The Rational Shammas Polynomials

By
Namir Clement Shammas

Contents

1/ Introduction

The polynomial curve fitting for data between two variables, X and Y, can become frustrating when one attempts to obtain a very good fit between these two variables, for the purpose of obtaining good projected Y values. The factors that influence (and limit) the goodness of fit are:

1. The order of the polynomial used in the curve fitting.
2. The nature, curvature, and source of data. The data could be values obtained from calculation-intensive special functions or ones observed as a set of discrete values.
3. The range of values for the independent variable X. Shorter ranges tend to give better fits than longer ones. Many researchers resort to chopping longer ranges of the independent variable into a set of smaller ranges.

4. The errors associated with the dependent variable Y (and even X values when we collect observations from experiments). This could be rounding/approximation errors or errors in the observed values of the data.
5. Optional transformations in the dependent and/or independent variables. Such transformations may favor one statistical model over another. Without such transformations, different models may offer the best/better fit.
6. Optional scaling of the dependent and/or independent variables. One can scale values of a variable to be in the range of (0, 1) or (1, 2). I personally prefer the latter since you can calculate logarithmic or reciprocal values in such ranges without generating runtime errors—it's always easier to work with small positive values. For example, raising 2 to the 7th power yields 128, whereas raising 10 to the 7th power yields 10,000,000! Big difference. I must admit that while scaling down values tends to give better model fits, de-scaling the projected values of Y tends to amplify prediction errors.

Typically, one uses regular polynomials to fit data involving two variables, X and Y. The order of such polynomials can range from 2 to 7, or even higher. Of course, higher polynomial powers can introduce rounding error calculations, especially if the independent variable X includes large values. The cure for this problem is to scale at least the values of the independent variable X to be in the range of (1, 2).

This study looks at comparing regular polynomial fit with those using special new rational polynomials (ones that do not require optimization algorithms and/or special schemes) that I present in the next section. Let me state at the get go that there is no one model that will emerge victorious in ALL cases! The factors that I listed above yield an infinite number of combinations that clearly prevent any one type of polynomial fit from consistently delivering the best results.

The study uses Excel's regression menu in the Data Analysis option. The regression menu generates a detailed regression ANOVA table that shows the regression coefficients along with many statistics that reflect the quality of the fit. You can use MATLAB, Python, and R library functions that generate regression ANOVA table with outputs like those in Excel. The output should include the Adjusted R Square, F value, and p-values for each regression coefficient.

In the next section I introduce the new rational polynomials to compete with regular polynomials in best data fitting.

2/ The Contenders

In the past I have regarded Padé rational polynomials as the obvious competitors for regular polynomials when performing curve fitting. A Padé polynomial is defined in general as:

$$Pade_{n,m}(x) = P_n(x) / Q_m(x) \quad (1)$$

Where $P_n(x)$ is a polynomial of order n and $Q_m(x)$ is a polynomial of order m. The latter polynomial always has a constant term of 1.

I never got the impression that using Padé rational polynomials followed any strict rule for selecting values for orders n and m. These values are often arbitrary. The closest that I have seen in several books is that the Padé rational polynomials used, had equal values of n and m. My approach to working with Padé rational polynomials is to employ nested loops to calculate the regression coefficients for various values of n and m, and then select the values of n and m that yield the best model (highest Adjusted R Square statistic). However, this study uses a simpler straight forward regression calculation.

In this study I try to put some order into selecting the values of the orders n and m. I present here the **Rational Shammas Polynomials** (RSP) that has the following general form:

$$P_n(x) = (a_0 + a_1x + a_3x^3 + a_5x^5 + \ldots + a_nx^n) / (1 + a_2x^2 + a_4x^4 + \ldots + a_{n-1}x^{n-1}) \quad (2)$$

As you can see, equation (2) presents a rational polynomial with the single order n. The polynomial in the numerator has odd–power terms, ranging from 1 to n. The polynomial in the denominator has even–power terms, ranging from 2 to n-1.

Next, I present the **Reversed Rational Shammas Polynomials** (RRSP) that has the following general form:

$$P_n(x) = (a_0 + a_2x^2 + a_4x^4 + a_6x^6 + \ldots + a_{n-1}x^{n-1}) / (1 + a_1x + a_3x^3 + \ldots + a_nx^n) \quad (3)$$

As you can see, equation (3) presents a rational polynomial with the orders of the terms for the numerator and denominator polynomials reversed!

Finally, I present the **Selective Rational Shammas Polynomials** (SRSP). This kind of rational polynomials derives the powers of the numerator and denominator polynomials using the following procedure:

1. Perform a polynomial regression of order n.
2. Examine the p-values of the regression coefficients. Select about (n – 1)/2 coefficients with the highest p-values.
3. Create a rational polynomial with the numerator polynomial having terms from the unselected regression coefficients. The denominator polynomial has the powers related to the regression coefficients with the highest p-values.
4. Perform the multiple regression on the edited data.
5. Compare the values in the ANOVA table in the original polynomial fit with that of the edited data to determine which one gives a better fit.

Using the Selective Rational Shammas Polynomials relies on polynomial regression calculations and the ANOVA regression table generated. This scheme seeks to identify (relatively) *weak* regression coefficients (with the highest p-values). The resulting rational polynomial model has the non–weak regression coefficients forming the numerator polynomial and the *weak* regression coefficients forming the denominator polynomial.

> My definition of the Rational Shammas Polynomial (in all their flavors) is they are rational polynomials where the powers used in the numerator and denominator polynomials are unique.

ALL common rational polynomial regression requires the following scheme to set up the regression variables. Some of these variables are terms with variable X raised to a power (1 or more). Other terms are the product of variable Y and X raised to a power (1 or more). The latter terms appear with negative signs. Why? Here is the explanation. Consider, for example, the polynomial in equation (2). We can replace $P_n(x)$ with y and write:

$$y = (a_0 + a_1x + a_3x^3 + a_5x^5 + \ldots + a_nx^n) / (1 + a_2x^2 + a_4x^4 + \ldots + a_{n-1}x^{n-1}) \qquad (4)$$

Multiplying both sides of equation (4) by the denominator polynomial, we get:

$$y\,(1 + a_2x^2 + a_4x^4 + \ldots + a_{n-1}x^{n-1}) = a_0 + a_1x + a_3x^3 + a_5x^5 + \ldots + a_nx^n \qquad (5)$$

After multiplying y with the even-power polynomial we have:

$$y + a_2yx^2 + a_4yx^4 + \ldots + a_{n-1}yx^{n-1} = a_0 + a_1x + a_3x^3 + a_5x^5 + \ldots + a_nx^n \quad (6)$$

Moving the terms that have both variables x and y to the right-hand side of the equation of we get:

$$y = a_0 + a_1x + a_3x^3 + a_5x^5 + \ldots + a_nx^n - a_2yx^2 - a_4yx^4 - \ldots - a_{n-1}yx^{n-1} \quad (7)$$

Equation (7) shows that when we calculate the terms that have both variables x and y, we take their negative values. Thus, when the regression model gives us values for coefficients like a_2, a_4, and so on, we can use these values in equations like those in (2) and (3) **without** having to take their negative values at that stage. Keep in mind that any constant a_i in equation (7) can be in itself either positive, negative, or even zero.

3/ Test Cases

As I mentioned before, there are virtually an infinite number of possible test cases. The next sections tests fitting the arctangent function using different schemes. The presentation shows how the various modeling schemes work. File **SRP Arctan(x).xlsx** has the worksheets for testing the arctangent approximations. I will be using the data in that Excel file in the next four sections. The Excel file has all the data and results. I will still describe how to fill in the data columns and perform the regression calculations, so you would know how I constructed the data and obtained the results.

4/ The Arctangent Function with X in Range (0, 10)

The following subsections show the various schemes for fitting the arctangent function with different types of polynomials.

The Polynomial Fit

The sheet **Poly 5** has the data and results for fitting the arctan function for x in the range of (0, 10) in increments of 0.1 for x. The regression uses a 5th order polynomial. The first two rows on this sheet are shown in Table 4.1.

Column	Heading	Formulas in the second row
A	X	none
B	Y	=ATAN(A2)
C	X	=A2
D	X^2	=A2^2

Column	Heading	Formulas in the second row
E	X^3	=A2^3
F	X^4	=A2^4
G	X^5	=A2^5

Table 4.1. The Poly 5 sheet configuration.

Enter the values of X in the sequence of 0, 0.1, 0.2, …, 10 in column A. Enter the headings and formulas in the above table into columns B to G of rows 1 and 2. You must copy the formulas from the second rows (in columns B to G) into rows 3 to 102. Please consult your favorite Excel book to learn how to copy these rows using simple and clever mouse clicks.

Using the Regression option in the Data→Data Analysis option using select the following:

- For **Input Y Range** text box, select the entire range for Y in column B.
- For **Input X Range** text box, select the entire ranges for columns C to G.
- Check the **Labels** check box.
- Check the **Confidence Level** check box and use the default of 95 %.
- Click the **Output Range** radio button. Click on the related text box and then click on cell I1.
- (Optional) Click on the **Residuals** check box.
- Click the **OK** button.

Excel displays the following regression ANOVA table.

SUMMARY OUTPUT					
Regression Statistics					
Multiple R	0.999696423				
R Square	0.999392938				
Adjusted R Squ	0.999360987				
Standard Error	0.008258837				
Observations	101				
ANOVA					
	df	*SS*	*MS*	*F*	*Significance F*
Regression	5	10.66754304	2.133508607	31279.26845	4.0879E-151
Residual	95	0.006479797	6.82084E-05		
Total	100	10.67402283			
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*
Intercept	0.011969715	0.004532739	2.640724608	0.009672465	0.002971091
X	1.0581459	0.009318314	113.5555137	2.8785E-103	1.039646709
X^2	-0.35733649	0.005846822	-61.1163657	4.52974E-78	-0.3689439
X^3	0.061809741	0.001490717	41.46310512	1.19624E-62	0.058850294
X^4	-0.0052207	0.000164657	-31.7064999	2.51869E-52	-0.00554759
X^5	0.000170263	6.5527E-06	25.98358887	5.88196E-45	0.000157254

Table 4.2. Regression ANOVA table for sheet Poly 5.

Take note of statistics like the Adjusted R Square, F statistic, Standard Error, and Significance F.

The Rational Shammas Polynomial Fit

The sheet **Poly 5 RSP** has the same data in columns A and B as does sheet **Poly 5**. Please make a copy of sheet **Poly 5** and label its tag as **Poly 5 RSP**. The first two rows on this sheet are shown in Table 4.3.

Column	Heading	Formulas in the second row
A	X	none
B	Y	=ATAN(A2)
C	X	=A2
D	Y*X^2	=–B2*A2^2
E	X^3	=A2^3
F	Y*X^4	=–B2*A2^4
G	X^5	=A2^5

Table 4.3. The Poly 5 RSP sheet configuration.

Enter the headings and formulas in the above table into columns D and F of row 2. You must copy the formulas from the second rows (in columns C to G) into rows 3 to 102. Sheet **Poly 5 RSP** fits the following model:

$P_5(x) = (a_0 + a_1x + a_3x^3 + a_5x^5) / (1 + a_2x^2 + a_4x^4)$

Using the Regression option in the Data→Data Analysis option using select the following:

- For **Input Y Range** text box, select the entire range for Y in column B.
- For **Input X Range** text box, select the entire ranges for columns C to G.
- Check the **Labels** check box.
- Check the **Confidence Level** check box and use the default of 95 %.
- Click the Output Range radio button. Click on the related text box and then click on cell I1.
- (Optional) Click on the **Residuals** check box.
- Click the **OK** button.

Excel displays the following regression ANOVA table.

SUMMARY OUTPUT					
Regression Statistics					
Multiple R	0.999385405				
R Square	0.998771188				
Adjusted R Squ	0.998706514				
Standard Error	0.011750192				
Observations	101				
ANOVA					
	df	*SS*	*MS*	*F*	*Significance F*
Regression	5	10.66090647	2.132181293	15443.08886	1.4391E-136
Residual	95	0.013116367	0.000138067		
Total	100	10.67402283			
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*
Intercept	0.041188177	0.006017109	6.845176766	7.46209E-10	0.029242704
X	0.853674079	0.008856968	96.38446126	1.46359E-96	0.836090775
Y*X^2	0.244983739	0.005774517	42.42497549	1.51225E-63	0.233519873
X^3	0.064504168	0.002137277	30.18053353	1.79579E-50	0.060261136
Y*X^4	0.003731202	0.000160073	23.30937193	4.45114E-41	0.003413416
X^5	0.000193412	9.9303E-06	19.47696908	6.11537E-35	0.000173698

Table 4.4. Regression ANOVA table for sheet Poly 5 RSP.

Take note of statistics like the R Square, F, Standard Error, and Significance F. Sheet **Poly 5** has an Adjusted R Square value of 0.999360 while sheet **Poly 5 RSP** has an Adjusted R Square value of 0.998706. Since the latter is a smaller value, we conclude that the Rational Shammas Polynomial did NOT give better results for this studied case.

☛	
	For convenience, I do not group together the columns with the headings that start with “Y*” (which belong to the denominator polynomials).

The Reverse Rational Shammas Polynomial Fit

The sheet **Poly 5 RRSP** has the same data in columns A and B as does sheet **Poly 5**. Please make a copy of sheet **Poly 5** and label its tag as **Poly 5 RRSP**. The first two rows on this sheet are shown in Table 4.5.

Column	Heading	Formulas in the second row
A	X	none
B	Y	=ATAN(A2)
C	Y*X	=–B2*A2
D	X^2	=A2^2
E	Y*X^3	=–B2*A2^3
F	X^4	= A2^4
G	Y*X^5	=–B2*A2^5

Table 4.5. The Poly 5 RRSP sheet configuration.

Enter the headings and formulas in the above table into columns C, E, and G of row 2. You must copy the formulas from the second rows (in columns C to G) into rows 3 to 102.

	The character – that appears in the tables of the various worksheet configurations is a MINUS SIGN!

Sheet **Poly 5 RRSP** fits the following model:

$P_5(x) = (a_0 + a_2x^2 + a_4x^4) \ / \ (1 + a_1x + a_3^3 + a_5x^5)$

Using the Regression option in the Data→Data Analysis option using select the following:

- For **Input Y Range** text box, select the entire range for Y in column B.
- For **Input X Range** text box, select the entire ranges for columns C to G.
- Check the **Labels** check box.
- Check the **Confidence Level** check box and use the default of 95 %.
- Click the Output Range radio button. Click on the related text box and then click on cell I1.
- (Optional) Click on the **Residuals** check box.
- Click the **OK** button.

Excel displays the following regression ANOVA table.

SUMMARY OUTPUT							
Regression Statistics							
Multiple R	0.991222721						
R Square	0.982522482						
Adjusted R Squ	0.981602613						
Standard Error	0.044314121						
Observations	101						
ANOVA							
	df	*SS*	*MS*	*F*	*Significance F*		
Regression	5	10.48746741	2.097493482	1068.110888	8.21963E-82		
Residual	95	0.186555425	0.001963741				
Total	100	10.67402283					
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*
Intercept	0.242636398	0.016945494	14.31863826	1.86021E-25	0.208995339	0.276277457	0.208995339
Y*X	-1.46485009	0.07585861	-19.3102681	1.17566E-34	-1.61544847	-1.31425171	-1.61544847
X^2	-0.75597306	0.051272755	-14.7441476	2.70704E-26	-0.85776235	-0.65418378	-0.85776235
Y*X^3	-0.09885944	0.008457849	-11.6884856	4.34962E-20	-0.1156504	-0.08206849	-0.1156504
X^4	-0.0131971	0.001310949	-10.0668325	1.18928E-16	-0.01579966	-0.01059454	-0.01579966
Y*X^5	-0.00028454	3.1958E-05	-8.90357235	3.64036E-14	-0.00034799	-0.0002211	-0.00034799

Table 4.6. Regression ANOVA table for sheet Poly 5 RRSP.

Take note of statistics like the R Square, F, Standard Error, and Significance F. Sheet **Poly 5** has an Adjusted R Square value of 0.999360 while sheet **Poly 5 RRSP** has an Adjusted R Square value of 0.981602. Since the latter is a lower value, we conclude that the Reverse Rational Shammas Polynomial does NOT give better results for this studied case.

The Selected Rational Shammas Polynomial Fit

The sheet **Poly 5 SRSP** has the same data in columns A and B as does sheet **Poly 5**. Please make a copy of sheet **Poly 5** and label its tag as **Poly 5 SRSP**. Looking at the ANOVA table in Table 4.2, we find that the coefficients for X^4 and X^5 have the highest p–values. So, we selected these two columns for the SRSP fit. The first two rows on this sheet are shown in Table 4.7.

Column	Heading	Formulas in the second row
A	X	none
B	Y	=ATAN(A2)
C	X	=A2
D	X^2	=A2^2
E	X^3	=A2^3
F	Y*X^4	=–B2*A2^4
G	Y*X^5	=–B2*A2^5

Table 4.7. The Poly 5 SRSP sheet configuration.

Enter the headings and formulas in the above table into columns F and G of row 2. You must copy the formulas from the second rows (in columns C to G) into rows 3 to 102. Sheet **Poly 5 SRSP** fits the following model:

$P_5(x) = (a_0 + a_1x + a_2x^2 + a_3x^3)\ /\ (1+ a_4x^4\ + a_5x^5)$

Using the Regression option in the Data→Data Analysis option using select the following:

- For **Input Y Range** text box, select the entire range for Y in column B.
- For **Input X Range** text box, select the entire ranges for columns C to G.
- Check the **Labels** check box.
- Check the **Confidence Level** check box and use the default of 95 %.
- Click the Output Range radio button. Click on the related text box and then click on cell I1.
- (Optional) Click on the **Residuals** check box.
- Click the **OK** button.

Excel displays the following regression ANOVA table.

SUMMARY OUTPUT						
Regression Statistics						
Multiple R	0.999698014					
R Square	0.99939612					
Adjusted R Squ	0.999364337					
Standard Error	0.008237162					
Observations	101					
ANOVA						
	df	*SS*	*MS*	*F*	*Significance F*	
Regression	5	10.667577	2.133515401	31444.1984	3.1848E-151	
Residual	95	0.00644583	6.78508E-05			
Total	100	10.67402283				
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	0.011811691	0.004523295	2.611302671	0.010482964	0.002831816	0.020791567
X	1.058880709	0.009312056	113.710732	2.5305E-103	1.040393941	1.077367478
X^2	-0.3571487	0.005824776	-61.3154438	3.35106E-78	-0.36871234	-0.34558506
X^3	0.0585078	0.00138394	42.27624447	2.07637E-63	0.055760331	0.061255269
Y*X^4	0.003251554	0.000101794	31.94242826	1.32212E-52	0.003049467	0.003453641
Y*X^5	-0.00010828	4.1562E-06	-26.0531779	4.70558E-45	-0.00011653	-0.00010003

Table 4.8. Regression ANOVA table for sheet Poly 5 SRSP.

Take note of statistics like the Adjusted R Square, F statistic, Standard Error, and Significance F. Sheet **Poly 5** has an Adjusted R Square value of 0.9993609 while sheet **Poly 5 SRSP** has an Adjusted R Square value of 0.9993643. Since the latter is a slightly higher value, we conclude that the Selected Rational Shammas Polynomial gives slightly better results for this studied case.

The ranking of the various curve fitting models appears in the next table.

Model	***Adjusted R Square***	***Rank***
Regular polynomial	0.999360987013234	2
Rational Shammas Polynomial	0.998706513707398	3
Reversed Rational Shammas Polynomial	0.98160261274546	4
Selected Rational Shammas Polynomial	0.999364336716669	1

Table 4.9. The ranks of the various polynomial fits

5/ The Arctangent Function with X in Range (0, 10) with Random Errors

In this section we have the values of the dependent variable Y calculated with +/- 10% random errors that are uniformly distributed. We look at how the four curve fitting models perform. Remember that random numbers generate different results.

The Polynomial Fit

The sheet **Poly 5 Err** has the data and results for fitting the arctan function for x in the range of (0, 10) in increments of 0.1 for x. The first two rows on this sheet are shown in Table 5.1.

Column	**Heading**	**Formulas in the second row**
A	X	none
B	Y	Pasted value from cell H3.
C	X	=A2
D	X^2	=A2^2
E	X^3	=A2^3
F	X^4	=A2^4
G	X^5	=A2^5
H	Y Calc	=ARCTAN(A2)*(1+0.2*(RAND()-0.5)

Table 5.1. The Poly 5 Err sheet configuration.

Enter the values of X in the sequence of 0, 0.1, 0.2, …, 10 in column A. Column H has the values of Y with random errors. After populating column H, row 3 to row 102, with copies of cell H2, copy cells H2:H102 and paste them (strictly by value) into cells B2:B102. Enter the headings and formulas in the above table into columns C to G of rows 1 and 2. You must also copy the formulas at the second rows (in columns C to G) into rows 3 to 102. As you perform calculations, the values in column H will automatically change. Ignore these changes, because you already have an initial snapshot of a set of values in column B. You can also delete column H (unless you want to generate new sets of random values).

Using the Regression option in the Data→Data Analysis option using select the following:

- For **Input Y Range** text box, select the entire range for Y in column B.
- For **Input X Range** text box, select the entire ranges for columns C to G.
- Check the **Labels** check box.
- Check the **Confidence Level** check box and use the default of 95 %.
- Click the Output Range radio button. Click on the related text box and then click on cell I1.
- (Optional) Click on the **Residuals** check box.
- Click the **OK** button.

Excel displays the following regression ANOVA table.

SUMMARY OUTPUT						
Regression Statistics						
Multiple R	0.977866888					
R Square	0.95622365					
Adjusted R Squ	0.953919632					
Standard Error	0.072373452					
Observations	101					
ANOVA						
	df	*SS*	*MS*	*F*	*Significance F*	
Regression	5	10.86931345	2.17386269	415.0243095	6.89674E-63	
Residual	95	0.49760207	0.005237917			
Total	100	11.36691552				
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	0.045307742	0.039721082	1.140647231	0.256884165	-0.03354858	0.12416406
X	0.962826739	0.081657806	11.79099444	2.65277E-20	0.800715497	1.12493798
X^2	-0.29229664	0.051236591	-5.70484175	1.3156E-07	-0.39401413	-0.19057915
X^3	0.043803293	0.013063378	3.35313681	0.00114866	0.017869209	0.069737376
X^4	-0.00305599	0.001442916	-2.11792541	0.036792066	-0.00592054	-0.00019144
X^5	7.73791E-05	5.74223E-05	1.347544179	0.181010069	-3.6619E-05	0.000191377

Table 5.2. Regression ANOVA table for sheet Poly 5 Err.

Take note of statistics like the Adjusted R Square, F statistic, Standard Error, and Significance F.

The Rational Shammas Polynomial Fit

The sheet **Poly 5 Err RSP** has the same data in columns A and B as does sheet **Poly 5**. Please make a copy of sheet **Poly Err 5** and label its tag as **Poly 5 Err RSP**. The first two rows on this sheet are shown in Table 5.3.

Column	Heading	Formulas in the second row
A	X	none
B	Y	none
C	X	=A2
D	Y*X^2	=–B2*A2^2
E	X^3	=A2^3
F	Y*X^4	=–B2*A2^4
G	X^5	=A2^5

Table 5.3. The Poly 5 Err RSP sheet configuration.

Enter the headings and formulas in the above table into columns D and F of row 2. You must also copy the formulas at the second rows (in columns C to G) into rows 3 to 102. Sheet **Poly 5 Err RSP** fits the following model:

$P_5(x) = (a_0 + a_1x + a_3x^3 + a_5x^5) / (1 + a_2x^2 + a_4x^4)$

Using the Regression option in the Data→Data Analysis option using select the following:

- For **Input Y Range** text box, select the entire range for Y in column B.
- For **Input X Range** text box, select the entire ranges for columns C to G.
- Check the **Labels** check box.
- Check the **Confidence Level** check box and use the default of 95 %.
- Click the Output Range radio button. Click on the related text box and then click on cell I1.
- (Optional) Click on the **Residuals** check box.
- Click the **OK** button.

Excel displays the following regression ANOVA table.

SUMMARY OUTPUT						
Regression Statistics						
Multiple R	0.961359468					
R Square	0.924212027					
Adjusted R Squ	0.920223186					
Standard Error	0.095226903					
Observations	101					
ANOVA						
	df	*SS*	*MS*	*F*	*Significance F*	
Regression	5	10.50544003	2.101088007	231.6994086	1.37747E-51	
Residual	95	0.861475486	0.009068163			
Total	100	11.36691552				
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	0.314304486	0.035542798	8.842986661	4.90187E-14	0.243743111	0.384865861
X	0.395604087	0.029244139	13.52763657	7.08612E-24	0.337547131	0.453661044
Y*X^2	0.015174987	0.00638715	2.375862129	0.019515341	0.002494892	0.027855082
X^3	-0.00515832	0.000776091	-6.64654036	1.8813E-09	-0.00669906	-0.00361758
Y*X^4	-0.00039236	7.73731E-05	-5.07101554	1.95287E-06	-0.00054597	-0.00023876
X^5	-1.0713E-05	1.00128E-05	-1.06996543	0.287345871	-3.0591E-05	9.1646E-06

Table 5.4. Regression ANOVA table for sheet Poly 5 Err RSP.

Take note of statistics like the Adjusted R Square, F statistic, Standard Error, and Significance F. Sheet **Poly 5 Err** has an Adjusted R Square value of 0.95391 while sheet **Poly 5 RSP** has an Adjusted R Square value of 0.92022. Since the latter is a smaller value, we conclude that the Rational Shammas Polynomial did NOT give better results for this studied case.

The Reverse Rational Shammas Polynomial Fit

The sheet **Poly 5 Err RRSP** has the same data in columns A and B as does sheet **Poly 5**. Please make a copy of sheet **Poly Err 5** and label its tag as **Poly 5 Err RRSP**. The first two rows on this sheet are shown in Table 5.5.

Column	Heading	Formulas in the second row
A	X	none
B	Y	one
C	Y*X	=–B2*A2
D	X^2	=A2^2
E	Y*X^3	=–B2*A2^3
F	X^4	= A2^4
G	Y*X^5	=–B2*A2^5

Table 5.5. The Poly 5 Err RRSP sheet configuration.

Enter the headings and formulas in the above table into columns C, E, and G of row 2. You must copy the formulas from the second rows (in columns C to G) into rows 3 to 102. Sheet **Poly 5 Err RSP** fits the following model:

$P_5(x) = (a_0 + a_2x^2 + a_4x^4) \;/\; (1 + a_1x + a_3{}^3 + a_5x^5)$

Using the Regression option in the Data→Data Analysis option using select the following:

- For **Input Y Range** text box, select the entire range for Y in column B.
- For **Input X Range** text box, select the entire ranges for columns C to G.
- Check the **Labels** check box.
- Check the **Confidence Level** check box and use the default of 95 %.
- Click the Output Range radio button. Click on the related text box and then click on cell I1.
- (Optional) Click on the **Residuals** check box.
- Click the **OK** button.

Excel displays the following regression ANOVA table.

SUMMARY OUTPUT					
Regression Statistics					
Multiple R	0.960476469				
R Square	0.922515047				
Adjusted R Squ	0.918436891				
Standard Error	0.096287121				
Observations	101				
ANOVA					
	df	*SS*	*MS*	*F*	*Significance F*
Regression	5	10.4861506	2.09723012	226.2088981	3.93301E-51
Residual	95	0.880764918	0.00927121		
Total	100	11.36691552			
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*
Intercept	0.456596219	0.027066884	16.86918328	2.5035E-30	0.402861662
Y*X	-0.37826655	0.028055272	-13.4829044	8.72449E-24	-0.4339633
X^2	-0.0728599	0.011189099	-6.51168611	3.50681E-09	-0.09507307
Y*X^3	0.001336428	0.000386772	3.455341873	0.000823213	0.000568589
X^4	0.000817789	0.000154716	5.285737354	7.96499E-07	0.000510639
Y*X^5	2.35643E-05	8.10945E-06	2.905784817	0.004557799	7.46502E-06

Table 5.6. Regression ANOVA table for sheet Poly 5 Err RRSP.

Take note of statistics like the Adjusted R Square, F statistic, Standard Error, and Significance F. Sheet **Poly 5 Err** has an Adjusted R Square value of 0.92251 while sheet **Poly 5 RRSP** has an Adjusted R Square value of 0.91843. Since the latter is a lower value, we conclude that the Reversed Rational Shammas Polynomial does NOT give better results for this studied case.

The Selected Rational Shammas Polynomial Fit

The sheet **Poly 5 Err SRSP** has the same data in columns A and B as does sheet **Poly 5**. Please make a copy of sheet **Poly Err 5** and label its tag as **Poly 5 Err SRSP**. Looking at the ANOVA table in Table 5.2, we find that the coefficients for X^3 and X^4 have the highest p–values. So, we selected these two columns for the SRSP fit. The first two rows on this sheet are shown in Table 5.7.

Column	Heading	Formulas in the second row
A	X	none
B	Y	none

Column	Heading	Formulas in the second row
C	X	=A2
D	X^2	=A2^2
E	Y*X^3	=–B2*A2^3
F	Y*X^4	=–B2*A2^4
G	X^5	=A2^5

Table 7. The Poly 5.7 SRSP sheet configuration.

Enter the headings and formulas in the above table into columns E and F of row 2. You must copy the formulas from the second rows (in columns C to G) into rows 3 to 102. Sheet **Poly 5 Err SRSP** fits the following model:

$P_5(x) = (a_0 + a_1x + a_2x^2 + a_3x^5) \ / \ (1+ a_4x^3 + a_5x^4)$

Using the Regression option in the Data→Data Analysis option using select the following:

- For **Input Y Range** text box, select the entire range for Y in column B.
- For **Input X Range** text box, select the entire ranges for columns C to G.
- Check the **Labels** check box.
- Check the **Confidence Level** check box and use the default of 95 %.
- Click the Output Range radio button. Click on the related text box and then click on cell I1.
- (Optional) Click on the **Residuals** check box.
- Click the **OK** button.

Excel displays the following regression ANOVA table.

SUMMARY OUTPUT					
Regression Statistics					
Multiple R	0.984524528				
R Square	0.969288546				
Adjusted R Squ	0.967672153				
Standard Error	0.060619131				
Observations	101				
ANOVA					
	df	*SS*	*MS*	*F*	*Significance F*
Regression	5	11.01782101	2.203564202	599.6616798	3.42914E-70
Residual	95	0.349094508	0.003674679		
Total	100	11.36691552			
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*
Intercept	0.199006862	0.02247367	8.855111765	4.61856E-14	0.154390985
X	0.642849027	0.022888867	28.08566423	8.43416E-48	0.597408881
X^2	-0.13296396	0.009379372	-14.1762103	3.56338E-25	-0.15158436
Y*X^3	-0.01197665	0.001509322	-7.93511826	4.1232E-12	-0.01497303
Y*X^4	-0.00110178	0.000166976	-6.59841345	2.35066E-09	-0.00143327
X^5	6.79415E-05	9.77931E-06	6.947471996	4.62003E-10	4.85271E-05

Table 5.8. Regression ANOVA table for sheet Poly 5 Err SRSP.

Take note of statistics like the Adjusted R Square, F statistic, Standard Error, and Significance F. Sheet **Poly Err 5** has an Adjusted R Square value of 0.953919while sheet **Poly 5 Err SRSP** has an Adjusted R Square value of 0.96767215. Since the latter is a slightly higher value, we conclude that the Selected Rational Shammas Polynomial gives slightly better results for this studied case.

The ranking of the various curve fitting models appears in the next table.

Model	***Adjusted R Square***	***Rank***
Regular polynomial	0.953919631776644	2
Rational Shammas Polynomial	0.92022318635697	3
Reversed Rational Shammas Polynomial	0.918436891329002	4
Selected Rational Shammas Polynomial	0.967672153209918	1

Table 5.9. The ranks of the various polynomial fits.

6/ The Arctangent Function with Scaled X Values

In this section we have the values of the independent variable X scaled onto the range (1, 2). We look at how the four curve fitting models perform.

The Polynomial Fit

The sheet **Poly 5 Scale** has the data and results for fitting the arctan function for x in the range of (0, 10) in increments of 0.1 for x. The first two rows on this sheet are shown in Table 6.1.

Column	Heading	Formulas in the second row
A	X	=(H2–MIN($H$2:$H$102))/ (MAX($H$2:$H$102) – MIN($H$2:$H$102)) + 1
B	Y	=ATAN(A2)
C	X	=A2
D	X^2	=A2^2
E	X^3	=A2^3
F	X^4	=A2^4
G	X^5	=A2^5
H	X	Values from 0 to 10 in increments of 0.1.

Table 6.1. The Poly 5 Scale sheet configuration.

You must first enter the values for X in column H in the sequence 0, 0.1, 0.2, …, 10. Enter the headings and formulas in the above table into columns A to G of rows 1 and 2. You must then copy the formulas at the second rows (in columns A to G) into rows 3 to 102.

Using the Regression option in the Data→Data Analysis option using select the following:

- For **Input Y Range** text box, select the entire range for Y in column B.
- For **Input X Range** text box, select the entire ranges for columns C to G.
- Check the **Labels** check box.
- Check the **Confidence Level** check box and use the default of 95 %.
- Click the Output Range radio button. Click on the related text box and then click on cell I1.
- (Optional) Click on the **Residuals** check box.
- Click the **OK** button.

Excel displays the following regression ANOVA table.

SUMMARY OUTPUT					
Regression Statistics					
Multiple R	0.9999999999653				
R Square	0.9999999999306				
Adjusted R Squar	0.9999999999269				
Standard Error	7.9785E-07				
Observations	101				
ANOVA					
	df	*SS*	*MS*	*F*	*Significance F*
Regression	5	0.87128228	0.174256456	2.73745E+11	0
Residual	95	6.04736E-11	6.36564E-13		
Total	100	0.87128228			
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*
Intercept	-0.078072069	0.000428219	-182.3178978	1.0393E-122	-0.078922192
X	1.360204936	0.001496836	908.720002	6.3183E-189	1.357233341
X^2	-0.653142922	0.0020669	-316.00125	2.3118E-145	-0.657246237
X^3	0.180658903	0.001409683	128.1557292	3.1691E-108	0.177860329
X^4	-0.025513795	0.000475041	-53.70857826	6.88832E-73	-0.026456872
X^5	0.001260323	6.33027E-05	19.90946792	1.13965E-35	0.001134651

Table 6.2. Regression ANOVA table for sheet Poly 5 Scale.

Take note of statistics like the Adjusted R Square, F statistic, Standard Error, and Significance F.

The Rational Shammas Polynomial Fit

The sheet **Poly 5 Scale RSP** has the same data in columns A and B as does sheet **Poly 5 Scale.** Please make a copy of sheet **Poly 5 Scale** and label its tag as **Poly 5 Scale RSP**. The first two rows on this sheet are shown in Table 6.3.

Column	Heading	Formulas in the second row
A	X	=(H2–MIN($H$2:$H$102))/(MAX($H$2:$H$102)–MIN($H$2:$H$102)) + 1
B	Y	=ATAN(A2)
C	X	=A2
D	Y*X^2	=–B2*A2^2
E	X^3	=A2^3
F	Y*X^4	=–B2*A2^4

Column	Heading	Formulas in the second row
G	X^5	=A2^5
H	X	Values from 0 to 10 in increments of 0.1.

Table 6.3. The Poly 5 Scale RSP sheet configuration.

Enter the headings and formulas in the above table into columns D and F of row 2. You must then copy the formulas at the second rows (in columns C to G) into rows 3 to 102. Sheet **Poly 5 Scale RSP** fits the following model:

$P_5(x) = (a_0 + a_1x + a_3x^3 + a_5x^5) / (1 + a_2x^2 + a_4x^4)$

Using the Regression option in the Data→Data Analysis option using select the following:

- For **Input Y Range** text box, select the entire range for Y in column B.
- For **Input X Range** text box, select the entire ranges for columns C to G.
- Check the **Labels** check box.
- Check the **Confidence Level** check box and use the default of 95 %.
- Click the Output Range radio button. Click on the related text box and then click on cell I1.
- (Optional) Click on the **Residuals** check box.
- Click the **OK** button.

Excel displays the following regression ANOVA table.

SUMMARY OUTPUT					
Regression Statistics					
Multiple R	0.9999999999772				
R Square	0.9999999999543				
Adjusted R Squ:	0.9999999999519				
Standard Error	6.47283E-07				
Observations	101				
ANOVA					
	df	*SS*	*MS*	*F*	*Significance F*
Regression	5	0.87128228	0.174256456	4.15911E+11	0
Residual	95	3.98026E-11	4.18975E-13		
Total	100	0.87128228			
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*
Intercept	0.007909634	0.000143367	55.1705318	5.80964E-74	0.007625015
X	0.981555422	0.000290694	3376.592095	4.448E-243	0.980978322
Y*X^2	0.649742377	0.001582532	410.571311	3.7187E-156	0.646600653
X^3	0.341624157	0.001290442	264.7341208	4.5626E-138	0.339062305
Y*X^4	0.055649427	0.000332684	167.2743436	3.6209E-119	0.054988967
X^5	0.008324298	6.70044E-05	124.2351662	5.9607E-107	0.008191278

Table 6.4. Regression ANOVA table for sheet Poly 5 Scale RSP.

Take note of statistics like the Adjusted R Square, F statistic, Standard Error, and Significance F. Sheet **Poly 5 Scale** has an Adjusted R Square value of 0.9999999999269 while sheet **Poly 5 Scale RSP** has an Adjusted R Square value of 0.999999999951913. Since the latter is slightly bigger value, we conclude that the Rational Shammas Polynomial did gives slightly better results for this studied case.

The Reverse Rational Shammas Polynomial Fit

The sheet **Poly 5 Scale RRSP** has the same data in columns A and B as does sheet **Poly 5 Scale.** Please make a copy of sheet **Poly 5 Scale** and label its tag as **Poly 5 Scale RRSP**. The first two rows on this sheet are shown in Table 6.6.

Column	Heading	Formulas in the second row
A	X	=(H2–MIN($H$2:$H$102))/ (MAX($H$2:$H$102)–MIN($H$2:$H$102)) + 1
B	Y	=ATAN(A2)

Column	Heading	Formulas in the second row
C	Y*X	=–B2*A2
D	X^2	=A2^2
E	Y*X^3	=–B2*A2^3
F	X^4	=A2^4
G	Y*X^5	=–B2*A2^5
H	X	Values from 0 to 10 in increments of 0.1.

Table 6.6. The Poly 5 Scale RRSP sheet configuration.

Enter the headings and formulas in the above table into columns C, E, and G of row 2. You must copy the formulas from the second rows (in columns C to G) into rows 3 to 102. Sheet **Poly 5 Scale RRSP** fits the following model:

$P_5(x) = (a_0 + a_2x^2 + a_4x^4) \ / \ (1 + a_1x + a_3{}^3 + a_5x^5)$

Using the Regression option in the Data→Data Analysis option using select the following:

- For **Input Y Range** text box, select the entire range for Y in column B.
- For **Input X Range** text box, select the entire ranges for columns C to G.
- Check the **Labels** check box.
- Check the **Confidence Level** check box and use the default of 95 %.
- Click the Output Range radio button. Click on the related text box and then click on cell I1.
- (Optional) Click on the **Residuals** check box.
- Click the **OK** button.

Excel displays the following regression ANOVA table.

SUMMARY OUTPUT					
Regression Statistics					
Multiple R	0.999999993				
R Square	0.999999985				
Adjusted R Squ:	0.999999984				
Standard Error	1.1712E-05				
Observations	101				
ANOVA					
	df	*SS*	*MS*	*F*	*Significance F*
Regression	5	0.871282267	0.174256453	1,270,358,999.76	0
Residual	95	1.30312E-08	1.37171E-10		
Total	100	0.87128228			
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*
Intercept	0.301918851	0.000783843	385.1775547	1.5957E-153	0.300362726
Y*X	-7.43097267	0.086242556	-86.16364125	5.4756E-92	-7.602185791
X^2	-6.502940407	0.083515132	-77.86541515	7.22189E-88	-6.668738906
Y*X^3	-2.554992146	0.040559656	-62.99343744	2.73989E-79	-2.635513244
X^4	-0.90585019	0.016330813	-55.46877588	3.53463E-74	-0.938270952
Y*X^5	-0.062835624	0.001295028	-48.52068631	7.63984E-69	-0.065406579

Table 6.6. Regression ANOVA table for sheet Poly 5 Scale RRSP.

Take note of statistics like the Adjusted R Square, F statistic, Standard Error, and Significance F. Sheet **Poly 5 Scale** has an Adjusted R Square value of 0.9999999999269 while sheet **Poly 5 Scale RRSP** has an Adjusted R Square value of 0.999999984256419. Since the latter is a lower value, we conclude that the Reverse Rational Shammas Polynomial does NOT give better results for this studied case. This is somewhat an expected result since the Rational Shammas Polynomial did better.

The Selected Rational Shammas Polynomial Fit

The sheet **Poly 5 Scale SRSP** has the same data in columns A and B as does sheet **Poly 5 Scale.** Please make a copy of sheet **Poly 5 Scale** and label its tag as **Poly 5 Scale SRSP**. Looking at the ANOVA table in Table 6.2, we find that the coefficients for X^4 and X^5 have the highest p–values. So, we selected these two columns for the SRSP fit. The first two rows on this sheet are shown in Table 6.7.

Column	Heading	Formulas in the second row
A	X	=(H2–MIN($H$2:$H$102))/ (MAX($H$2:$H$102)–MIN($H$2:$H$102)

Column	Heading	Formulas in the second row
		) + 1
B	Y	=ATAN(A2)
C	X	=A2
D	X^2	=A2^2
E	X^3	=A2^3
F	Y*X^4	=–B2*A2^4
G	Y*X^5	=–B2*A2^5
H	X	Values from 0 to 10 in increments of 0.1.

Table 6.7. The Poly 5 Scale SRSP sheet configuration.

Enter the headings and formulas in the above table into columns F and G of row 2. You must copy the formulas from the second rows (in columns C to G) into rows 3 to 102. Sheet **Poly 5 Scale SRSP** fits the following model:

$P_5(x) = (a_0 + a_1x + a_2x^2 + a_3x^3) \;/\; (1+ a_4x^4 + a_5x^5)$

Using the Regression option in the Data→Data Analysis option using select the following:

- For **Input Y Range** text box, select the entire range for Y in column B.
- For **Input X Range** text box, select the entire ranges for columns C to G.
- Check the **Labels** check box.
- Check the **Confidence Level** check box and use the default of 95 %.
- Click the Output Range radio button. Click on the related text box and then click on cell I1.
- (Optional) Click on the **Residuals** check box.
- Click the **OK** button.

Excel displays the following regression ANOVA table.

SUMMARY OUTPUT					
Regression Statistics					
Multiple R	0.9999999999636				
R Square	0.9999999999271				
Adjusted R Squ	0.9999999999233				
Standard Error	8.17569E-07				
Observations	101				
ANOVA					
	df	*SS*	*MS*	*F*	*Significance F*
Regression	5	0.87128228	0.174256456	2.60699E+11	0
Residual	95	6.34998E-11	6.68419E-13		
Total	100	0.87128228			
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*
Intercept	-0.078436056	0.000447513	-175.270923	4.3484E-121	-0.07932448
X	1.360118402	0.001542011	882.0418049	1.0711E-187	1.357057123
X^2	-0.646815552	0.002019244	-320.32567	6.3635E-146	-0.65082426
X^3	0.161832365	0.001107347	146.1442494	1.2851E-113	0.159634003
Y*X^4	0.01512091	0.000272903	55.407537	3.91351E-74	0.014579128
Y*X^5	-0.000728419	3.99939E-05	-18.2132317	9.44798E-33	-0.00080782

Table 6.8. Regression ANOVA table for sheet Poly 5 Scale SRSP.

Take note of statistics like the Adjusted R Square, F statistic, Standard Error, and Significance F. Sheet **Poly 5 Scale** has an Adjusted R Square value of 0.9999999999269 while sheet **Poly 5 Scale SRSP** has an Adjusted R Square value of 0.999999999923283. Since the latter is less, we conclude that the Selected Rational Shammas Polynomial does NOT yield better results for this studied case.

The ranking of the various curve fitting models appears in the next table.

Model	***Adjusted R Square***	***Rank***
Regular polynomial	0.999999999926939	2
Rational Shammas Polynomial	0.999999999951913	1
Reversed Rational Shammas Polynomial	0.999999984256419	4
Selected Rational Shammas Polynomial	0.999999999923283	3

Table 6.9. The ranks of the various polynomial fits.

7/ The Arctangent Function with X in Range (0, 20)

The Polynomial Fit

The sheet **Poly 5 (0,20)** has the data and results for fitting the arctan function for x in the range of (0, 20) in increments of 0.1 for x. The first two rows on this sheet are shown in Table 7.1.

Column	Heading	Formulas in the second row
A	X	none
B	Y	=ATAN(A2)
C	X	=A2
D	X^2	=A2^2
E	X^3	=A2^3
F	X^4	=A2^4
G	X^5	=A2^5

Table 7.1. The Poly 5 (0,20) sheet configuration.

You must populate cells A2 to A202 with values of X in the sequence of 0, 0.1, 0.2, …, 20. Enter the headings and formulas in the above table into columns B to G of rows 1 and 2. You must also copy the formulas at the second rows (in columns B to G) into rows 3 to 202.

Using the Regression option in the Data→Data Analysis option using select the following:

- For **Input Y Range** text box, select the entire range for Y in column B.
- For **Input X Range** text box, select the entire ranges for columns C to G.
- Check the **Labels** check box.
- Check the **Confidence Level** check box and use the default of 95 %.
- Click the Output Range radio button. Click on the related text box and then click on cell I1.
- (Optional) Click on the **Residuals** check box.
- Click the **OK** button.

Excel displays the following regression ANOVA table.

SUMMARY OUTPUT					
Regression Statistics					
Multiple R	0.9930504				
R Square	0.986149097				
Adjusted R Squ	0.985793945				
Standard Error	0.031829895				
Observations	201				
ANOVA					
	df	*SS*	*MS*	*F*	*Significance F*
Regression	5	14.06596445	2.813192891	2776.700824	4.5008E-179
Residual	195	0.197562737	0.001013142		
Total	200	14.26352719			
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*
Intercept	0.159664983	0.012904927	12.372405	2.49509E-26	0.134213834
X	0.684599541	0.013152215	52.05203587	2.5204E-116	0.65866069
X^2	-0.14199674	0.004100584	-34.6284216	3.14489E-85	-0.15008393
X^3	0.013930226	0.0005213	26.72206646	4.2013E-67	0.012902115
X^4	-0.00063925	2.87622E-05	-22.2251804	2.41153E-55	-0.00069597
X^5	1.10495E-05	5.72334E-07	19.30606997	3.94968E-47	9.92076E-06

Table 7.2. Regression ANOVA table for sheet Poly 5 (0,20).

Take note of statistics like the Adjusted R Square, F statistic, Standard Error, and Significance F.

The Rational Shammas Polynomial Fit

The sheet **Poly 5 (0,20) RSP** has the same data in columns A and B as does sheet **Poly 5 (0,20)**. Please make a copy of sheet **Poly 5 (0,20)** and label its tag as **Poly 5 (0,20) RSP**. The first two rows on this sheet are shown in Table 7.3.

Column	**Heading**	**Formulas in the second row**
A	X	none
B	Y	=ATAN(A2)
C	X	=A2
D	Y*X^2	=–B2*A2^2
E	X^3	=A2^3
F	Y*X^4	=–B2*A2^4
G	X^5	=A2^5

Table 7.3. The Poly 5 (0,20) RSP sheet configuration.

Enter the headings and formulas in the above table into columns D and F of row 2. You must copy the formulas from the second rows (in columns C to G) into rows 3 to 202. Sheet **Poly 5 (0,20) RSP** fits the following model:

$P_5(x) = (a_0 + a_1x + a_3x^3 + a_5x^5) / (1 + a_2x^2 + a_4x^4)$

Using the Regression option in the Data→Data Analysis option using select the following:

- For **Input Y Range** text box, select the entire range for Y in column B.
- For **Input X Range** text box, select the entire ranges for columns C to G.
- Check the **Labels** check box.
- Check the **Confidence Level** check box and use the default of 95 %.
- Click the Output Range radio button. Click on the related text box and then click on cell I1.
- (Optional) Click on the **Residuals** check box.
- Click the **OK** button.

Excel displays the following regression ANOVA table.

SUMMARY OUTPUT						
Regression Statistics						
Multiple R	0.992456068					
R Square	0.984969046					
Adjusted R Squ	0.984583637					
Standard Error	0.033158084					
Observations	201					
ANOVA						
	df	*SS*	*MS*	*F*	*Significance F*	
Regression	5	14.04913277	2.809826555	2555.645743	1.3019E-175	
Residual	195	0.214394417	0.001099459			
Total	200	14.26352719				
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	0.178287026	0.013086158	13.62409236	3.89866E-30	0.152478452	0.2040956
X	0.59074565	0.01106476	53.38983149	2.4695E-118	0.568923686	0.612567613
Y*X^2	0.090193729	0.002732239	33.010926	9.14588E-82	0.084805197	0.095582261
X^3	0.01351306	0.000527388	25.62259911	2.49278E-64	0.012472942	0.014553177
Y*X^4	0.000406968	1.9197E-05	21.199598	1.65148E-52	0.000369108	0.000444829
X^5	1.10542E-05	5.99913E-07	18.42640113	1.42749E-44	9.87109E-06	1.22374E-05

Table 7.4. Regression ANOVA table for sheet Poly 5 (0,20) RSP.

Take note of statistics like the R Square, F, Standard Error, and Significance F. Sheet **Poly 5 (0,20)** has an Adjusted R Square value of 0.985793945271375 while sheet **Poly 5 (0,20) RSP** has an Adjusted R Square value of 0.984583637240211. Since the latter is a smaller value, we conclude that the Rational Shammas Polynomial did NOT give better results for this studied case.

The Reverse Rational Shammas Polynomial Fit

The sheet **Poly 5 (0,20) RRSP** has the same data in columns A and B as does sheet **Poly 5 (0,20)**. Please make a copy of sheet **Poly 5 (0,20)** and label its tag as **Poly 5 (0,20) RRSP**. The first two rows on this sheet are shown in Table 7.5.

Column	Heading	Formulas in the second row
A	X	none
B	Y	=ATAN(A2)
C	Y*X	=–B2*A2
D	X^2	=A2^2
E	Y*X^3	=–B2*A2^3
F	X^4	=A2^4
G	Y*X^5	=–B2*A2^5

Table 7.5. The Poly 5 (0,20) RRSP sheet configuration.

Enter the headings and formulas in the above table into columns C, E, and G of row 2. You must copy the formulas from the second rows (in columns C to G) into rows 3 to 202. Sheet **Poly 5 (0,20) RSP** fits the following model:

$$P_5(x) = (a_0 + a_2x^2 + a_4x^4) \ / \ (1 + a_1x + a_3^3 + a_5x^5)$$

Using the Regression option in the Data→Data Analysis option using select the following:

- For **Input Y Range** text box, select the entire range for Y in column B.
- For **Input X Range** text box, select the entire ranges for columns C to G.
- Check the **Labels** check box.
- Check the **Confidence Level** check box and use the default of 95 %.
- Click the Output Range radio button. Click on the related text box and then click on cell I1.
- (Optional) Click on the **Residuals** check box.
- Click the **OK** button.

Excel displays the following regression ANOVA table.

SUMMARY OUTPUT						
Regression Statistics						
Multiple R	0.992456068					
R Square	0.984969046					
Adjusted R Squ	0.984583637					
Standard Error	0.033158084					
Observations	201					
ANOVA						
	df	*SS*	*MS*	*F*	*Significance F*	
Regression	5	14.04913277	2.809826555	2555.645743	1.3019E-175	
Residual	195	0.214394417	0.001099459			
Total	200	14.26352719				
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	0.178287026	0.013086158	13.62409236	3.89866E-30	0.152478452	0.2040956
X	0.59074565	0.01106476	53.38983149	2.4695E-118	0.568923686	0.612567613
Y*X^2	0.090193729	0.002732239	33.010926	9.14588E-82	0.084805197	0.095582261
X^3	0.01351306	0.000527388	25.62259911	2.49278E-64	0.012472942	0.014553177
Y*X^4	0.000406968	1.9197E-05	21.199598	1.65148E-52	0.000369108	0.000444829
X^5	1.10542E-05	5.99913E-07	18.42640113	1.42749E-44	9.87109E-06	1.22374E-05

Table 7.6. Regression ANOVA table for sheet Poly 5 (0,20) RRSP.

Take note of statistics like the R Square, F, Standard Error, and Significance F. Sheet **Poly 5 (0,20)** has an Adjusted R Square value of 0.985793945271375while sheet **Poly 5 (0,20) RRSP** has an Adjusted R Square value of 0.955464209565189. Since the latter is a lower value, we conclude that the Reverse Rational Shammas Polynomial does NOT give better results for this studied case.

The Selected Rational Shammas Polynomial Fit

The sheet **Poly 5 (0,20) SRSP** has the same data in columns A and B as does sheet **Poly 5 (0,20)**. Please make a copy of sheet **Poly 5 (0,20)** and label it**Poly 5 (0,20) SRSP**. Looking at the ANOVA table in Table 7.2, we find that the coefficients for X^4 and X^5 have the highest p–values. So, we selected these two columns for the SRSP fit. The first two rows on this sheet are shown in Table 7.7.

Column	Heading	Formulas in the second row
A	X	none
B	Y	=ATAN(A2)
C	X	=A2
D	X^2	=A2^2

Column	Heading	Formulas in the second row
E	X^3	=A2^3
F	Y*X^4	=–B2*A2^4
G	Y*X^5	=–B2*A2^5

Table 7.7. The Poly 5 (0,20) SRSP sheet configuration.

Enter the headings and formulas in the above table into columns F and G of row 2. You must copy the formulas from the second rows (in columns C to G) into rows 3 to 202. Sheet **Poly 5 (0,20) SRSP** fits the following model:

$P_5(x) = (a_0 + a_1x + a_2x^2 + a_3x^3) \;/\; (1+ a_4x^4 + a_5x^5)$

Using the Regression option in the Data→Data Analysis option using select the following:

- For **Input Y Range** text box, select the entire range for Y in column B.
- For **Input X Range** text box, select the entire ranges for columns C to G.
- Check the **Labels** check box.
- Check the **Confidence Level** check box and use the default of 95 %.
- Click the Output Range radio button. Click on the related text box and then click on cell I1.
- (Optional) Click on the **Residuals** check box.
- Click the **OK** button.

Excel displays the following regression ANOVA table.

SUMMARY OUTPUT						
Regression Statistics						
Multiple R	0.993051945					
R Square	0.986152165					
Adjusted R Squ	0.985797092					
Standard Error	0.03182637					
Observations	201					
ANOVA						
	df	*SS*	*MS*	*F*	*Significance F*	
Regression	5	14.06600822	2.813201643	2777.324675	4.4046E-179	
Residual	195	0.197518974	0.001012918			
Total	200	14.26352719				
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	0.159622789	0.012904412	12.36962932	2.54387E-26	0.134172656	0.185072922
X	0.684741778	0.013155848	52.04846951	2.552E-116	0.65879576	0.710687796
X^2	-0.142	0.004100118	-34.6331459	3.07369E-85	-0.15008627	-0.13391373
X^3	0.01352783	0.000503293	26.87863427	1.71294E-67	0.012535233	0.014520426
Y*X^4	0.000402481	1.80772E-05	22.26460065	1.88116E-55	0.000366829	0.000438133
Y*X^5	-7.0344E-06	3.64306E-07	-19.3090615	3.8719E-47	-7.7529E-06	-6.3159E-06

Table 7.8. Regression ANOVA table for sheet Poly 5 (0,20) SRSP.

Take note of statistics like the Adjusted R Square, F statistic, Standard Error, and Significance F. Sheet **Poly 5 (0,20)** has an Adjusted R Square value of 0.985793945271375 while sheet **Poly 5 (0,20) SRSP** has an Adjusted R Square value of 0.985797092093277. Since the latter is a slightly higher value, we conclude that the Selected Rational Shammas Polynomial gives slightly better results for this studied case.

The ranking of the various curve fitting models appears in the next table.

Model	***Adjusted R Square***	***Rank***
Regular polynomial	0.985793945271375	2
Rational Shammas Polynomial	0.984583637240211	3
Reversed Rational Shammas Polynomial	0.955464209565189	4
Selected Rational Shammas Polynomial	0.985797092093277	1

Table 7.9. The ranks of the various polynomial fits.

8/ The Arctangent Function with X in Range (0, 10) and 7th Order Polynomial

The Polynomial Fit

The sheet **Poly 7** has the data and results for fitting the arctan function for x in the range of (0, 10) in increments of 0.1 for x. The fit uses a 7th order polynomial. Since we are using a higher polynomial order, we should expect better curve fitting results than those in sheet **Poly 5**. The first two rows on this sheet are shown in Table 8.1.

Column	Heading	Formulas in the second row
A	X	none
B	Y	=ATAN(A2)
C	X	=A2
D	X^2	=A2^2
E	X^3	=A2^3
F	X^4	=A2^4
G	X^5	=A2^5
H	X^6	=A2^6
I	X^7	=A2^7

Table 8.1. The Poly 7 sheet configuration.

Enter the values of X in the sequence of 0, 0.1, 0.2, …, 20 in column A. Enter the headings and formulas in the above table into columns B to I of rows 1 and 2. You must copy the formulas from the second rows (in columns B to I) into rows 3 to 202.

Using the Regression option in the Data→Data Analysis option using select the following:

- For **Input Y Range** text box, select the entire range for Y in column B.
- For **Input X Range** text box, select the entire ranges for columns C to I.
- Check the **Labels** check box.
- Check the **Confidence Level** check box and use the default of 95 %.
- Click the **Output Range** radio button. Click on the related text box and then click on cell K1.
- (Optional) Click on the **Residuals** check box.
- Click the **OK** button.

Excel displays the following regression ANOVA table.

SUMMARY OUTPUT						
Regression Statistics						
Multiple R	0.999972852					
R Square	0.999945705					
Adjusted R Squ	0.999941618					
Standard Error	0.002496329					
Observations	101					
ANOVA						
	df	*SS*	*MS*	*F*	*Significance F*	
Regression	7	10.67344329	1.524777613	244682.4797	2.2557E-195	
Residual	93	0.000579544	6.23166E-06			
Total	100	10.67402283				
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-0.01370072	0.001713907	-7.99385329	3.49092E-12	-0.0171042	-0.01029724
X	1.184219852	0.006448718	183.6364884	7.0432E-121	1.171413977	1.197025727
X^2	-0.49586208	0.007688715	-64.4921892	5.30823E-79	-0.51113034	-0.48059381
X^3	0.123101898	0.004024975	30.58451268	2.54295E-50	0.115109095	0.131094701
X^4	-0.01840428	0.001072501	-17.160148	1.43235E-30	-0.02053405	-0.0162745
X^5	0.001619788	0.000151875	10.66525627	8.11118E-18	0.001318194	0.001921382
X^6	-7.7019E-05	1.08809E-05	-7.07838205	2.71037E-10	-9.8627E-05	-5.5412E-05
X^7	1.52092E-06	3.10077E-07	4.904973456	3.959E-06	9.05168E-07	2.13667E-06

Table 8.2. Regression ANOVA table for sheet Poly 7.

Take note of statistics like the Adjusted R Square, F statistic, Standard Error, and Significance F.

The Rational Shammas Polynomial Fit

The sheet **Poly 7 RSP** has the same data in columns A and B as does sheet **Poly 7**. Please make a copy of sheet **Poly 7** and label its tag as **Poly 7 RSP**. The first two rows on this sheet are shown in Table 8.3.

Column	Heading	Formulas in the second row
A	X	none
B	Y	=ATAN(A2)
C	X	=A2
D	Y*X^2	=–B2*A2^2
E	X^3	=A2^3
F	Y*X^4	=–B2*A2^4
G	X^5	=A2^5

Column	Heading	Formulas in the second row
H	Y*X^6	=–B2*A2^6
I	X^7	=A2^7

Table 8.3. The Poly 7 RSP sheet configuration.

Enter the headings and formulas in the above table into columns D, F, and H of row 2. You must copy the formulas from the second rows (in columns C to I) into rows 3 to 202. Sheet **Poly 7 RSP** fits the following model:

$P_7(x) = (a_0 + a_1x + a_3x^3 + a_5x^5 + a_7x^7) / (1 + a_2x^2 + a_4x^4 + a_6x^6)$

Using the Regression option in the Data→Data Analysis option using select the following:

- For **Input Y Range** text box, select the entire range for Y in column B.
- For **Input X Range** text box, select the entire ranges for columns C to I.
- Check the **Labels** check box.
- Check the **Confidence Level** check box and use the default of 95 %.
- Click the **Output Range** radio button. Click on the related text box and then click on cell K1.
- (Optional) Click on the **Residuals** check box.
- Click the **OK** button.

Excel displays the following regression ANOVA table.

SUMMARY OUTPUT							
Regression Statistics							
Multiple R	0.999979357						
R Square	0.999958714						
Adjusted R Squ	0.999955607						
Standard Error	0.002176824						
Observations	101						
ANOVA							
	df	*SS*	*MS*	*F*	*Significance F*		
Regression	7	10.67358215	1.52479745	321784.6623	6.6343E-201		
Residual	93	0.000440687	4.73856E-06				
Total	100	10.67402283					
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*
Intercept	0.006853959	0.00132039	5.19086011	1.22459E-06	0.004231926	0.009475992	0.004231926
X	0.965144816	0.00291483	331.1152558	1.1912E-144	0.95935654	0.970933091	0.95935654
Y*X^2	0.480434464	0.006366028	75.46848341	3.12707E-85	0.467792795	0.493076134	0.467792795
X^3	0.207416372	0.004132736	50.18862891	3.71728E-69	0.199209577	0.215623168	0.199209577
Y*X^4	0.027942374	0.000794257	35.18053875	1.57435E-55	0.026365138	0.029519611	0.026365138
X^5	0.004605739	0.000162582	28.32875753	1.59515E-47	0.004282883	0.004928594	0.004282883
Y*X^6	0.000178335	7.53228E-06	23.6761406	3.7394E-41	0.000163378	0.000193293	0.000163378
X^7	6.79265E-06	3.29361E-07	20.62374848	1.83395E-36	6.13861E-06	7.44669E-06	6.13861E-06

Table 8.4. Regression ANOVA table for sheet Poly 7 RSP.

Take note of statistics like the R Square, F, Standard Error, and Significance F. Sheet **Poly 7** has an Adjusted R Square value of 0.999941618465448 while sheet **Poly 7 RSP** has an Adjusted R Square value of 0.999955606571228. Since the latter is a higher value, we conclude that the Rational Shammas Polynomial gives better results for this studied case.

The Reverse Rational Shammas Polynomial Fit

The sheet **Poly 7 RRSP** has the same data in columns A and B as does sheet **Poly 7**. Please make a copy of sheet **Poly 7** and label its tag as **Poly 7 RRSP**. The first two rows on this sheet are shown in Table 8.5.

Column	Heading	Formulas in the second row
A	X	none
B	Y	=ATAN(A2)
C	Y*X	=–B2*A2
D	X^2	=A2^2
E	Y*X^3	=–B2*A2^3
F	X^4	=A2^4
G	Y*X^5	=–B2*A2^5
H	X^6	=A2^6

Column	Heading	Formulas in the second row
I	Y*X^7	=–B2*A2^7

Table 8.5. The Poly 7 RRSP sheet configuration.

Enter the headings and formulas in the above table into columns C, E, G, and I of row 2. You must copy the formulas from the second rows (in columns C to I) into rows 3 to 202. Sheet **Poly 7 RRSP** fits the following model:

$P_7(x) = (a_0 + a_2x^2 + a_4x^4 + a_6x^6) \ / \ (1 + a_1x + a_3^3 + a_5x^5 + a_7x^7)$

Using the Regression option in the Data→Data Analysis option using select the following:

- For **Input Y Range** text box, select the entire range for Y in column B.
- For **Input X Range** text box, select the entire ranges for columns C to I.
- Check the **Labels** check box.
- Check the **Confidence Level** check box and use the default of 95 %.
- Click the **Output Range** radio button. Click on the related text box and then click on cell K1.
- (Optional) Click on the **Residuals** check box.
- Click the **OK** button.

Excel displays the following regression ANOVA table.

SUMMARY OUTPUT						
Regression Statistics						
Multiple R	0.996502966					
R Square	0.99301816					
Adjusted R Squ	0.992492646					
Standard Error	0.028307892					
Observations	101					
ANOVA						
	df	*SS*	*MS*	*F*	*Significance F*	
Regression	7	10.59949852	1.514214074	1889.610227	2.65594E-97	
Residual	93	0.074524316	0.000801337			
Total	100	10.67402283				
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	0.169045894	0.012522179	13.4997192	1.23439E-23	0.144179329	0.193912459
Y*X	-1.72584937	0.05883415	-29.3341431	8.6008E-49	-1.84268233	-1.6090164
X^2	-0.6844011	0.026567701	-25.7606445	4.08561E-44	-0.73715929	-0.63164291
Y*X^3	0.171424726	0.015728455	10.89901844	2.62163E-18	0.140191131	0.202658321
X^4	-0.13738134	0.013898187	-9.88483933	3.58579E-16	-0.16498039	-0.1097823
Y*X^5	0.001779299	0.000219181	8.117935631	1.92088E-12	0.001344048	0.002214549
X^6	-0.00142624	0.000191335	-7.45414771	4.60442E-11	-0.00180619	-0.00104629
Y*X^7	1.08422E-06	1.59446E-07	6.799945307	9.92392E-10	7.67595E-07	1.40085E-06

Table 8.6. Regression ANOVA table for sheet Poly 7 RRSP.

Take note of statistics like the R Square, F, Standard Error, and Significance F. Sheet **Poly 7** has an Adjusted R Square value of 0.999941618465448 while sheet **Poly 7 RRSP** has an Adjusted R Square value of 0.992492645565934. Since the latter is a lower value, we conclude that the Reverse Rational Shammas Polynomial does NOT give better results for this studied case.

The Selected Rational Shammas Polynomial Fit

The sheet **Poly 7 SRSP** has the same data in columns A and B as does sheet **Poly 7**. Please make a copy of sheet **Poly 7** and label its tag as **Poly 7 SRSP**. Looking at the ANOVA table in Table 8.2, we find that the coefficients for X^2, X^4, and X^6 have the highest p–values. As such, the SRSP model is the same as the RSP model. So, we selected these three columns for the SRSP fit. The first two rows on this sheet are shown in Table 8.7.

Column	Heading	Formulas in the second row
A	X	none
B	Y	=ATAN(A2)

Column	Heading	Formulas in the second row
C	X	=A2
D	X^3	=A2^3
E	X^5	=A2^5
F	X^7	=A2^7
G	Y*X^2	=–B2*A2^2
H	Y*X^4	=–B2*A2^4
I	Y*X^6	=–B2*A2^6

Table 8.7. The Poly 7 SRSP sheet configuration.

Enter the headings and formulas in the above table into columns G, H, and I of row 2. You must copy the formulas from the second rows (in columns C to I) into rows 3 to 202. Sheet **Poly 7 SRSP** fits the following model:

$P_7(x) = (a_0 + a_1x + a_3x^3 + a_5x^5 + a_7x^7) \;/\; (1+ a_2x^2 + a_4x^4 + a_6x^6)$

Using the Regression option in the Data→Data Analysis option using select the following:

- For **Input Y Range** text box, select the entire range for Y in column B.
- For **Input X Range** text box, select the entire ranges for columns C to I.
- Check the **Labels** check box.
- Check the **Confidence Level** check box and use the default of 95 %.
- Click the **Output Range** radio button. Click on the related text box and then click on cell K1.
- (Optional) Click on the **Residuals** check box.
- Click the **OK** button.

Excel displays the following regression ANOVA table.

SUMMARY OUTPUT						
Regression Statistics						
Multiple R	0.999979357					
R Square	0.999958714					
Adjusted R Squa	0.999955607					
Standard Error	0.002176824					
Observations	101					
ANOVA						
	df	*SS*	*MS*	*F*	*Significance F*	
Regression	7	10.67358215	1.52479745	321784.6623	6.6343E-201	
Residual	93	0.000440687	4.73856E-06			
Total	100	10.67402283				
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	0.006853959	0.00132039	5.190860113	1.22459E-06	0.004231926	0.009475992
X	0.965144816	0.00291483	331.1152558	1.1912E-144	0.95935654	0.970933091
X^3	0.207416372	0.004132736	50.18862891	3.71728E-69	0.199209577	0.215623168
X^5	0.004605739	0.000162582	28.32875753	1.59515E-47	0.004282883	0.004928594
X^7	6.79265E-06	3.29361E-07	20.62374848	1.83395E-36	6.13861E-06	7.44669E-06
Y*X^2	0.480434464	0.006366028	75.46848341	3.12707E-85	0.467792795	0.493076134
Y*X^4	0.027942374	0.000794257	35.18053875	1.57435E-55	0.026365138	0.029519611
Y*X^6	0.000178335	7.53228E-06	23.6761406	3.7394E-41	0.000163378	0.000193293

Table 8.8. Regression ANOVA table for sheet Poly 7 SRSP.

Take note of statistics like the Adjusted R Square, F statistic, Standard Error, and Significance F. Sheet **Poly 7** has an Adjusted R Square value of 0.999941618465448 while sheet **Poly 7 SRSP** has an Adjusted R Square value of 0.999955606571228. Since the latter is a slightly higher value, we conclude that the Selected Rational Shammas Polynomial gives slightly better results for this studied case.

The ranking of the various curve fitting models appears in the next table.

Model	***Adjusted R Square***	***Rank***
Regular polynomial	0.999941618465448	2
Rational Shammas Polynomial	0.999955606571228	1
Reversed Rational Shammas Polynomial	0.992492645565934	3
Selected Rational Shammas Polynomial	0.999955606571228	1

Table 8.9. The ranks of the various polynomial fits.

9/ Conclusion

The following table summarizes the ranks of the four polynomial models tested in sections 4 through 8 and calculates their averages.

	Sec 4	Sec 5	Sec 6	Sec 7	Sec 8		Mean
Poly	2	2	2	2	2		2
RSP	3	3	1	3	1		2.2
RRSP	4	4	4	4	3		3.8
SRSP	1	1	3	1	1		1.4

The Selected Rational Shammas Polynomial has been shown to be in the lead, followed by regular polynomials. The Rational Shammas Polynomial comes in third place, close to the regular polynomials. The Reversed Rational Shammas Polynomial comes in last place.

These results are merely a snapshot of a single case-one in a vast number of cases. The purpose of this study is to show you how to compare fitting data with regular polynomials and the three types of rational Shammas polynomials. This new tool is now in your hands, go out and make the best of it.

The next two sections offer a brief version (to prevent this document from going over 60 pages) of the calculations presented earlier in this study. The examples focus on polynomial curve fitting for basic data calculated from functions.

10/ Fitting the ln(x+1) Curve

The file **SRP ln(x+1).xlsx** contains four worksheets for fitting the values of function ln(x+1) with the four types of tested polynomials. The Excel file has all the data and results. I will still describe how to fill in the data columns and perform the regression calculations,

The Polynomial Fit

The sheet **Poly 7** has the data and results for fitting the ln(x+1) function for x in the range of (0, 10) in increments of 0.05 for x. The fit uses a 7^{th} order polynomial. The first two rows on this sheet are shown in Table 10.1.

Column	Heading	Formulas in the second row
A	X	none
B	Y	=LN(A2+1)

Column	Heading	Formulas in the second row
C	X	=A2
D	X^2	=A2^2
E	X^3	=A2^3
F	X^4	=A2^4
G	X^5	=A2^5
H	X^6	=A2^6
I	X^7	=A2^7

Table 10.1. The Poly 7 sheet configuration.

Enter the values of X in the sequence of 0, 0.05, 0.1, 0.15 …, 10 in column A. Enter the headings and formulas in the above table into columns B to I of rows 1 and 2. You must copy the formulas from the second rows (in columns B to I) into rows 3 to 202.

Using the Regression option in the Data→Data Analysis option using select the following:

- For **Input Y Range** text box, select the entire range for Y in column B.
- For **Input X Range** text box, select the entire ranges for columns C to I.
- Check the **Labels** check box.
- Check the **Confidence Level** check box and use the default of 95 %.
- Click the **Output Range** radio button. Click on the related text box and then click on cell K1.
- (Optional) Click on the **Residuals** check box.
- Click the **OK** button.

Excel displays the following regression ANOVA table.

SUMMARY OUTPUT						
Regression Statistics						
Multiple R	0.99909437					
R Square	0.99818955					
Adjusted R Square	0.99812389					
Standard Error	0.00537185					
Observations	201					
ANOVA						
	df	*SS*	*MS*	*F*	*Significance F*	
Regression	7	3.07065949	0.43866564	15,201.50	6.817E-261	
Residual	193	0.00556935	2.8857E-05			
Total	200	3.07622884				
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-0.03002488	0.00302678	-9.91975278	5.28327E-19	-0.03599469	-0.02405507
X	0.33052205	0.01359012	24.320754	1.11508E-60	0.30371782	0.35732628
X^2	0.19567566	0.02007823	9.74566228	1.67174E-18	0.15607473	0.2352766
X^3	-0.23091814	0.0136195	-16.9549587	4.40639E-40	-0.25778032	-0.20405596
X^4	0.07902987	0.0043203	18.2926612	5.10942E-44	0.07050879	0.08755094
Y*X^5	0.01805497	0.00098112	18.4023982	2.44795E-44	0.01611987	0.01999006
Y*X^6	-0.00055603	3.156E-05	-17.6180763	4.83236E-42	-0.00061827	-0.00049378
Y*X^7	1.3726E-05	8.307E-07	16.5239241	8.44448E-39	1.2088E-05	1.5365E-05

Table 10.2. Regression ANOVA table for sheet Poly 7.

Take note of statistics like the Adjusted R Square, F statistic, Standard Error, and Significance F.

The Rational Shammas Polynomial Fit

The sheet **Poly 7 RSP** has the same data in columns A and B as does sheet **Poly 7**. Please make a copy of sheet **Poly 7** and label its tag as **Poly 7 RSP**. The first two rows on this sheet are shown in Table 10.3.

Column	**Heading**	**Formulas in the second row**
A	X	none
B	Y	=LN(A2+1)
C	X	=A2
D	Y*X^2	=–B2*A2^2
E	X^3	=A2^3
F	Y*X^4	=–B2*A2^4
G	X^5	=A2^5
H	Y*X^6	=–B2*A2^6

Column	Heading	Formulas in the second row
I	X^7	=A2^7

Table 10.3. The Poly 7 RSP sheet configuration.

Enter the headings and formulas in the above table into columns D, F, and H of row 2. You must copy the formulas from the second rows (in columns C to I) into rows 3 to 202. Sheet **Poly 7 RSP** fits the following model:

$P_7(x) = (a_0 + a_1x + a_3x^3 + a_5x^5 + a_7x^7) / (1 + a_2x^2 + a_4x^4 + a_6x^6)$

Using the Regression option in the Data→Data Analysis option using select the following:

- For **Input Y Range** text box, select the entire range for Y in column B.
- For **Input X Range** text box, select the entire ranges for columns C to I.
- Check the **Labels** check box.
- Check the **Confidence Level** check box and use the default of 95 %.
- Click the **Output Range** radio button. Click on the related text box and then click on cell K1.
- (Optional) Click on the **Residuals** check box.
- Click the **OK** button.

Excel displays the following regression ANOVA table.

SUMMARY OUTPUT							
Regression Statistics							
Multiple R	0.99879446						
R Square	0.99759037						
Adjusted R Square	0.99750297						
Standard Error	0.00619735						
Observations	201						
ANOVA							
	df	*SS*	*MS*	*F*	*Significance F*		
Regression	7	3.06881626	0.43840232	11414.59748	6.53E-249		
Residual	193	0.00741258	3.8407E-05				
Total	200	3.07622884					
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*
Intercept	-0.04884794	0.00256504	-19.0437502	3.40362E-46	-0.05390705	-0.04378884	-0.05390705
X	0.44508663	0.00391472	113.695785	8.6757E-179	0.43736551	0.45280774	0.43736551
Y*X^2	0.04013926	0.03342163	1.20099661	0.231224362	-0.02577927	0.10605779	-0.02577927
X^3	-0.04552043	0.01301249	-3.49820948	0.000581396	-0.07118538	-0.01985548	-0.07118538
Y*X^4	-0.02261246	0.00541853	-4.17317545	4.54003E-05	-0.03329958	-0.01192533	-0.03329958
X^5	-0.0019711	0.00042388	-4.65013765	6.14225E-06	-0.00280713	-0.00113507	-0.00280713
Y*X^6	-0.00037421	8.0685E-05	-4.63796894	6.4765E-06	-0.00053335	-0.00021508	-0.00053335
X^7	-1.1547E-06	2.613E-07	-4.41910665	1.65094E-05	-1.6701E-06	-6.3934E-07	-1.6701E-06

Table 10.4. Regression ANOVA table for sheet Poly 7 RSP.

Take note of statistics like the R Square, F, Standard Error, and Significance F. Sheet **Poly 7** has an Adjusted R Square value of 0.996980310863072 while sheet **Poly 7 RSP** has an Adjusted R Square value of 0.997502970913142. Since the latter is a higher value, we conclude that the Rational Shammas Polynomial gives better results for this studied case.

The Reverse Rational Shammas Polynomial Fit

The sheet **Poly 7 RRSP** has the same data in columns A and B as does sheet **Poly 7**. Please make a copy of sheet **Poly 7** and label its tag as **Poly 7 RRSP**. The first two rows on this sheet are shown in Table 10.5.

Column	Heading	Formulas in the second row
A	X	none
B	Y	=LN(A2+1)
C	Y*X	=–B2*A2
D	X^2	=A2^2
E	Y*X^3	=–B2*A2^3
F	X^4	= A2^4
G	Y*X^5	=–B2*A2^5
H	X^6	= A2^6

Column	Heading	Formulas in the second row
I	Y*X^7	=–B2*A2^7

Table 10.5. The Poly 7 RRSP sheet configuration.

Enter the headings and formulas in the above table into columns C, E, G, and I of row 2. You must copy the formulas from the second rows (in columns C to I) into rows 3 to 202. Sheet **Poly 7 RRSP** fits the following model:

$P_7(x) = (a_0 + a_2x^2 + a_4x^4 + a_6x^6) \ / \ (1 + a_1x + a_3{}^3 + a_5x^5 + a_7x^7)$

Using the Regression option in the Data→Data Analysis option using select the following:

- For **Input Y Range** text box, select the entire range for Y in column B.
- For **Input X Range** text box, select the entire ranges for columns C to I.
- Check the **Labels** check box.
- Check the **Confidence Level** check box and use the default of 95 %.
- Click the **Output Range** radio button. Click on the related text box and then click on cell K1.
- (Optional) Click on the **Residuals** check box.
- Click the **OK** button.

Excel displays the following regression ANOVA table.

SUMMARY OUTPUT							
Regression Statistics							
Multiple R	0.9963515						
R Square	0.99271631						
Adjusted R Square	0.99245214						
Standard Error	0.01077473						
Observations	201						
ANOVA							
	df	*SS*	*MS*	*F*	*Significance F*		
Regression	7	3.05382256	0.43626037	3757.796545	1.473E-202		
Residual	193	0.02240628	0.00011609				
Total	200	3.07622884					
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*
Intercept	-0.00816899	0.00487964	-1.67409588	0.095731533	-0.01779327	0.00145528	-0.01779327
Y*X	3.4038886	0.17560023	19.3843063	3.57882E-47	3.05754671	3.75023049	3.05754671
X^2	1.75849403	0.07563517	23.2496868	7.35418E-58	1.60931638	1.90767167	1.60931638
Y*X^3	1.14942039	0.03924752	29.2864446	6.10829E-73	1.07201126	1.22682953	1.07201126
X^4	0.1761345	0.00498845	35.3084744	3.74008E-86	0.16629562	0.18597337	0.16629562
Y*X^5	0.04595678	0.00126767	36.2528346	4.45579E-88	0.04345651	0.04845705	0.04345651
X^6	0.00123498	3.504E-05	35.2453042	5.04575E-86	0.00116587	0.00130409	0.00116587
Y*X^7	0.00017807	5.1456E-06	34.6062652	1.06695E-84	0.00016792	0.00018822	0.00016792

Table 10.6. Regression ANOVA table for sheet Poly 7 RRSP.

Take note of statistics like the R Square, F, Standard Error, and Significance F. Sheet **Poly 7** has an Adjusted R Square value of 0.9969803108630721while sheet **Poly 7 RRSP** has an Adjusted R Square value of 0.992452139719689. Since the latter is a lower value, we conclude that the Reverse Rational Shammas Polynomial does NOT give better results for this studied case.

The Selected Rational Shammas Polynomial Fit

The sheet **Poly 7 SRSP** has the same data in columns A and B as does sheet **Poly 7**. Please make a copy of sheet **Poly 7** and label its tag as **Poly 7 SRSP**. Looking at the ANOVA table in Table 10.2, we find that the coefficients for X^2, X^3, and X^7 have the highest p–values. So, we selected these three columns for the SRSP fit. The first two rows on this sheet are shown in Table 10.7.

Column	**Heading**	**Formulas in the second row**
A	X	none
B	Y	=LN(A2+1)
C	X	=A2
D	Y*X^2	=–B2*A2^2
E	Y*X^3	=–B2*A2^3

Column	Heading	Formulas in the second row
F	X^4	=A2^4
G	X^5	=A2^5
H	X^6	=A2^6
I	Y*X^7	=–B2*A2^7

Table 10.7. The Poly 7 SRSP sheet configuration.

Enter the headings and formulas in the above table into columns D, E, and I of row 2. You must copy the formulas from the second rows (in columns C to I) into rows 3 to 202. Sheet **Poly 7 SRSP** fits the following model:

$P_7(x) = (a_0 + a_1x + a_4x^4 + a_5x^5 + a_6x^6) \ / \ (1+ a_2x^2 + a_3x^3 + a_7x^7)$

Using the Regression option in the Data→Data Analysis option using select the following:

- For **Input Y Range** text box, select the entire range for Y in column B.
- For **Input X Range** text box, select the entire ranges for columns C to I.
- Check the **Labels** check box.
- Check the **Confidence Level** check box and use the default of 95 %.
- Click the **Output Range** radio button. Click on the related text box and then click on cell K1.
- (Optional) Click on the **Residuals** check box.
- Click the **OK** button.

Excel displays the following regression ANOVA table.

SUMMARY OUTPUT							
Regression Statistics							
Multiple R	0.99891766						
R Square	0.99783648						
Adjusted R Square	0.99775801						
Standard Error	0.00587233						
Observations	201						
ANOVA							
	df	*SS*	*MS*	*F*	*Significance F*		
Regression	7	3.06957337	0.43851048	12716.23108	1.995E-253		
Residual	193	0.00665547	3.4484E-05				
Total	200	3.07622884					
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*
Intercept	-0.04406448	0.00262742	-16.7709908	1.55125E-39	-0.04924662	-0.03888233	-0.04924662
X	0.42538311	0.0054386	78.215592	4.1135E-148	0.41465639	0.43610982	0.41465639
Y*X^2	0.10387509	0.01001497	10.3719858	2.57754E-20	0.08412226	0.12362793	0.08412226
Y*X^3	0.04294319	0.00716952	5.98969044	1.01047E-08	0.02880252	0.05708385	0.02880252
X^4	0.01372033	0.00181486	7.55999828	1.58961E-12	0.01014083	0.01729983	0.01014083
X^5	-0.00340675	0.00044906	-7.58641065	1.35894E-12	-0.00429245	-0.00252106	-0.00429245
X^6	0.00047313	6.3174E-05	7.48938226	2.41397E-12	0.00034853	0.00059773	0.00034853
Y*X^7	5.9845E-05	8.0478E-06	7.43614603	3.30321E-12	4.3972E-05	7.5717E-05	4.3972E-05

Table 10.8. Regression ANOVA table for sheet Poly 7 SRSP.

Take note of statistics like the Adjusted R Square, F statistic, Standard Error, and Significance F. Sheet **Poly 7** has an Adjusted R Square value of 0.996980310863072 while sheet **Poly 7 SRSP** has an Adjusted R Square value of 0.997758013861809. Since the latter is a slightly higher value, we conclude that the Selected Rational Shammas Polynomial gives slightly better results for this studied case.

The conclusion for the calculations in this section is that the Selected Rational Shammas Polynomial model, followed by the Rational Shammas Polynomial model, gave the best two curve fitting models.

11/ Fitting A Special Arbitrary Function

This section looks at fitting the following arbitrary function:

$F(x) = \sqrt{[\ln(x^2+1)]}*\exp(-x)*x^2$

The file **SRP fx1.xlsx** contains four worksheets for fitting the values of the above function with the four types of tested polynomials. The Excel file has all the data and results. I will still describe how to fill in the data columns and perform the regression calculations,

The Polynomial Fit

The sheet **Poly 7** has the data and results for fitting the special function for x in the range of (0, 10) in increments of 0.05 for x. The fit uses a 7th order polynomial. The first two rows on this sheet are shown in Table 11.1.

Column	Heading	Formulas in the second row
A	X	none
B	Y	=SQRT(LN(A2^2+1))*EXP(-A2)*A2^2
C	X	=A2
D	X^2	=A2^2
E	X^3	=A2^3
F	X^4	=A2^4
G	X^5	=A2^5
H	X^6	=A2^6
I	X^7	=A2^7

Table 11.1. The Poly 7 sheet configuration.

Enter the values of X in the sequence of 0, 0.05, 0.1, 0.15 …, 10 in column A. Enter the headings and formulas in the above table into columns C to I of rows 1 and 2. You must copy the formulas from the second rows (in columns C to I) into rows 3 to 202.

Using the Regression option in the Data→Data Analysis option using select the following:

- For **Input Y Range** text box, select the entire range for Y in column B.
- For **Input X Range** text box, select the entire ranges for columns C to I.
- Check the **Labels** check box.
- Check the **Confidence Level** check box and use the default of 95 %.
- Click the **Output Range** radio button. Click on the related text box and then click on cell K1.
- (Optional) Click on the **Residuals** check box.
- Click the **OK** button.

Excel displays the following regression ANOVA table.

SUMMARY OUTPUT						
Regression Statistics						
Multiple R	0.99965364					
R Square	0.99930741					
Adjusted R Squ	0.99928229					
Standard Error	0.0067129					
Observations	201					
ANOVA						
	df	*SS*	*MS*	*F*	*Significance F*	
Regression	7	12.5486625	1.79266606	39,781.36	3.67E-301	
Residual	193	0.00869715	4.5063E-05			
Total	200	12.5573596				
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	0.00033902	0.00350958	0.09659944	0.92314473	-0.00658304	0.00726108
x	-0.16542052	0.01298685	-12.7375367	2.2726E-27	-0.19103491	-0.13980614
x^2	0.8676759	0.01528651	56.760872	2.279E-122	0.83752582	0.89782598
x^3	-0.49853839	0.00794943	-62.7136967	2.67E-130	-0.51421731	-0.48285947
x^4	0.12248372	0.00211086	58.0254064	4.098E-124	0.1183204	0.12664705
x^5	-0.01540635	0.0002984	-51.630018	6.46E-115	-0.0159949	-0.01481781
x^6	0.0009784	2.1364E-05	45.7965838	1.27E-105	0.00093626	0.00102054
X^7	-2.4919E-05	6.0883E-07	-40.9288418	4.4322E-97	-2.6119E-05	-2.3718E-05

Table 11.2. Regression ANOVA table for sheet Poly 7.

Take note of statistics like the Adjusted R Square, F statistic, Standard Error, and Significance F.

The Rational Shammas Polynomial Fit

The sheet **Poly 7 RSP** has the same data in columns A and B as does sheet **Poly 7**. Please make a copy of sheet **Poly 7** and label its tag as **Poly 7 RSP**. The first two rows on this sheet are shown in Table 11.3.

Column	Heading	Formulas in the second row
A	X	none
B	Y	=SQRT(LN(A2^2+1))*EXP(-A2)*A2^2
C	X	=A2
D	Y*X^2	=–B2*A2^2
E	X^3	=A2^3
F	Y*X^4	=–B2*A2^4

Column	Heading	Formulas in the second row
G	X^5	=A2^5
H	Y*X^6	=–B2*A2^6
I	X^7	=A2^7

Table 11.3. The Poly 7 RSP sheet configuration.

Enter the headings and formulas in the above table into columns D, F and H of row 2. You must copy the formulas from the second rows (in columns C to I) into rows 3 to 202. Sheet **Poly 7 RSP** fits the following model:

$P_7(x) = (a_0 + a_1x + a_3x^3 + a_5x^5 + a_7x^7) / (1 + a_2x^2 + a_4x^4 + a_6x^6)$

Using the Regression option in the Data→Data Analysis option using select the following:

- For **Input Y Range** text box, select the entire range for Y in column B.
- For **Input X Range** text box, select the entire ranges for columns C to I.
- Check the **Labels** check box.
- Check the **Confidence Level** check box and use the default of 95 %.
- Click the **Output Range** radio button. Click on the related text box and then click on cell K1.
- (Optional) Click on the **Residuals** check box.
- Click the **OK** button.

Excel displays the following regression ANOVA table.

SUMMARY OUTPUT						
Regression Statistics						
Multiple R	0.99703976					
R Square	0.99408829					
Adjusted R Squ	0.99387387					
Standard Error	0.01961224					
Observations	201					
ANOVA						
	df	*SS*	*MS*	*F*	*Significance F*	
Regression	7	12.4831241	1.78330344	4,636.29	2.65E-211	
Residual	193	0.07423549	0.00038464			
Total	200	12.5573596				
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-0.10547215	0.00726967	-14.5085231	9.7846E-33	-0.11981035	-0.09113395
X	0.45324407	0.00940851	48.1738189	1.59E-109	0.43468736	0.47180078
Y*X^2	-0.03844074	0.00926066	-4.15097081	4.9637E-05	-0.05670583	-0.02017564
X^3	-0.0267211	0.00275362	-9.70397964	2.2005E-18	-0.03215216	-0.02129004
Y*X^4	0.00498836	0.00020142	24.7660075	7.8306E-62	0.00459109	0.00538562
X^5	0.00032254	4.1783E-05	7.71937773	6.1462E-13	0.00024013	0.00040495
Y*X^6	-0.00025528	3.4179E-05	-7.46877551	2.7259E-12	-0.00032269	-0.00018786
X^7	-1.2007E-06	1.7557E-07	-6.83894445	1.0248E-10	-1.547E-06	-8.5443E-07

Table 11.4. Regression ANOVA table for sheet Poly 7 RSP.

Take note of statistics like the R Square, F, Standard Error, and Significance F. Sheet **Poly 7** has an Adjusted R Square value of 0.999282286051075 while sheet **Poly 7 RSP** has an Adjusted R Square value of 0.993873873675832. Since the latter is a smaller value, we conclude that the Rational Shammas Polynomial did NOT give better results for this studied case.

The Reverse Rational Shammas Polynomial Fit

The sheet **Poly 7 RRSP** has the same data in columns A and B as does sheet **Poly 7**. Please make a copy of sheet **Poly 7** and label its tag as **Poly 7 RRSP**. The first two rows on this sheet are shown in Table 11.5.

Column	**Heading**	**Formulas in the second row**
A	X	none
B	Y	=SQRT(LN(A2^2+1))*EXP(-A2)*A2^2
C	Y*X	=–B2*A2

Column	Heading	Formulas in the second row
D	X^2	=A2^2
E	Y*X^3	=–B2*A2^3
F	X^4	= A2^4
G	Y*X^5	=–B2*A2^5
H	X^6	= A2^6
I	Y*X^7	=–B2*A2^7

Table 11.5. The Poly 7 RRSP sheet configuration.

Enter the headings and formulas in the above table into columns E, G, and I of row 2. You must copy the formulas from the second rows (in columns C to I) into rows 3 to 202. Sheet **Poly 7 RRSP** fits the following model:

$P_7(x) = (a_0 + a_2x^2 + a_4x^4 + a_6x^6) \;/\; (1 + a_1x + a_3{}^3 + a_5x^5 + a_7x^7)$

Using the Regression option in the Data→Data Analysis option using select the following:

- For **Input Y Range** text box, select the entire range for Y in column B.
- For **Input X Range** text box, select the entire ranges for columns C to I.
- Check the **Labels** check box.
- Check the **Confidence Level** check box and use the default of 95 %.
- Click the **Output Range** radio button. Click on the related text box and then click on cell K1.
- (Optional) Click on the **Residuals** check box.
- Click the **OK** button.

Excel displays the following regression ANOVA table.

SUMMARY OUTPUT							
Regression Statistics							
Multiple R	0.99976333						
R Square	0.99952671						
Adjusted R Squ	0.99950954						
Standard Error	0.00554926						
Observations	201						
ANOVA							
	df	*SS*	*MS*	*F*	*Significance F*		
Regression	7	12.5514163	1.79305947	58,226.98	0		
Residual	193	0.0059433	3.0794E-05				
Total	200	12.5573596					
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*
Intercept	-0.01853611	0.00194215	-9.54410721	6.2918E-18	-0.02236668	-0.01470554	-0.02236668
Y*X	-0.3431875	0.01730183	-19.8353266	1.8487E-48	-0.37731245	-0.30906255	-0.37731245
X^2	0.46713294	0.00792936	58.9117918	2.567E-125	0.45149361	0.48277228	0.45149361
Y*X^3	0.08434498	0.00070764	119.192326	1.087E-182	0.08294928	0.08574068	0.08294928
X^4	-0.00520844	9.7895E-05	-53.2044443	2.863E-117	-0.00540152	-0.00501536	-0.00540152
Y*X^5	0.01277973	0.00045867	27.8627952	1.4716E-69	0.01187509	0.01368437	0.01187509
X^6	1.6547E-05	3.5692E-07	46.3597548	1.459E-106	1.5843E-05	1.7251E-05	1.5843E-05
Y*X^7	9.2693E-05	1.2773E-06	72.5677111	4.905E-142	9.0174E-05	9.5213E-05	9.0174E-05

Table 11.6. Regression ANOVA table for sheet Poly 7 RRSP.

Take note of statistics like the R Square, F, Standard Error, and Significance F. Sheet **Poly 7** has an Adjusted R Square value of 0.999282286051075 while sheet **Poly 7 RRSP** has an Adjusted R Square value of 0.999509541660192. Since the latter is a higher value, we conclude that the Reverse Rational Shammas Polynomial gives better results for this studied case.

The Selected Rational Shammas Polynomial Fit

The sheet **Poly 7 SRSP** has the same data in columns A and B as does sheet **Poly 7**. Please make a copy of sheet **Poly 7** and label its tag as **Poly 7 SRSP**. Looking at the ANOVA table in Table 11.2, we find that the coefficients for X, X^6, and X^7 have the highest p–values. So, we selected these three columns for the SRSP fit. The first two rows on this sheet are shown in Table 11.7.

Column	**Heading**	**Formulas in the second row**
A	X	none
B	Y	=SQRT(LN(A2^2+1))*EXP(-A2)*A2^2
C	Y*X	=–B2*A2

Column	Heading	Formulas in the second row
D	X^2	=A2^2
E	X^3	=A2^3
F	X^4	=A2^4
G	X^5	=A2^5
H	Y*X^6	=–B2*A2^6
I	Y*X^7	=–B2*A2^7

Table 11.7. The Poly 7 SRSP sheet configuration.

Enter the headings and formulas in the above table into columns C, H, and I of row 2. You must copy the formulas from the second rows (in columns C to I) into rows 3 to 202. Sheet **Poly 7 SRSP** fits the following model:

$P_7(x) = (a_0 + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5) \; / \; (1+ a_1x + a_6x^6 + a_7x^7)$

Using the Regression option in the Data→Data Analysis option using select the following:

- For **Input Y Range** text box, select the entire range for Y in column B.
- For **Input X Range** text box, select the entire ranges for columns C to I.
- Check the **Labels** check box.
- Check the **Confidence Level** check box and use the default of 95 %.
- Click the **Output Range** radio button. Click on the related text box and then click on cell K1.
- (Optional) Click on the **Residuals** check box.
- Click the **OK** button.

Excel displays the following regression ANOVA table.

SUMMARY OUTPUT						
Regression Statistics						
Multiple R	0.99925084					
R Square	0.99850224					
Adjusted R Squ	0.99844792					
Standard Error	0.00987169					
Observations	201					
ANOVA						
	df	*SS*	*MS*	*F*	*Significance F*	
Regression	7	12.5385517	1.79122167	18,380.89	7.722E-269	
Residual	193	0.01880789	9.745E-05			
Total	200	12.5573596				
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-0.02655951	0.00341643	-7.77404337	4.4266E-13	-0.03329786	-0.01982117
Y*X	0.10499688	0.01872296	5.60792152	7.0159E-08	0.06806899	0.14192476
X^2	0.59226438	0.01337147	44.2931381	4.567E-103	0.56589141	0.61863735
X^3	-0.24518813	0.00406466	-60.3218856	3.376E-127	-0.25320499	-0.23717127
X^4	0.0286098	0.00042057	68.0256817	8.041E-137	0.02778029	0.02943931
X^5	-0.00104573	1.5762E-05	-66.343251	8.294E-135	-0.00107682	-0.00101464
Y*X^6	-0.00026271	1.5347E-05	-17.1181048	1.4466E-40	-0.00029298	-0.00023244
Y*X^7	-1.945E-05	2.3166E-06	-8.39588019	9.8046E-15	-2.4019E-05	-1.4881E-05

Table 11.8. Regression ANOVA table for sheet Poly 7 SRSP.

Take note of statistics like the Adjusted R Square, F statistic, Standard Error, and Significance F. Sheet **Poly 7** has an Adjusted R Square value of 0.999282286051075 while sheet **Poly 7 SRSP** has an Adjusted R Square value of 0.998447918675428. Since the latter is a lower value, we conclude that the Selected Rational Shammas Polynomial did NOT give better results for this studied case.

The conclusion for the calculations in this section is that the Reversed Rational Shammas Polynomial model followed by the regular polynomial model, gave the best two curve fitting models. This case shows that the Reversed Rational Shammas Polynomial model *can* excel!

12/ Final Comments

This study showed you alternatives to using regular polynomials for curve fitting between two variables. The study gave you examples for the various flavors of rational Shammas polynomials and how they competed with regular polynomials. I selected the examples somewhat arbitrarily. My purpose was not to attain excellent curve fitting (where the Adjusted R Square has a long sequence of 9s after the decimal place) but instead to demonstrate how to work with the new rational Shammas polynomials.

Document History

Date	Version	Comments
11/15/2023	1.0.0	Initial release.

