
    Residual    1.89775164471939e-08    91    2.08544136782351e-10

A = 0.000000, B = 0.100000, C = 0.000000
List of powers: [0.100000, 0.200000, 0.300000, 0.400000, 0.500000, 0.600000, 0.700000]
Fitting digamma(x) in range (2.000000, 100.000000)
MSS of errors squared = 1.391496e-06
Corrected MSS of errors squared = 2.410142e-06
Particle swarm AICc =  -1.900589e+03
AIC =  -1.918189e+03
AICc =  -1.900589e+03
```

## Using Power A+B*sqrt(i)+C*(i–1)

```
Fitting digamma(x) in range (2.000000, 100.000000)
Shammas polynomial power is A+B*sqrt(i)+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat                pValue
                ________________    ________________    ________________    ____________________

    (Intercept)    -33075.1214995678    523.790472161324    -63.1457104652733    6.09831264439729e-77
    x1               1842383.56427166    31535.0181086343     58.4234186238571    6.07944430867739e-74
    x2              -12461085.6224837    219808.065464544    -56.6907569845011    8.75354783817037e-73
    x3               35709366.896732    644172.415728992     55.4344862102185    6.35564478272536e-72
```

| | | | | |
|---|---|---|---|---|
| x4 | -53932428.788949 | 991029.339669774 | -54.4206176649826 | 3.24738715224008e-71 |
| x5 | 45208767.357697 | 844085.512452269 | 53.5594636926711 | 1.32749501040969e-70 |
| x6 | -19975002.6346251 | 378275.728040915 | -52.805403978932 | 4.63571168815731e-70 |
| x7 | 3641073.77945367 | 69843.9771430453 | 52.1315355796034 | 1.43758256271111e-69 |

Number of observations: 99, Error degrees of freedom: 91
Root Mean Squared Error: 6.25e-06
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 2.82e+11, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 77.1021513112867 | 98 | 0.786756646033537 | | |
| Model | 77.1021513077353 | 7 | 11.0145930439622 | 282238620798.308 | 0 |
| Residual | 3.55134943674784e-09 | 91 | 3.902581798624e-11 | | |

A = 0.000000, B = 0.100000, C = 0.000000
List of powers: [0.100000, 0.141421, 0.173205, 0.200000, 0.223607, 0.244949, 0.264575]
Fitting digamma(x) in range (2.000000, 100.000000)
MSS of errors squared = 6.017223e-07
Corrected MSS of errors squared = 1.042214e-06
Particle swarm AICc =  -2.066580e+03
AIC =  -2.084180e+03
AICc =  -2.066580e+03

## Using Power A+B*sqrt(i)+C*sqrt(i–1)

Fitting digamma(x) in range (2.000000, 100.000000)
Shammas polynomial power is A+B*sqrt(i)+C*sqrt(i-1)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | -33075.1214995678 | 523.790472161324 | -63.1457104652733 | 6.09831264439729e-77 |
| x1 | 1842383.56427166 | 31535.0181086343 | 58.4234186238571 | 6.07944430867739e-74 |
| x2 | -12461085.6224837 | 219808.065464544 | -56.6907569845011 | 8.75354783817037e-73 |
| x3 | 35709366.896732 | 644172.415728992 | 55.4344862102185 | 6.35564478272536e-72 |
| x4 | -53932428.788949 | 991029.339669774 | -54.4206176649826 | 3.24738715224008e-71 |
| x5 | 45208767.357697 | 844085.512452269 | 53.5594636926711 | 1.32749501040969e-70 |
| x6 | -19975002.6346251 | 378275.728040915 | -52.805403978932 | 4.63571168815731e-70 |
| x7 | 3641073.77945367 | 69843.9771430453 | 52.1315355796034 | 1.43758256271111e-69 |

Number of observations: 99, Error degrees of freedom: 91
Root Mean Squared Error: 6.25e-06
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 2.82e+11, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 77.1021513112867 | 98 | 0.786756646033537 | | |
| Model | 77.1021513077353 | 7 | 11.0145930439622 | 282238620798.308 | 0 |
| Residual | 3.55134943674784e-09 | 91 | 3.902581798624e-11 | | |

A = 0.000000, B = 0.100000, C = 0.000000
List of powers: [0.100000, 0.141421, 0.173205, 0.200000, 0.223607, 0.244949, 0.264575]
Fitting digamma(x) in range (2.000000, 100.000000)

 

```
MSS of errors squared = 6.017223e-07
Corrected MSS of errors squared = 1.042214e-06
Particle swarm AICc =  -2.066580e+03
AIC =  -2.084180e+03
AICc =  -2.066580e+03
```

## Comments

The following table shows the summary results for the seven models that fit the digamma function.

| *Power* | *A* | *B* | *C* | *F* | *AICc* | *MSSE* | *Pwr Rng* |
|---|---|---|---|---|---|---|---|
| A+B*i | 0.000 | 0.100 | | 52816602631.1 | -1.900589e+03 | 1.391496e-06 | 0.1, 0.7 |
| A+B/i | 0.000 | 0.365 | | 488236255427.9 | -2.120803e+03 | 4.575766e-07 | 0.05, 0.36 |
| A+B*√(i) | 0.000 | 0.100 | | 282238620798.3 | -2.066580e+03 | 6.017223e-07 | 0.1, 0.26 |
| A+B*√(i+C) | 0.000 | 0.100 | 0.000 | 282238620798.3 | -2.066580e+03 | 6.017223e-07 | 0.1, 0.26 |
| A+B*i+C*(i-1) | 0.000 | 0.100 | 0.000 | 52816602631.1 | -1.900589e+03 | 1.391496e-06 | 0.1, 0.7 |
| A+B*i-C*(i-1) | 0.000 | 0.100 | 0.250 | 142202739455115 | -2.686094e+03 | 2.632725e-08 | 0.1, -0.8 |
| A+B*i+C*√(i-1) | 0.000 | 0.100 | 0.000 | 52816602631.1 | -1.900589e+03 | 1.391496e-06 | 0.1, 0.7 |
| A+B*√(i)+C*(i-1) | 0.000 | 0.100 | 0.000 | 282238620798.3 | -2.066580e+03 | 6.017223e-07 | 0.1, 0.26 |
| A+B*√(i)+C*√(i-1) | 0.000 | 0.100 | 0.000 | 282238620798.3 | -2.066580e+03 | 6.017223e-07 | 0.1, 0.26 |

The above table shows that the top two Shammas polynomials use the following power equations:

0.365/i

And,

0.1*√ (i)

 

# RESULTS FOR THE TRIGAMMA FUNCTION

## Using Power A+B* i

```
Fitting tigamma(x) in range (1.000000, 100.000000)
Shammas polynomial power is A+B*i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat                pValue
                ________________    ________________    ________________    ____________________

    (Intercept)     10879.6938560571    246.348089321621     44.1639059836712    9.90660823651141e-64
    x1             -55926.7660785436    1346.35262199356    -41.5394638558599    2.12823184764884e-61
    x2              123182.306371465    3140.45519459563     39.2243476625452    3.14831866970076e-59
    x3             -150636.121357045    4053.12202244195    -37.1654543147182    3.35582559033769e-57
    x4              110421.590187513    3126.15120886454     35.3218967382001    2.67187686094085e-55
    x5             -48509.1634985356    1441.08875687869    -33.6614682940164    1.63637628769082e-53
    x6              11823.3238023248    367.659833130833     32.1583233654936    7.90018836530069e-52
    x7             -1233.21854992755    40.0507480581857    -30.7913986559234    3.06985743874365e-50


Number of observations: 100, Error degrees of freedom: 92
Root Mean Squared Error: 0.000528
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.65e+06, p-value = 1.41e-231
                    SumSq            DF          MeanSq                F                pValue
                ________________    __    ____________________    ____________    ____________________

    Total           3.21395269078251    99      0.0324641685937627
    Model           3.21392706838735     7       0.45913243834105    1648565.0178784    1.40745932179894e-231
    Residual    2.56223951553558e-05    92    2.78504295166911e-07

A = 0.000000, B = 0.100000
List of powers: [0.100000, 0.200000, 0.300000, 0.400000, 0.500000, 0.600000, 0.700000]
Fitting tigamma(x) in range (1.000000, 100.000000)
MSS of errors squared = 5.061857e-05
Corrected MSS of errors squared = 7.158547e-05
Particle swarm AICc =  -1.200351e+03
AIC =  -1.217934e+03
AICc =  -1.200351e+03
```

## Using Power A+B/i

```
Fitting trigamma(x) in range (1.000000, 100.000000)
Shammas polynomial power is A+B/i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat                pValue
                ________________    ________________    ________________    ____________________

    (Intercept)     35085159.3354475    831287.329415492     42.2058151182422    5.29098622311785e-62
    x1             -18519.8333267921    518.491291491416    -35.7186969785368    1.02443394278935e-55
    x2              11548859.5885607    298552.683820818     38.6828195304113     1.0522548960795e-58
    x3             -386343610.354907    9710155.71729676    -39.7875813326774    9.11606601804515e-60
    x4              3136766696.4222    77710369.0068161     40.3648410953636    2.60096379831928e-60
    x5             -9417937621.18196    231287557.156902    -40.7196034968407    1.21295042804442e-60
```

 

| | | | | |
|---|---|---|---|---|
| x6 | 11616293497.0324 | 283602821.450337 | 40.9597247221556 | 7.26227274438869e-61 |
| x7 | -4995394459.36358 | 121444799.104829 | -41.1330455991915 | 5.02345350098921e-61 |

Number of observations: 100, Error degrees of freedom: 92
Root Mean Squared Error: 0.000214
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.01e+07, p-value = 1.02e-267

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 3.21390824230147 | 99 | 0.0324637196192068 | | |
| Model | 3.21390404478054 | 7 | 0.459129149254363 | 10063054.4734724 | 1.02301947886847e-267 |
| Residual | 4.1975209259526e-06 | 92 | 4.56252274560065e-08 | | |

A = 0.000000, B = 0.314866
List of powers: [0.314866, 0.157433, 0.104955, 0.078716, 0.062973, 0.052478, 0.044981]
Fitting trigamma(x) in range (1.000000, 100.000000)
MSS of errors squared = 2.047385e-05
Corrected MSS of errors squared = 2.895440e-05
Particle swarm AICc =  -1.381385e+03
AIC =  -1.398968e+03
AICc =  -1.381385e+03

## Using Power A+B*sqrt(i)

Fitting trigamma(x) in range (1.000000, 100.000000)
Shammas polynomial power is A+B*sqrt(i)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 393644.363953996 | 9204.28606100436 | 42.7675065013181 | 1.6620794070165e-62 |
| x1 | -22993353.0573087 | 569562.391858352 | -40.3702094555201 | 2.57099798106793e-60 |
| x2 | 158384453.80737 | 4014072.98098758 | 39.4572930182258 | 1.88203937756331e-59 |
| x3 | -460081722.302858 | 11862235.0290157 | -38.7854161696739 | 8.36257124385208e-59 |
| x4 | 702675026.37209 | 18376725.983637 | 38.2372260977155 | 2.87224603373665e-58 |
| x5 | -594725154.720812 | 15747028.330485 | -37.767453149841 | 8.37283541625011e-58 |
| x6 | 265034136.499637 | 7095396.83768133 | 37.352968771546 | 2.17311095573234e-57 |
| x7 | -48687029.3172067 | 1316574.33691671 | -36.9800838069099 | 5.1663196722718e-57 |

Number of observations: 100, Error degrees of freedom: 92
Root Mean Squared Error: 0.000278
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 5.95e+06, p-value = 3.23e-257

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 3.21396787579423 | 99 | 0.0324643219777195 | | |
| Model | 3.21396077582161 | 7 | 0.459137253688801 | 5949407.07667655 | 3.23330926787845e-257 |
| Residual | 7.09997261827411e-06 | 92 | 7.71736154160229e-08 | | |

A = 0.000000, B = 0.100000
List of powers: [0.100000, 0.141421, 0.173205, 0.200000, 0.223607, 0.244949, 0.264575]
Fitting trigamma(x) in range (1.000000, 100.000000)
MSS of errors squared = 2.664423e-05
Corrected MSS of errors squared = 3.768062e-05

 

```
Particle swarm AICc =  -1.328700e+03
AIC =  -1.346283e+03
AICc =  -1.328700e+03
```

## Using Power A+B*sqrt(i+C)

```
Fitting trigamma(x) in range (1.000000, 100.000000)
Shammas polynomial power is A+B*sqrt(i+C)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat                pValue
                ________________    ________________    ________________    ____________________

    (Intercept)     393604.436666247    9201.41338629477     42.7765192304597    1.63166665829006e-62
    x1             -22993330.0394703    569444.136789341    -40.3785526164375    2.52511768206208e-60
    x2              158383885.277787     4013231.914633     39.4654205505268    1.84864563468042e-59
    x3             -460078274.342527     11859710.836324    -38.7933804366796    8.21493075212916e-59
    x4              702667526.824381    18372767.7142326     38.2450558213967    2.82175976655739e-58
    x5             -594717287.21586     15743604.4306073    -37.7751670424126    8.22624696800143e-58
    x6              265030073.713865    7093842.55070824     37.3605802242404    2.13520306347203e-57
    x7             -48686197.0099853    1316284.18914864    -36.9876029898036    5.07650145180321e-57


Number of observations: 100, Error degrees of freedom: 92
Root Mean Squared Error: 0.000278
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 5.95e+06, p-value = 3.18e-257
                    SumSq            DF           MeanSq                F                pValue
                ____________________    __    ____________________    ________________    ____________________

    Total           3.21394789485871    99       0.032464120150088
    Model           3.21394079755677     7       0.459134399650968    5951608.81359994    3.1787429332272e-257
    Residual    7.09730193815261e-06    92    7.71445862842675e-08

A = 0.000000, B = 0.100000, C = 0.000047
List of powers: [0.100002, 0.141423, 0.173206, 0.200001, 0.223608, 0.244950, 0.264576]
Fitting trigamma(x) in range (1.000000, 100.000000)
MSS of errors squared = 2.664007e-05
Corrected MSS of errors squared = 4.614196e-05
Particle swarm AICc =  -1.328732e+03
AIC =  -1.346314e+03
AICc =  -1.328732e+03
```

## Using Power A+B*i+C*(i–1)

```
Fitting trigamma(x) in range (1.000000, 100.000000)
Shammas polynomial power is A+B*i+C*(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat                pValue
                ________________    ________________    ________________    ____________________

    (Intercept)     10879.6938560571    246.348089321621     44.1639059836712    9.90660823651141e-64
    x1             -55926.7660785436    1346.35262199356    -41.5394638558599    2.12823184764884e-61
    x2              123182.306371465    3140.45519459563     39.2243476625452    3.14831866970076e-59
    x3             -150636.121357045    4053.12202244195    -37.1654543147182    3.35582559033769e-57
    x4              110421.590187513    3126.15120886454     35.3218967382001    2.67187686094085e-55
```

| | | | | |
|---|---|---|---|---|
| x5 | -48509.1634985356 | 1441.08875687869 | -33.6614682940164 | 1.63637628769082e-53 |
| x6 | 11823.3238023248 | 367.659833130833 | 32.1583233654936 | 7.90018836530069e-52 |
| x7 | -1233.21854992755 | 40.0507480581857 | -30.7913986559234 | 3.06985743874365e-50 |

Number of observations: 100, Error degrees of freedom: 92
Root Mean Squared Error: 0.000528
R-squared: 1, Adjusted R-Squared: 1
F-statistic vs. constant model: 1.65e+06, p-value = 1.41e-231

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 3.21395269078251 | 99 | 0.0324641685937627 | | |
| Model | 3.21392706838735 | 7 | 0.45913243834105 | 1648565.0178784 | 1.40745932179894e-231 |
| Residual | 2.56223951553558e-05 | 92 | 2.78504295166911e-07 | | |

A = 0.000000, B = 0.100000, C = 0.000000
List of powers: [0.100000, 0.200000, 0.300000, 0.400000, 0.500000, 0.600000, 0.700000]
Fitting trigamma(x) in range (1.000000, 100.000000)
MSS of errors squared = 5.061857e-05
Corrected MSS of errors squared = 8.767393e-05
Particle swarm AICc = -1.200351e+03
AIC = -1.217934e+03
AICc = -1.200351e+03

## Using Power A+B*i–C*(i–1)

Fitting trigamma(x) in range (1.000000, 100.000000)
Shammas polynomial power is A+B*i-C*(i-1)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 3810.11293079206 | 96.7140780282016 | 39.3956392747811 | 2.15603063237099e-59 |
| x1 | -311.835301128181 | 8.32084744142582 | -37.476387269846 | 1.63427356039291e-57 |
| x2 | -5904.09852234501 | 146.035735302018 | -40.4291354450656 | 2.26412732590355e-60 |
| x3 | 5806.40447276657 | 132.350648903385 | 43.8713713976968 | 1.77644754056287e-63 |
| x4 | -6947.2659578512 | 144.950299228943 | -47.9286072178319 | 7.2818117467495e-67 |
| x5 | 5722.20650115584 | 108.421509065587 | 52.7774105938178 | 1.39835187252431e-70 |
| x6 | -2764.4101119591 | 47.1046903419557 | -58.6865149073461 | 1.06301317016374e-74 |
| x7 | 590.530919826391 | 8.92534845575618 | 66.1633461991663 | 2.21421093456334e-79 |

Number of observations: 100, Error degrees of freedom: 92
Root Mean Squared Error: 3.38e-05
R-squared: 1, Adjusted R-Squared: 1
F-statistic vs. constant model: 4.03e+08, p-value = 0

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 3.21395269140545 | 99 | 0.032464168600055 | | |
| Model | 3.21395258646034 | 7 | 0.459136083780049 | 402501107.018228 | 0 |
| Residual | 1.04945101941922e-07 | 92 | 1.1407076298035e-09 | | |

A = 0.000000, B = 0.100000, C = 0.250000
List of powers: [0.100000, -0.050000, -0.200000, -0.350000, -0.500000, -0.650000, -0.800000]
Fitting trigamma(x) in range (1.000000, 100.000000)
MSS of errors squared = 3.239523e-06

 

```
Corrected MSS of errors squared = 5.611019e-06
Particle swarm AICc =  -1.750130e+03
AIC =  -1.767712e+03
AICc =  -1.750130e+03
```

## Using Power A+B*i+C*sqrt(i–1)

```
Fitting trigamma(x) in range (1.000000, 100.000000)
Shammas polynomial power is A+B*i+C*sqrt(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat                pValue
                ________________    ________________    ________________    ____________________

    (Intercept)     10879.6938560571    246.348089321621     44.1639059836712    9.90660823651141e-64
    x1             -55926.7660785436    1346.35262199356    -41.5394638558599    2.12823184764884e-61
    x2              123182.306371465    3140.45519459563     39.2243476625452    3.14831866970076e-59
    x3             -150636.121357045    4053.12202244195    -37.1654543147182    3.35582559033769e-57
    x4              110421.590187513    3126.15120886454     35.3218967382001    2.67187686094085e-55
    x5             -48509.1634985356    1441.08875687869    -33.6614682940164    1.63637628769082e-53
    x6              11823.3238023248    367.659833130833     32.1583233654936    7.90018836530069e-52
    x7             -1233.21854992755    40.0507480581857    -30.7913986559234    3.06985743874365e-50


Number of observations: 100, Error degrees of freedom: 92
Root Mean Squared Error: 0.000528
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.65e+06, p-value = 1.41e-231
                    SumSq            DF           MeanSq                F                pValue
                ____________________    __    ____________________    ________________    ____________________

    Total           3.21395269078251    99      0.0324641685937627
    Model           3.21392706838735     7       0.45913243834105    1648565.0178784    1.40745932179894e-231
    Residual    2.56223951553558e-05    92    2.78504295166911e-07

A = 0.000000, B = 0.100000, C = 0.000000
List of powers: [0.100000, 0.200000, 0.300000, 0.400000, 0.500000, 0.600000, 0.700000]
Fitting trigamma(x) in range (1.000000, 100.000000)
MSS of errors squared = 5.061857e-05
Corrected MSS of errors squared = 8.767393e-05
Particle swarm AICc =  -1.200351e+03
AIC =  -1.217934e+03
AICc =  -1.200351e+03
```

## Using Power A+B*sqrt(i)+C*(i–1)

```
Fitting trigamma(x) in range (1.000000, 100.000000)
Shammas polynomial power is A+B*sqrt(i)+C*sqrt(i-1)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                    Estimate                SE                tStat                pValue
                ________________    ________________    ________________    ____________________

    (Intercept)     393644.363953996    9204.28606100436     42.7675065013181     1.6620794070165e-62
    x1             -22993353.0573087    569562.391858352    -40.3702094555201    2.57099798106793e-60
    x2              158384453.80737    4014072.98098758     39.4572930182258    1.88203937756331e-59
    x3             -460081722.302858    11862235.0290157    -38.7854161696739    8.36257124385208e-59
```

| | | | | |
|---|---|---|---|---|
| x4 | 702675026.37209 | 18376725.983637 | 38.2372260977155 | 2.87224603373665e-58 |
| x5 | -594725154.720812 | 15747028.330485 | -37.767453149841 | 8.37283541625011e-58 |
| x6 | 265034136.499637 | 7095396.83768133 | 37.352968771546 | 2.17311095573234e-57 |
| x7 | -48687029.3172067 | 1316574.33691671 | -36.9800838069099 | 5.1663196722718e-57 |

Number of observations: 100, Error degrees of freedom: 92
Root Mean Squared Error: 0.000278
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 5.95e+06, p-value = 3.23e-257

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 3.21396787579423 | 99 | 0.0324643219777195 | | |
| Model | 3.21396077582161 | 7 | 0.459137253688801 | 5949407.07667655 | 3.23330926787845e-257 |
| Residual | 7.09997261827411e-06 | 92 | 7.71736154160229e-08 | | |

A = 0.000000, B = 0.100000, C = 0.000000
List of powers: [0.100000, 0.141421, 0.173205, 0.200000, 0.223607, 0.244949, 0.264575]
Fitting trigamma(x) in range (1.000000, 100.000000)
MSS of errors squared = 2.664423e-05
Corrected MSS of errors squared = 4.614915e-05
Particle swarm AICc =  -1.328700e+03
AIC =  -1.346283e+03
AICc =  -1.328700e+03

## Using Power A+B*sqrt(i)+C*sqrt(i–1)

Fitting trigamma(x) in range (1.000000, 100.000000)
Shammas polynomial power is A+B*sqrt(i)+C*(i-1)

Linear regression model:
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|---|---|---|---|---|
| (Intercept) | 393644.363953996 | 9204.28606100436 | 42.7675065013181 | 1.6620794070165e-62 |
| x1 | -22993353.0573087 | 569562.391858352 | -40.3702094555201 | 2.57099798106793e-60 |
| x2 | 158384453.80737 | 4014072.98098758 | 39.4572930182258 | 1.88203937756331e-59 |
| x3 | -460081722.302858 | 11862235.0290157 | -38.7854161696739 | 8.36257124385208e-59 |
| x4 | 702675026.37209 | 18376725.983637 | 38.2372260977155 | 2.87224603373665e-58 |
| x5 | -594725154.720812 | 15747028.330485 | -37.767453149841 | 8.37283541625011e-58 |
| x6 | 265034136.499637 | 7095396.83768133 | 37.352968771546 | 2.17311095573234e-57 |
| x7 | -48687029.3172067 | 1316574.33691671 | -36.9800838069099 | 5.1663196722718e-57 |

Number of observations: 100, Error degrees of freedom: 92
Root Mean Squared Error: 0.000278
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 5.95e+06, p-value = 3.23e-257

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| Total | 3.21396787579423 | 99 | 0.0324643219777195 | | |
| Model | 3.21396077582161 | 7 | 0.459137253688801 | 5949407.07667655 | 3.23330926787845e-257 |
| Residual | 7.09997261827411e-06 | 92 | 7.71736154160229e-08 | | |

A = 0.000000, B = 0.100000, C = 0.000000
List of powers: [0.100000, 0.141421, 0.173205, 0.200000, 0.223607, 0.244949, 0.264575]
Fitting trigamma(x) in range (1.000000, 100.000000)

 

```
MSS of errors squared = 2.664423e-05
Corrected MSS of errors squared = 4.614915e-05
Particle swarm AICc =  -1.328700e+03
AIC =  -1.346283e+03
AICc =  -1.328700e+03
```

### Comments

The following table shows the summary results for the seven models that fit the trigamma function.

| Power | A | B | C | F | AICc | MSSE | Pwr Rng |
|---|---|---|---|---|---|---|---|
| A+B*i | 0.000 | 0.100 | | 1648565.0 | -1.200351e+03 | 5.061857e-05 | 0.1, 0.7 |
| A+B/i | 0.000 | 0.314 | | 10063054.4 | -1.381385e+03 | 2.047385e-05 | 0.04, 0.31 |
| A+B*√(i) | 0.000 | 0.100 | | 5949407.0 | -1.328700e+03 | 2.664423e-05 | 0.1, 0.26 |
| A+B*√(i+C) | 0.000 | 0.100 | 0.000 | 5951608.8 | -1.328732e+03 | 2.664007e-05 | 0.1, 0.26 |
| A+B*i+C*(i-1) | 0.000 | 0.100 | 0.000 | 1648565.0 | -1.200351e+03 | 5.061857e-05 | 0.1, 0.7 |
| A+B*i-C*(i-1) | 0.000 | 0.100 | 0.250 | 402501107.0 | -1.750130e+03 | 3.239523e-06 | 0.1, -0.8 |
| A+B*i+C*√(i-1) | 0.000 | 0.100 | 0.000 | 1648565.0 | -1.200351e+03 | 5.061857e-05 | 0.1, 0.7 |
| A+B*√(i)+C*(i-1) | 0.000 | 0.100 | 0.000 | 5949407.0 | -1.328700e+03 | 2.664423e-05 | 0.1, 0.26 |
| A+B*√(i)+C*√(i-1) | 0.000 | 0.100 | 0.000 | 5949407.0 | -1.328700e+03 | 2.664423e-05 | 0.1, 0.26 |

The above table shows that the top two Shammas polynomials use the following power equations:

0.314/i

And,

0.1*√ (i)

## CONCLUSIONS

This study shows that the Shammas polynomials perform good fitting for a variety of common and special functions. The approach of using the swarm optimization function works well. Different functions are approximated by Shammas polynomials with different optimum powers—there is no single Shammas polynomial pattern that is a clear winner.

## DOCUMENT HISTORY

| Date | Version | Comments |
|---|---|---|
| 9/7/2020 | 1.00.00 | Initial release. |

 