

Pade-Shammas Polynomial Approximations

Part II

By

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Combo Powers 5	251
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Combo Powers 8	254
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Shammas polynomial power for Q(x) is A+B*i	259

Shammas polynomial power for D(x) is A+B/i				259
Linear regression model:.....				259
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12 + x13 + x14.....				259
Estimated Coefficients:				259
Estimate	SE	tStat	pValue.....	259
.....				259
(Intercept)	0.0107484015313061	0	Inf	
0				259
x1	-0.460719401037082	0	-Inf	
0				259
x2	1.67915743473963	0	Inf	
0				259
x3	-2.72306926670345	0	-Inf	
0				259
x4	2.45294930112733	0	Inf	
0				259
x5	-1.27825237794406	0	-Inf	
0				259
x6	0.362869290002897	0	Inf	
0				259
x7	-0.0436832062771219	0	-Inf	
0				259
x8	26.834770544752	0	Inf	
0				259
x9	-904.624680551182	0	-Inf	
0				259
x10	3183.40806197793	0	Inf	
0				259
x11		0	0	NaN
NaN.....				259

```

x12          -5386.17266462574      0      -Inf
0 .....259
x13                      0      0      NaN
NaN.....259
x14          3081.55451252861      0      Inf
0 .....259
Number of observations: 301, Error degrees of freedom:
288.....259
R-squared: 1, Adjusted R-Squared: 1.....259
F-statistic vs. constant model: Inf, p-value = 0.....259
SumSq          DF          MeanSq          F
pValue .....259
-----
----- .....259
Total          18.4699692867817      300
0.0615665642892724 .....260
Model          18.4699692867817      12
1.53916410723181      Inf      0 .....260
Residual          0      288
0 .....260
for Q(x) A = 0.500000, B = 0.263855 .....260
for D(x) A = 0.260488, B = 0.192381 .....260
order Q(x) = 7.000000, order D(x) = 7.000000.....260
Xmin = 0.000000 and Xmax = 30.000000.....260
Ymin = -0.274347 and Ymax = 0.399625.....260
List of powers for Q(x): [0.763855, 1.027709,
1.291564, 1.555418, 1.819273, 2.083128, 2.346982].....260
List of powers for D(x): [0.452869, 0.356678,
0.324615, 0.308583, 0.298964, 0.292551, 0.287971].....260
Fitting J4(x) in range (1.000000, 2.000000) .....260

```

MSS of errors squared = 6.187011e-02.....	260
Corrected MSS of errors squared = 1.237402e-01.....	260
R-squared = 1.00000000.....	260
R-squared Adjusted = 1.00000000.....	260
Particle swarm AICc = -1.000000e+99.....	260
AIC = -Inf.....	260
AICc = -Inf.....	260
Combo Powers 2.....	260
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Combo Powers 7	300
Combo Powers 8	301
Combo Powers 9	302
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Combo Powers 3	307

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Combo Powers 7	311
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INTRODUCTION

This study looks at the collaboration of two mathematical frameworks—the Pade polynomials and the Shammas Polynomials. The study combines both frameworks to approximate several common functions. This study uses x and y data (calculated from the functions being approximated) that are transformed into the range of (1, 2). This transformation should limit rounding errors in calculations. Please do not be intimidated by the page count of this study. Most of the pages contain calculation results. You can quickly browse through this document. You are probably best served by focusing on the tables that summarize the results of fitting various functions with Pade-Shammas polynomials.

The Pade Polynomials

Pade polynomials are polynomial ratios defined as:

$$y = P_{m,n}(x) = \frac{Q_m(x)}{D_n(x)} \quad (1)$$

Where $Q_m(x)$ is defined as:

$$Q_m(x) = a_0 + \sum_{i=1}^m a_i x^i \quad (2)$$

And $D_n(x)$ is defined as:

$$D_n(x) = 1 + \sum_{i=1}^n b_i x^i \quad (3)$$

The Pade polynomials have more flexibility than ordinary legacy polynomials. The trick is to find the optimum orders of the polynomials $Q_m(x)$ and $D_n(x)$. The general multiple regression model used to fit Pade polynomials is:

$$y = a_0 + \sum_{i=1}^m a_i x^i - \sum_{i=1}^n b_i y x^i \quad (4)$$

The Shammas Polynomials

In the HHC 2008 conference in Corvallis, Oregon, I introduced the Shammas polynomials as polynomials with non-integer powers. I explained that the powers of such polynomial change using some math expressions that involve the polynomial term sequence number. Moreover, the study explained that such expressions must never produce the same powers of two or more terms. In the

2008 presentation I showed that the Shammas polynomials can have multiple terms with a smaller power range compared to regular polynomials. This feature allows better curve fitting with Shammas polynomials that avoids the instability of raising large or small numbers to high powers. If you are not familiar with Shammas polynomials, then you can download my 2008 presentation using the following link:

<http://www.namirshammas.com/NEW/ShammasPolynomials.pdf>.

A Shammas polynomial has the following general form:

$$y = a_0 + \sum_{i=1}^n a_i x^{gx(i,A,B,C)} \quad (5)$$

Where A, B, and C are fixed constants. Using C is optional. The function $gx(i, A, B, C)$ supplies the math expressions used to calculate the powers of the Shammas polynomial. The powers can be integers. In most cases, the powers are non-integers.

The Pade-Shammas Polynomials

The Pade-Shammas polynomials combine Pade and Shammas polynomials concepts. They have the following multiple regression general form:

$$y = a_0 + \sum_{i=1}^m a_i x^{gx1(i,A,B,C)} + \sum_{i=1}^n b_i y x^{gx2(i,A,B,C)} \quad (6)$$

Notice that the integral powers in normal Pade polynomials are replaced with power functions $gx1()$ and $gx2()$ that can be identical or different. The latter type gives the Pade-Shammas polynomials, in general, more flexibility in fitting a given set of (x, y) data. This statement is truer when the values of A, B, and C (when used) are different for the $gx1()$ and $gx2()$ functions. this study uses this scheme to give more flexibility to Pade-Shammas polynomials. Also note that Part IB conducts the Pade-Shammas polynomial fitting on the same functions but without normalizing the x and y data.

 I am appending my last name to the name Pade in an effort to make sure it is a unique combined name. Using a more general descriptive name risks conflicts with other mathematicians that may use the same descriptive name for different types of Pade polynomials. I certainly would like to avoid unpleasant communications with these fine mathematicians!

This study looks at Pade-Shammas polynomials used to approximate common functions that include:

- Trigonometric functions and their inverses.
- Hyperbolic functions.
- Logarithmic functions.
- Exponential functions.
- Bessel functions $J_0(x)$ to $J_5(x)$.
- The sine and cosine integrals.
- The Fresnel sine and cosine.
- Inverse student-t functions.
- The common logarithm of the gamma function.
- The digamma function.
- The trigamma function.

The digamma function is defined as:

$$\psi(x) = \Gamma'(x) / \Gamma(x) = \frac{d \ln(\Gamma(x))}{dx} \quad (7)$$

The following Matlab function implements the code for the digamma function:

```
function y = digamma(x)
%DIGAMMA Summary of this function goes here
% Detailed explanation goes here
h = 0.001;
fp = gammaln(x+h);
fm = gammaln(x-h);
y = (fp - fm)/2/h;
end
```

The above implementation of the digamma function was suggested by Albert Chan, a member of the hp museum web site, in a post he wrote on that site. The above code gives slightly more accurate results than the expression $(\gamma(x+h) - \gamma(x-h))/(2h)/\gamma(x)$.

The trigamma function is defined as:

$$\psi_1(x) = \frac{d^2}{dx^2} \ln(\Gamma(x)) \quad (8)$$

The following Matlab function implements the code for the trigamma function:

```
function y = trigamma(x)
%DIGAMMA Summary of this function goes here
% Detailed explanation goes here
h = 0.001;
fp = gammaln(x+h);
fm = gammaln(x-h);
```

```
f0 = gamma1n(x);
y = (fp -2*f0 + fm)/h/h;
end
```

The study works with Pade-Shammas Polynomials that use the following ten somewhat arbitrary power combinations to calculate the powers of Shammas polynomial terms.

Combination #	Powers Q(x)	Powers D(x)
1	$A+B*i$	$A+B/i$
2	$A+B*i$	$A+B*\sqrt{i}$
3	$A+B*\sqrt{i}+C*i$	$A+B*i+C*\sqrt{i}$
4	$A+B/i$	$A+B*i$
5	$A+B*\sqrt{i}$	$A+B*i$
6	$A+B*i+C*\sqrt{i}$	$A+B*\sqrt{i}+C*i$
7	$A+B*i$	$A+B*\log_{10}(i)^4$
8	$A+B/i$	$A+B*\log_{10}(i)^4$
9	$A+B*\log_{10}(i)^4$	$A+B*i$
10	$A+B*\log_{10}(i)^4$	$A+B/i$

Where A, B, and C are constants, and i a variable that represents the term number in a Pade-Shammas polynomial. The last equation introduces a new power expression not used in the Shammas Polynomial approximation study.

There are several choices that we can use to determine the best Pade-Shammas polynomial orders and the best values for parameters A, B, and C. The best one is to use an optimization function to determine the best Pade-Shammas polynomial order and the best values for parameters A, B, and C. The function would use ranges for the polynomial orders as well as the values for A, B, C.

The study uses the Matlab particle swarm optimization function to select the best Shammas polynomial orders (for $Q_n(x)$ and $D_m(x)$) along with the best values for A, B, and C.

 The approximations that I obtain are typically for a defined and suitable interval. It is your responsibility to implement expanded versions of the approximation functions that take wider ranges if arguments and map them onto the interval used. For example, given that my approximation for $\log_{10}(x)$ uses the range (1, 10), to calculate $\log_{10}(235)$ use:

$\log_{10}(235) =$

$$\log_{10}(2.35 * 100) =$$

$$\log_{10}(2.35) + \log_{10}(100) =$$

$$\log_{10}(2.35) + 2$$

The argument of the $\log_{10}(x)$ function in the last line falls in the interval (1, 10).

 Please note that I use the following model to perform the multiple regression:

$$y = a_0 + \sum_{i=1}^m a_i x^{gx1(i,A,B,C)} + \sum_{i=1}^n b_i y x^{gx2(i,A,B,C)} \quad (9)$$

I added the second set of summations instead of subtracting them (as in equation 6). Therefore, when you use the regression coefficients, use the following equation:

$$y = [a_0 + \sum_{i=1}^m a_i x^{gx1(i,A,B,C)}] / [1 - \sum_{i=1}^n b_i x^{gx2(i,A,B,C)}] \quad (10)$$

Notice the minus sign after the constant 1 in equation 10. I use equation 10 in the Matlab functions PadeShamPoly2() and PadeShamPoly3() that I present in the next section.

 The functions that perform the Pade-Shammas approximations convert both the x and y data into values that fall in the range of (1, 2). The rational behind the transformation of data onto the range (1, 2) is to prevent the curve fitting process from wild swings and excessive rounding errors. The reports for the curve fitting results contain the values of minimum values for x and y (xmin and ymin, respectively) and maximum values for x and y (xmax and ymax, respectively). To use the Pade-Shammas polynomials perform the following:

1. Transform your x value(s) into the range of (1, 2) by using the expression $(x - \text{xmin})/(\text{xmax} - \text{xmin}) + 1$. Use the xmin and xmax in the associated approximation results. You can still use your transformed x value if it reasonably falls outside the range (1, 2).
2. Calculate the value for yc. If the result is reasonably outside the range (1, 2) then you are fine.
3. Reverse map the value of y using the expression $(yc - 1) * (\text{ymax} - \text{ymin}) + \text{ymin}$. Use the ymin and ymax in the associated approximation results.

MATLAB CODE

The algorithm in this study uses particle swarm optimization to obtain the best values for parameters A and B in a prespecified range of Pade-Shammas polynomial orders for $Q_n(x)$ and $D_m(x)$.

This study uses the function PadeShamPoly2 () to perform various Pade-Shammas polynomial curve fitting:

```
function PadeShamPoly2 (fx,gxQ,gxD,xRange,Lb,Ub,runNum,sFxName,diaryFilename)
%PADESHAMPOLY2 Summary of this function goes here
% Detailed explanation goes here
clc
global bDeleteIfExists
global bUseDiary
global xdata
global ydata
global orderQ
global orderD
global ggxQ
global ggxD

warning('off','all')
if isempty(sFxName)
    sFxName = getFuncName(fx);
end
xdata = xRange';
ydata = xdata;
for i=1:length(xdata)
    ydata(i)=fx(xdata(i));
end
ggxQ = gxQ;
ggxD = gxD;

xmin = min(xdata);
xmax = max(xdata);
ymin = min(ydata);
ymax = max(ydata);
xdata = (xdata - xmin)/(xmax - xmin) + 1;
ydata = (ydata - ymin)/(ymax - ymin) + 1;
fprintf('Fitting %s in range (%f, %f)\n', sFxName, xmin ,xmax);

options = optimoptions('particleswarm', 'Display', 'iter');
[x,psAICc] = particleswarm(@optimFunc,6,Lb,Ub,options);
qA = x(1);
qB = x(2);
dA = x(3);
dB = x(4);
orderQ = round(x(5));
orderD = round(x(6));
if bUseDiary
    diaryFilename = strrep(diaryFilename, ".txt", strcat("_Q",
num2str(orderQ), "_D", num2str(orderD), "_run", num2str(runNum), ".txt"));
    if exist(diaryFilename, 'file')==2
```

```

        if bDeleteIfExists
            delete(diaryFilename);
        else
            return;
        end
    end
end
X = [];
for i=1:orderQ
    xs = xdata.^gxQ(i,qA,qB);
    X = [X;xs'];
end
for i=1:orderD
    xs = ydata.*(xdata.^gxD(i,dA,dB));
    X = [X;xs'];
end
X = X';
lm = fitlm(X,ydata);
if bUseDiary
    diary(diaryFilename)
end
fprintf('Fitting %s in range (%f, %f)\n', sFxName, xmin ,xmax);
sp = getFuncName(gxQ);
fprintf('Shammas polynomial power for Q(x) is %s\n', sp);
sp = getFuncName(gxD);
fprintf('Shammas polynomial power for D(x) is %s\n', sp);
format long
disp(lm);
anva = anova(lm,'summary');
disp(anva);
format short
fprintf('for Q(x) A = %f, B = %f\n', qA, qB);
fprintf('for D(x) A = %f, B = %f\n', dA, dB);
fprintf('order Q(x) = %f, order D(x) = %f\n',orderQ, orderD);
fprintf("Xmin = %f and Xmax = %f\n", xmin, xmax);
fprintf("Ymin = %f and Ymax = %f\n", ymin, ymax);
lstPwrQ = zeros(orderQ,1);
for i=1:orderQ
    lstPwrQ(i) = gxQ(i,qA,qB);
end
lstPwrD= zeros(orderD,1);
for i=1:orderD
    lstPwrD(i) = gxD(i,dA,dB);
end
fprintf('List of powers for Q(x): [');
for i=1:orderQ-1
    fprintf('%f, ', lstPwrQ(i));
end
fprintf('%f]\n', lstPwrQ(orderQ));
fprintf('List of powers for D(x): [');
for i=1:orderD-1
    fprintf('%f, ', lstPwrD(i));
end
fprintf('%f]\n', lstPwrD(orderD));
fprintf('Fitting %s in range (%f, %f)\n', sFxName, min(xdata),max(xdata));
n = length(xdata);
sumsq = 0;

```

```

for i=1:n
    qx = lm.Coefficients{1,1};
    for j=2:orderQ+1
        qx = qx + lm.Coefficients{j,1} * xdata(i)^1stPwrQ(j-1);
    end
    dx = 1;
    k = 1;
    for j=orderQ+2:orderQ+orderD+1
        dx = dx - lm.Coefficients{j,1} * xdata(i)^1stPwrD(k);
        k = k + 1;
    end
    yc = qx/dx;
    sumsqr = sumsqr + (ydata(i) - yc)^2;
end
k = orderQ + orderD + 1;
fprintf('MSS of errors squared = %e\n', sqrt(sumsqr)/n);
fprintf('Corrected MSS of errors squared = %e\n', sqrt(sumsqr)/n*sqrt(4));
fprintf('R-squared = %12.8f\n', lm.Rsquared.Ordinary);
fprintf('R-squared Adjusted = %12.8f\n', lm.Rsquared.Adjusted);
AIC = lm.ModelCriterion.AIC;
AICc = AIC + 2*k*(1 + (k+1)/(n-k-1));
fprintf('Particle swarm AICc = %e\n', psAICc);
fprintf('AIC = %e\n', AIC);
fprintf('AICc = %e\n', AICc);

if bUseDiary
    diary off
end
end

function AICc = optimFunc(x)
    global xdata
    global ydata
    global orderQ
    global orderD
    global ggxQ
    global ggxD

    qA = x(1);
    qB = x(2);
    dA = x(3);
    dB = x(4);
    orderQ = round(x(5));
    orderD = round(x(6));
    X = [];
    for i=1:orderQ
        xs = xdata.^ggxQ(i,qA,qB);
        X = [X;xs'];
    end
    for i=1:orderD
        xs = ydata.*(xdata.^ggxD(i,dA,dB));
        X = [X;xs'];
    end
    X = X';
    lm = fitlm(X,ydata);

    n = length(xdata);

```

```

k = orderQ + orderD + 1;
AIC = lm.ModelCriterion.AIC;
AICc = AIC + 2*k*(1 + (k+1)/(n-k-1));
if isinf(AICc), AICc = -1e+99; end
end

function sFx = getFuncName(fx)
sFx = func2str(fx);
if sFx(1:2)=='@('
    i = strfind(sFx,')');
    sFx = sFx(i(1)+1:end);
elseif sFx(1)=='@'
    sFx = strcat(sFx(2:end),'.m');
else
    % return sFx as is
end
end

```

The parameters of function PadeShamPoly2 () are:

- The parameter fx is the handle (or inline function) for the function being approximated. An example is @(x)cos(x) which also shows the **recommended format** for the argument of parameter fx.
- The parameter gx is the handle (or inline function) for the function that calculates the powers of the Shammas polynomial. An example is @(i,A,B)A+B*sqrt(i) which also shows the **recommended format** for the argument of parameter gx.
- The parameter xRange is the array that specifies the minimum value, increment value, and maximum value for the range of approximation.
- The parameter Lb is is the array of lower limits for the parameters A and B, and the Shammas polynomial order. An example is [0.1 0 3].
- The parameter Ub is is the array of upper limits for the parameters A and B, and the Shammas polynomial order. An example is [1 5 7].
- The parameter runNum specifies the run number. The arguments for this parameter have nothing to do with the calculations and serve in fine tuning the name of the diary files, when used.
- The optional parameter sFxName is the name of the approximated function. An example is “cos(x)”.
- The parameter diaryFilename is the name of the diary file. An example is “cos_1.txt”.

The above listing performs the following tasks:

1. Initialize the data for the curve fitting. The function uses the global variables `xdata` and `ydata` to store the data for the polynomial fitting.
2. Store the handle of function `gx` in the global handle `g gx`.
3. Normalize the `x` and `y` data to have their values in the range (1, 2).
4. Set the optimization options and then call the Matlab function `particleswarm()`. The function call returns the optimized values of `A` and `B` and the optimum value for *corrected* Akaike information criterion (`AICc`). The arguments for this function call are:
 - a. The handle to the local function `optimFunc()` that calculates the root mean sum of errors squared.
 - b. The number of optimized variables which is 2.
 - c. The lower and upper bounds arrays, `Lb` and `Ub`, respectively,
 - d. The optimization parameters for function `particleswarm()`.
5. Retrieve the optimum values and perform a Shammas polynomial fit for the best values of `A` and `B`.
6. Display the results of the regression and its associated ANOVA table.
7. Display the list of Shammas polynomial powers.
8. Display the range of the approximated function.
9. Calculate and display the value of the mean square root of the sum of squared errors. This statistic serves as a check that the Shammas polynomial performs well in checking the training data.
10. Calculate and display the value of the *corrected* mean square root of the sum of squared errors. This value equals the mean square root of the sum of squared errors multiplied by the square root of the number of Shammas polynomial parameters (i.e. `A`, `B`, and `C`, if used). In the case of `PadeShamPoly2()`, the function uses square root of 2.
11. Calculate and display the *corrected* Akaike information criterion. This statistic is calculated using:

$$AIC = n * \ln(SSE/n) + 2*k \quad (8)$$

$$AICc = AIC + 2*k*(k+1)/(n-k-1) \quad (9)$$

Where `n` is the number of observations, `k` is the total number of regression coefficients (including the intercept), and `SSE` is the sum of squared errors. The program obtains the value of `AIC` using `lm.ModelCriterion.AIC`. The program uses equation (3) to calculate the value for `AICc`.

12. Close the diary file, if one is used.

The function `optimFunc()` obtains the array `x` containing the current values of `A` and `B`, and the best Shammas polynomial order. The function calculates the transformed variables needed to perform a curve fit for a Shammas polynomial. This task calls the Matlab function `fitlm()`. The `optimFunc()` function returns the AICc as its result. I am using this statistic since the optimization is dealing with different Shammas polynomial orders and thus a varying number of polynomial coefficients.

The function `getFuncName()` returns a string-type function name given a handle of a function. The best way to take advantage of this function is to supply arguments like `@(x)cos(x)` and `@(x,A,B)A+B*sqrt(x)`. Such arguments allow the function to discard the part that declares the variable(s) and return the part that comes after the first closed parenthesis (e.g. `cos(x)` and `A+B*sqrt(x)` for the above examples). If you supply an argument like `@fx1` which refers to the file `fx1.m` that defines the function `fx1()` then the function `getFuncName()` returns `fx1.m`. This string value indicates that you are referencing a separate Matlab file that implements the code for `fx1`.

It is worth going over the code that estimates the values of `y` to calculate the sum of error squared:

```
n = length(xdata);
sumsq = 0;
for i=1:n
    qx = lm.Coefficients{1,1};
    for j=2:orderA+1
        qx = qx + lm.Coefficients{j,1} * xdata(i)^lstPwrQ(j-1);
    end
    dx = 1;
    k = 1;
    for j=orderA+2:orderA+1+orderB
        dx = dx + lm.Coefficients{j,1} * xdata(i)^lstPwrD(k);
        k = k + 1;
    end
    yc = qx/dx;
    sumsq = sumsq + (ydata(i) - yc)^2;
end
```

The above code used the arrays `lstPwrQ` and `lstPwrD` to store the powers used in the Shammas polynomials $Q_n(x)$ and $D_m(x)$, respectively. The first for loop calculates the value for Shammas polynomials $Q_n(x)$ using variable `qx`. Notice that the code initializes the value of `qx` using the intercept of the regression model. The loop accesses the powers of $Q_n(x)$ using `lstPwrQ(j-1)`. The second for loop calculates the value for Shammas polynomials $D_m(x)$ using variable `dx`. Notice that

the code initializes the value of dx using the constant 1. The loop accesses the powers of $Q_n(x)$ using $lstPwrD(k)$. Here k is initialized with the value 1 and is incremented in the for loop. Also noticed that the value of variable dx is decremented with the expression $lm.Coefficients\{j,1\} * xdata(i)^{lstPwrD(k)}$. I have explained the above code because you may need similar code to estimate other values of y given an array (or scalar) of x , as shown below:

```
n = length(your_xdata);
for i=1:n
    qx = lm.Coefficients{1,1};
    for j=2:orderA+1
        qx = qx + lm.Coefficients{j,1} * your_xdata(i)^lstPwrQ(j-1);
    end
    dx = 1;
    k = 1;
    for j=orderA+2:orderA+1+orderB
        dx = dx - lm.Coefficients{j,1} * your_xdata(i)^lstPwrD(k);
        k = k + 1;
    end
    your_y(i) = qx/dx;
end
```

Under the current calculation scheme, note that, the function `PadeShamPoly2()` does not explicitly iterate over different values of A and B . It delegates that task to the Matlab function `particleswarm()`.

The study also uses the function `PadeShamPoly3()` to work with Shammas polynomial powers that use parameters A , B , and C :

```
function PadeShamPoly3(fx,gxQ,gxD,xRange,Lb,Ub,runNum,sFxName,diaryFilename)
%PADESHAMPOLY2 Summary of this function goes here
% Detailed explanation goes here
clc
global bDeleteIfExists
global bUseDiary
global xdata
global ydata
global orderQ
global orderD
global ggxQ
global ggxD

warning('off','all')
if isempty(sFxName)
    sFxName = getFuncName(fx);
end
xdata = xRange';
ydata = xdata;
for i=1:length(xdata)
    ydata(i)=fx(xdata(i));
end
ggxQ = gxQ;
```

```

ggxD = gxD;

xmin = min(xdata);
xmax = max(xdata);
ymin = min(ydata);
ymax = max(ydata);
xdata = (xdata - xmin)/(xmax - xmin) + 1;
ydata = (ydata - ymin)/(ymax - ymin) + 1;
fprintf('Fitting %s in range (%f, %f)\n', sFxName, xmin, xmax);

options = optimoptions('particleswarm', 'Display', 'iter');
[x,psAICc] = particleswarm(@optimFunc,8,Lb,Ub,options);
qA = x(1);
qB = x(2);
qC = x(3);
dA = x(4);
dB = x(5);
dC = x(6);
orderQ = round(x(7));
orderD = round(x(8));
if bUseDiary
    diaryFilename = strrep(diaryFilename, ".txt", strcat("_Q",
num2str(orderQ), "_D", num2str(orderD), "_run", num2str(runNum), ".txt"));
    if exist(diaryFilename, 'file')==2
        if bDeleteIfExists
            delete(diaryFilename);
        else
            return;
        end
    end
end
X = [];
for i=1:orderQ
    xs = xdata.^gxQ(i,qA,qB,qC);
    X = [X;xs'];
end
for i=1:orderD
    xs = ydata.*(xdata.^gxD(i,dA,dB,dC));
    X = [X;xs'];
end
X = X';
lm = fitlm(X,ydata);
if bUseDiary
    diary(diaryFilename)
end
fprintf('Fitting %s in range (%f, %f)\n', sFxName, xmin, xmax);
sp = getFuncName(gxQ);
fprintf('Shammas polynomial power for Q(x) is %s\n', sp);
sp = getFuncName(gxD);
fprintf('Shammas polynomial power for D(x) is %s\n', sp);
format long
disp(lm);
anva = anova(lm,'summary');
disp(anva);
format short
fprintf('for Q(x) A = %f, B = %f, C = %f\n', qA, qB, qC);

```

```

fprintf('for D(x) A = %f, B = %f, C = %f\n', dA, dB, dC);
fprintf('order Q(x) = %f, order D(x) = %f\n', orderQ, orderD);
fprintf('Xmin = %f and Xmax = %f\n', xmin, xmax);
fprintf('Ymin = %f and Ymax = %f\n', ymin, ymax);
lstPwrQ = zeros(orderQ,1);
for i=1:orderQ
    lstPwrQ(i) = gxQ(i,qA,qB,qC);
end
lstPwrD= zeros(orderD,1);
for i=1:orderD
    lstPwrD(i) = gxD(i,dA,dB,dC);
end
fprintf('List of powers for Q(x): [');
for i=1:orderQ-1
    fprintf('%f, ', lstPwrQ(i));
end
fprintf('%f]\n', lstPwrQ(orderQ));
fprintf('List of powers for D(x): [');
for i=1:orderD-1
    fprintf('%f, ', lstPwrD(i));
end
fprintf('%f]\n', lstPwrD(orderD));
fprintf('Fitting %s in range (%f, %f)\n', sFxName, min(xdata),max(xdata));
n = length(xdata);
sumsq = 0;
for i=1:n
    qx = lm.Coefficients{1,1};
    for j=2:orderQ+1
        qx = qx + lm.Coefficients{j,1} * xdata(i)^lstPwrQ(j-1);
    end
    dx = 1;
    k = 1;
    for j=orderQ+2:orderQ+orderD+1
        dx = dx - lm.Coefficients{j,1} * xdata(i)^lstPwrD(k);
        k = k + 1;
    end
    yc = qx/dx;
    sumsq = sumsq + (ydata(i) - yc)^2;
end
k = orderQ + orderD + 1;
fprintf('MSS of errors squared = %e\n', sqrt(sumsq)/n);
fprintf('Corrected MSS of errors squared = %e\n', sqrt(sumsq)/n*sqrt(6));
fprintf('R-squared = %12.8f\n', lm.Rsquared.Ordinary);
fprintf('R-squared Adjusted = %12.8f\n', lm.Rsquared.Adjusted);
AIC = lm.ModelCriterion.AIC;
AICc = AIC + 2*k*(1 + (k+1)/(n-k-1));
fprintf('Particle swarm AICc = %e\n', psAICc);
fprintf('AIC = %e\n', AIC);
fprintf('AICc = %e\n', AICc);

if bUseDiary
    diary off
end
end

function AICc = optimFunc(x)
    global xdata

```

```

global ydata
global orderQ
global orderD
global ggxD
global ggxD

qA = x(1);
qB = x(2);
qC = x(3);
dA = x(4);
dB = x(5);
dC = x(6);
orderQ = round(x(7));
orderD = round(x(8));
X = [];
for i=1:orderQ
    xs = xdata.^ggxD(i,qA,qB,qC);
    X = [X;xs'];
end
for i=1:orderD
    xs = ydata.*(xdata.^ggxD(i,dA,dB,dC));
    X = [X;xs'];
end
X = X';
lm = fitlm(X,ydata);

n = length(xdata);
k = orderQ + orderD + 1;
AIC = lm.ModelCriterion.AIC;
AICc = AIC + 2*k*(1 + (k+1)/(n-k-1));
if isinf(AICc), AICc = -1e+99; end
end

function sFx = getFuncName(fx)
sFx = func2str(fx);
if sFx(1:2)=='@('
    i = strfind(sFx,')');
    sFx = sFx(i(1)+1:end);
elseif sFx(1)=='@'
    sFx = strcat(sFx(2:end),'.m');
else
    % return sFx as is
end
end
end

```

The parameters of function PadeShamPoly3() are:

- The parameter fx is the handle (or inline function) for the function being approximated. An example is @(x)cos(x) which also shows the **recommended format** for the argument of parameter fx.
- The parameter gx is the handle (or inline function) for the function that calculates the powers of the Shammas polynomial. An example is

@(i,A,B,c)A+B*sqrt(i+C) which also shows the **recommended format** for the argument of parameter gx.

- The parameter xRange is the array that specifies the minimum value, increment value, and maximum value for the range of approximation.
- The parameter Lb is is the array of lower limits for the parameters A, B, C and the Shammas polynomial order. An example is [0.1 0 - 3].
- The parameter Ub is is the array of upper limits for the parameters A, B and C, and the Shammas polynomial order. An example is [1 5 7].
- The parameter runNum specifies the run number. The arguments for this parameter have nothing to do with the calculations and serve in fine tuning the name of the diary files, when used.
- The optional parameter sFxName is the name of the approximated function. An example is “cos(x)”.
- The parameter diaryFilename is the name of the diary file. An example is “cos_1.txt”.

The above listing performs basically the same tasks as function PadeShamPoly2 () with the exception that it optimizes the parameter C and manages values related to that parameter. The function optimFunc() is the function to be optimized to get the best values for the Shammas polynomial power parameters A, B, and C, and the best Shammas polynomial order. The function getFuncName() returns a string-type function name given a handle of a function.

Using the calculation scheme, note that function PadeShamPoly3() does not explicitly iterate over different values of A, B and C. It delegates that task to the Matlab function particleswarm().

The following Matlab script goAll performs the various Pade-Shammas polynomial fittings for the various tested functions:

```
% Version 1.0.0 9/8/2020
global bUseDiary
global bDeleteIfExists

bUseDiary = true;
bDeleteIfExists = true; % or false
selIdx = 0; % note a zero value will execute all the models
runNum = 1;
bShutdown = false;

tic;

if selIdx==0 || selIdx==1
```

```

1stA = [0 0.5 0 2];
1stB = [0.1 0.5 0.1 2];
1stOrder = [3 7];
Lb = [1stA(1) 1stB(1) 1stA(3) 1stB(3) 1stOrder(1) 1stOrder(1)];
Ub = [1stA(2) 1stB(2) 1stA(4) 1stB(4) 1stOrder(2) 1stOrder(2)];
gx1 = @(i,A,B)A+B*i;
gx2 = @(i,A,B)A+B/i;

PadeShamPoly2(@(x)acos(x),gx1,gx2,[0:.01:1],Lb,Ub,runNum,"arccos(x)","arccos_
1.txt")

PadeShamPoly2(@(x)asin(x),gx1,gx2,[0:.01:1],Lb,Ub,runNum,"arcsin(x)","arcsin_
1.txt")

PadeShamPoly2(@(x)atan(x),gx1,gx2,[0:.01:1],Lb,Ub,runNum,"arctan(x)","arctan_
1.txt")

PadeShamPoly2(@(x)sin(x),gx1,gx2,[0:pi/1000:pi/2],Lb,Ub,runNum,"sin(x)","sin_
1.txt")

PadeShamPoly2(@(x)cos(x),gx1,gx2,[0:pi/1000:pi/2],Lb,Ub,runNum,"cos(x)","cos_
1.txt")

PadeShamPoly2(@(x)tan(x),gx1,gx2,[0:.01:1],Lb,Ub,runNum,"tan(x)","tan_1.txt")

PadeShamPoly2(@(x)sinh(x),gx1,gx2,[0:.01:5],Lb,Ub,runNum,"sinh(x)","sinh_1.tx
t")

PadeShamPoly2(@(x)cosh(x),gx1,gx2,[0:.01:5],Lb,Ub,runNum,"cosh(x)","cosh_1.tx
t")

PadeShamPoly2(@(x)tanh(x),gx1,gx2,[0:.01:3],Lb,Ub,runNum,"tanh(x)","tanh_1.tx
t")

PadeShamPoly2(@(x)erf(x),gx1,gx2,[0:.01:2.1],Lb,Ub,runNum,"erf(x)","erf_1.txt
")

PadeShamPoly2(@(x)exp(x),gx1,gx2,[0:.01:2],Lb,Ub,runNum,"exp(x)","exp_1.txt")

PadeShamPoly2(@(x)log(x),gx1,gx2,[1:.01:10],Lb,Ub,runNum,"ln(x)","ln_1.txt")

PadeShamPoly2(@(x)log10(x),gx1,gx2,[1:.01:10],Lb,Ub,runNum,"log(x)","log_1.tx
t")

PadeShamPoly2(@(x)10.^x,gx1,gx2,[0:.01:1],Lb,Ub,runNum,"10^x","pwr10_1.txt")

PadeShamPoly2(@(x)tinva(0.95,x),gx1,gx2,[2:0.1:100],Lb,Ub,runNum,"tinva(0.95,x)
","tinva_1.txt")

PadeShamPoly2(@(x)tinva(0.975,x),gx1,gx2,[2:0.1:100],Lb,Ub,runNum,"tinva(0.975,
x)","tinva2_1.txt")

PadeShamPoly2(@(x)log10(gamma(x)),gx1,gx2,[2:0.1:100],Lb,Ub,runNum,"log10Gamm
a(x)","log10Gamma_1.txt")

PadeShamPoly2(@(x)digamma(x),gx1,gx2,[2:0.1:100],Lb,Ub,runNum,"digamma(x)","d
igamma_1.txt")

```

```

PadeShamPoly2 (@ (x) trigamma (x) , gx1 , gx2 , [1:0.1:100] , Lb , Ub , runNum , "trigamma (x) " , "
trigamma_1.txt")

PadeShamPoly2 (@ (x) besselj (0 , x) , gx1 , gx2 , [2:0.1:30] , Lb , Ub , runNum , "J0 (x) " , "J0x_1
.txt")

PadeShamPoly2 (@ (x) besselj (1 , x) , gx1 , gx2 , [0:0.1:30] , Lb , Ub , runNum , "J1 (x) " , "J1x_1
.txt")

PadeShamPoly2 (@ (x) besselj (2 , x) , gx1 , gx2 , [0:0.1:30] , Lb , Ub , runNum , "J2 (x) " , "J2x_1
.txt")

PadeShamPoly2 (@ (x) besselj (3 , x) , gx1 , gx2 , [0:0.1:30] , Lb , Ub , runNum , "J3 (x) " , "J3x_1
.txt")

PadeShamPoly2 (@ (x) besselj (4 , x) , gx1 , gx2 , [0:0.1:30] , Lb , Ub , runNum , "J4 (x) " , "J4x_1
.txt")

PadeShamPoly2 (@ (x) besselj (5 , x) , gx1 , gx2 , [0:0.1:30] , Lb , Ub , runNum , "J5 (x) " , "J5x_1
.txt")

PadeShamPoly2 (@ (x) integral (@ (z) sin (z.^2) , 0 , x) , gx1 , gx2 , [1:0.01:5] , Lb , Ub , runNum
, "FresnelSine (x) " , "FresnelSine_1.txt")

PadeShamPoly2 (@ (x) integral (@ (z) cos (z.^2) , 0 , x) , gx1 , gx2 , [0.5:0.01:5] , Lb , Ub , runN
um , "FresnelCosine (x) " , "FresnelCosine_1.txt")

PadeShamPoly2 (@ (x) integral (@ (z) sin (z) ./z , 0 , x) , gx1 , gx2 , [1.3:0.1:20] , Lb , Ub , runN
um , "Si (x) " , "Si_1.txt")
    PadeShamPoly2 (@ (x) 0.57721566+log (x) -integral (@ (z) (1-
cos (z)) ./z , 0 , x) , gx1 , gx2 , [0.5:0.1:20] , Lb , Ub , runNum , "Ci (x) " , "CI_1.txt")

PadeShamPoly2 (@ (x) asinh (x) , gx1 , gx2 , [0:0.1:100] , Lb , Ub , runNum , "asinh (x) " , "asinh
_1.txt")

PadeShamPoly2 (@ (x) acosh (x) , gx1 , gx2 , [1:0.1:100] , Lb , Ub , runNum , "acosh (x) " , "acosh
_1.txt")

PadeShamPoly2 (@ (x) atanh (x) , gx1 , gx2 , [0:0.001:0.999] , Lb , Ub , runNum , "atanh (x) " , "a
tanh_1.txt")
end

if selIdx==0 || selIdx==2
    lstA = [0 0.5 0 2];
    lstB = [0.1 2 0.1 1];
    lstOrder = [3 7];
    Lb = [lstA(1) lstB(1) lstA(3) lstB(3) lstOrder(1) lstOrder(1)];
    Ub = [lstA(2) lstB(2) lstA(4) lstB(4) lstOrder(2) lstOrder(2)];
    gx1 = @(i,A,B)A+B*i;
    gx2 = @(i,A,B)A+B*sqrt(i);

PadeShamPoly2 (@ (x) acos (x) , gx1 , gx2 , [0:.01:1] , Lb , Ub , runNum , "arccos (x) " , "arccos_
2.txt")

PadeShamPoly2 (@ (x) asin (x) , gx1 , gx2 , [0:.01:1] , Lb , Ub , runNum , "arcsin (x) " , "arcsin_
2.txt")

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PadeShamPoly2 (@ (x) atan (x) ,gx1,gx2,[0:.01:1],Lb,Ub,runNum,"arctan(x)","arctan_
2.txt")

PadeShamPoly2 (@ (x) sin (x) ,gx1,gx2,[0:pi/1000:pi/2],Lb,Ub,runNum,"sin(x)","sin_
2.txt")

PadeShamPoly2 (@ (x) cos (x) ,gx1,gx2,[0:pi/1000:pi/2],Lb,Ub,runNum,"cos(x)","cos_
2.txt")

PadeShamPoly2 (@ (x) tan (x) ,gx1,gx2,[0:.01:1],Lb,Ub,runNum,"tan(x)","tan_2.txt")

PadeShamPoly2 (@ (x) sinh (x) ,gx1,gx2,[0:.01:5],Lb,Ub,runNum,"sinh(x)","sinh_2.tx
t")

PadeShamPoly2 (@ (x) cosh (x) ,gx1,gx2,[0:.01:5],Lb,Ub,runNum,"cosh(x)","cosh_2.tx
t")

PadeShamPoly2 (@ (x) tanh (x) ,gx1,gx2,[0:.01:3],Lb,Ub,runNum,"tanh(x)","tanh_2.tx
t")

PadeShamPoly2 (@ (x) erf (x) ,gx1,gx2,[0:.01:2.1],Lb,Ub,runNum,"erf(x)","erf_2.txt
")

PadeShamPoly2 (@ (x) exp (x) ,gx1,gx2,[0:.01:2],Lb,Ub,runNum,"exp(x)","exp_2.txt")

PadeShamPoly2 (@ (x) log (x) ,gx1,gx2,[1:.01:10],Lb,Ub,runNum,"ln(x)","ln_2.txt")

PadeShamPoly2 (@ (x) log10 (x) ,gx1,gx2,[1:.01:10],Lb,Ub,runNum,"log(x)","log_2.tx
t")

PadeShamPoly2 (@ (x) 10.^x,gx1,gx2,[0:.01:1],Lb,Ub,runNum,"10^x","pwr10_2.txt")

PadeShamPoly2 (@ (x) tinv(0.95,x) ,gx1,gx2,[2:0.1:100],Lb,Ub,runNum,"tinv(0.95,x)
","tinv1_2.txt")

PadeShamPoly2 (@ (x) tinv(0.975,x) ,gx1,gx2,[2:0.1:100],Lb,Ub,runNum,"tinv(0.975,
x)","tinv2_2.txt")

PadeShamPoly2 (@ (x) log10 (gamma (x) ) ,gx1,gx2,[2:0.1:100],Lb,Ub,runNum,"log10Gamm
a(x)","log10Gamma_2.txt")

PadeShamPoly2 (@ (x) digamma (x) ,gx1,gx2,[2:0.1:100],Lb,Ub,runNum,"digamma(x)","d
igamma_2.txt")

PadeShamPoly2 (@ (x) trigamma (x) ,gx1,gx2,[1:0.1:100],Lb,Ub,runNum,"trigamma(x)",
"trigamma_2.txt")

PadeShamPoly2 (@ (x) trigamma (x) ,gx1,gx2,[1:0.1:100],Lb,Ub,runNum,"trigamma(x)",
"trigamma_2.txt")

PadeShamPoly2 (@ (x) besselj (0,x) ,gx1,gx2,[2:0.1:30],Lb,Ub,runNum,"J0(x)","J0x_2
.txt")

PadeShamPoly2 (@ (x) besselj (1,x) ,gx1,gx2,[0:0.1:30],Lb,Ub,runNum,"J1(x)","J1x_2
.txt")

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PadeShamPoly2 (@ (x) besselj (2, x) , gx1, gx2, [0:0.1:30] , Lb, Ub, runNum, "J2 (x) " , "J2x_2
.txt")

PadeShamPoly2 (@ (x) besselj (3, x) , gx1, gx2, [0:0.1:30] , Lb, Ub, runNum, "J3 (x) " , "J3x_2
.txt")

PadeShamPoly2 (@ (x) besselj (4, x) , gx1, gx2, [0:0.1:30] , Lb, Ub, runNum, "J4 (x) " , "J4x_2
.txt")

PadeShamPoly2 (@ (x) besselj (5, x) , gx1, gx2, [0:0.1:30] , Lb, Ub, runNum, "J5 (x) " , "J5x_2
.txt")

PadeShamPoly2 (@ (x) integral (@ (z) sin (z.^2) , 0, x) , gx1, gx2, [1:0.01:5] , Lb, Ub, runNum
, "FresnelSine (x) " , "FresnelSine_2.txt")

PadeShamPoly2 (@ (x) integral (@ (z) cos (z.^2) , 0, x) , gx1, gx2, [0.5:0.01:5] , Lb, Ub, runN
um, "FresnelCosine (x) " , "FresnelCosine_2.txt")

PadeShamPoly2 (@ (x) integral (@ (z) sin (z) ./z, 0, x) , gx1, gx2, [1.3:0.1:20] , Lb, Ub, runN
um, "Si (x) " , "Si_2.txt")
    PadeShamPoly2 (@ (x) 0.57721566+log (x) -integral (@ (z) (1-
cos (z)) ./z, 0, x) , gx1, gx2, [0.5:0.1:20] , Lb, Ub, runNum, "Ci (x) " , "Ci_2.txt")

PadeShamPoly2 (@ (x) asinh (x) , gx1, gx2, [0:0.1:100] , Lb, Ub, runNum, "asinh (x) " , "asinh
_2.txt")

PadeShamPoly2 (@ (x) acosh (x) , gx1, gx2, [1:0.1:100] , Lb, Ub, runNum, "acosh (x) " , "acosh
_2.txt")

PadeShamPoly2 (@ (x) atanh (x) , gx1, gx2, [0:0.001:0.999] , Lb, Ub, runNum, "atanh (x) " , "a
tanh_2.txt")
end

if selIdx==0 || selIdx==3
    lstA = [0 0.55 0 0.34];
    lstB = [0.1 0.4 0.1 0.39];
    lstC = [0 0.4 0 0.36];
    lstOrder = [3 7];
    Lb = [lstA(1) lstB(1) lstC(1) lstA(3) lstB(3) lstC(3) lstOrder(1)
lstOrder(1)];
    Ub = [lstA(2) lstB(2) lstC(2) lstA(4) lstB(4) lstC(4) lstOrder(2)
lstOrder(2)];
    gx1 = @(i, A, B, C) A+B*sqrt(i)+C*i;
    gx2 = @(i, A, B, C) A+B*i+C*sqrt(i);

PadeShamPoly3 (@ (x) acos (x) , gx1, gx2, [0:.01:1] , Lb, Ub, runNum, "arccos (x) " , "arccos_
3.txt")

PadeShamPoly3 (@ (x) asin (x) , gx1, gx2, [0:.01:1] , Lb, Ub, runNum, "arcsin (x) " , "arcsin_
3.txt")

PadeShamPoly3 (@ (x) atan (x) , gx1, gx2, [0:.01:1] , Lb, Ub, runNum, "arctan (x) " , "arctan_
3.txt")

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PadeShamPoly3(@ (x) sin(x),gx1,gx2,[0:pi/1000:pi/2],Lb,Ub,runNum,"sin(x)","sin_
3.txt")

PadeShamPoly3(@ (x) cos(x),gx1,gx2,[0:pi/1000:pi/2],Lb,Ub,runNum,"cos(x)","cos_
3.txt")

PadeShamPoly3(@ (x) tan(x),gx1,gx2,[0:.01:1],Lb,Ub,runNum,"tan(x)","tan_3.txt")

PadeShamPoly3(@ (x) sinh(x),gx1,gx2,[0:.01:5],Lb,Ub,runNum,"sinh(x)","sinh_3.tx
t")

PadeShamPoly3(@ (x) cosh(x),gx1,gx2,[0:.01:5],Lb,Ub,runNum,"cosh(x)","cosh_3.tx
t")

PadeShamPoly3(@ (x) tanh(x),gx1,gx2,[0:.01:3],Lb,Ub,runNum,"tanh(x)","tanh_3.tx
t")

PadeShamPoly3(@ (x) erf(x),gx1,gx2,[0:.01:2.1],Lb,Ub,runNum,"erf(x)","erf_3.txt
")

PadeShamPoly3(@ (x) exp(x),gx1,gx2,[0:.01:2],Lb,Ub,runNum,"exp(x)","exp_3.txt")

PadeShamPoly3(@ (x) log(x),gx1,gx2,[1:.01:10],Lb,Ub,runNum,"ln(x)","ln_3.txt")

PadeShamPoly3(@ (x) log10(x),gx1,gx2,[1:.01:10],Lb,Ub,runNum,"log(x)","log_3.tx
t")

PadeShamPoly3(@ (x) 10.^x,gx1,gx2,[0:.01:1],Lb,Ub,runNum,"10^x","pwr10_3.txt")

PadeShamPoly3(@ (x) tinv(0.95,x),gx1,gx2,[2:0.1:100],Lb,Ub,runNum,"tinv(0.95,x)
","tinv1_3.txt")

PadeShamPoly3(@ (x) tinv(0.975,x),gx1,gx2,[2:0.1:100],Lb,Ub,runNum,"tinv(0.975,
x)","tinv2_3.txt")

PadeShamPoly3(@ (x) log10(gamma(x)),gx1,gx2,[2:0.1:100],Lb,Ub,runNum,"log10Gamm
a(x)","log10Gamma_3.txt")

PadeShamPoly3(@ (x) digamma(x),gx1,gx2,[2:0.1:100],Lb,Ub,runNum,"digamma(x)","d
igamma_3.txt")

PadeShamPoly3(@ (x) trigamma(x),gx1,gx2,[1:0.1:100],Lb,Ub,runNum,"trigamma(x)",
"trigamma_3.txt")

PadeShamPoly3(@ (x) besselj(0,x),gx1,gx2,[2:0.1:30],Lb,Ub,runNum,"J0(x)","J0x_3
.txt")

PadeShamPoly3(@ (x) besselj(1,x),gx1,gx2,[0:0.1:30],Lb,Ub,runNum,"J1(x)","J1x_3
.txt")

PadeShamPoly3(@ (x) besselj(2,x),gx1,gx2,[0:0.1:30],Lb,Ub,runNum,"J2(x)","J2x_3
.txt")

PadeShamPoly3(@ (x) besselj(3,x),gx1,gx2,[0:0.1:30],Lb,Ub,runNum,"J3(x)","J3x_3
.txt")

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PadeShamPoly3(@ (x)besselj(4,x),gx1,gx2,[0:0.1:30],Lb,Ub,runNum,"J4(x)","J4x_3
.txt")

PadeShamPoly3(@ (x)besselj(5,x),gx1,gx2,[0:0.1:30],Lb,Ub,runNum,"J5(x)","J5x_3
.txt")

PadeShamPoly3(@ (x)integral(@(z)sin(z.^2),0,x),gx1,gx2,[1:0.01:5],Lb,Ub,runNum
,"FresnelSine(x)","FresnelSine_3.txt")

PadeShamPoly3(@ (x)integral(@(z)cos(z.^2),0,x),gx1,gx2,[0.5:0.01:5],Lb,Ub,runN
um,"FresnelCosine(x)","FresnelCosine_3.txt")

PadeShamPoly3(@ (x)integral(@(z)sin(z)./z,0,x),gx1,gx2,[1.3:0.1:20],Lb,Ub,runN
um,"Si(x)","Si_3.txt")
PadeShamPoly3(@ (x)0.57721566+log(x)-integral(@(z)(1-
cos(z))./z,0,x),gx1,gx2,[0.5:0.1:20],Lb,Ub,runNum,"Ci(x)","Ci_3.txt")

PadeShamPoly3(@ (x)asinh(x),gx1,gx2,[0:0.1:100],Lb,Ub,runNum,"asinh(x)","asinh
_3.txt")

PadeShamPoly3(@ (x)acosh(x),gx1,gx2,[1:0.1:100],Lb,Ub,runNum,"acosh(x)","acosh
_3.txt")

PadeShamPoly3(@ (x)atanh(x),gx1,gx2,[0:0.001:0.999],Lb,Ub,runNum,"atanh(x)","a
tanh_3.txt")
end

if selIdx==0 || selIdx==4
    lstA = [0.1 0.5 0.1 2];
    lstB = [0 0.5 0 2];
    lstOrder = [3 7];
    Lb = [lstA(1) lstB(1) lstA(3) lstB(3) lstOrder(1) lstOrder(1)];
    Ub = [lstA(2) lstB(2) lstA(4) lstB(4) lstOrder(2) lstOrder(2)];
    gx1 = @(i,A,B)A+B/i;
    gx2 = @(i,A,B)A+B*i;

PadeShamPoly2(@ (x)acos(x),gx1,gx2,[0:.01:1],Lb,Ub,runNum,"arccos(x)","arccos_
4.txt")

PadeShamPoly2(@ (x)asin(x),gx1,gx2,[0:.01:1],Lb,Ub,runNum,"arcsin(x)","arcsin_
4.txt")

PadeShamPoly2(@ (x)atan(x),gx1,gx2,[0:.01:1],Lb,Ub,runNum,"arctan(x)","arctan_
4.txt")

PadeShamPoly2(@ (x)sin(x),gx1,gx2,[0:pi/1000:pi/2],Lb,Ub,runNum,"sin(x)","sin_
4.txt")

PadeShamPoly2(@ (x)cos(x),gx1,gx2,[0:pi/1000:pi/2],Lb,Ub,runNum,"cos(x)","cos_
4.txt")

PadeShamPoly2(@ (x)tan(x),gx1,gx2,[0:.01:1],Lb,Ub,runNum,"tan(x)","tan_4.txt")

PadeShamPoly2(@ (x)sinh(x),gx1,gx2,[0:.01:5],Lb,Ub,runNum,"sinh(x)","sinh_4.tx
t")

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PadeShamPoly2 (@ (x) cosh (x) ,gx1,gx2,[0:.01:5],Lb,Ub,runNum,"cosh(x)","cosh_4.txt")

PadeShamPoly2 (@ (x) tanh (x) ,gx1,gx2,[0:.01:3],Lb,Ub,runNum,"tanh(x)","tanh_4.txt")

PadeShamPoly2 (@ (x) erf (x) ,gx1,gx2,[0:.01:2.1],Lb,Ub,runNum,"erf(x)","erf_4.txt")

PadeShamPoly2 (@ (x) exp (x) ,gx1,gx2,[0:.01:2],Lb,Ub,runNum,"exp(x)","exp_4.txt")

PadeShamPoly2 (@ (x) log (x) ,gx1,gx2,[1:.01:10],Lb,Ub,runNum,"ln(x)","ln_4.txt")

PadeShamPoly2 (@ (x) log10 (x) ,gx1,gx2,[1:.01:10],Lb,Ub,runNum,"log(x)","log_4.txt")

PadeShamPoly2 (@ (x) 10.^x,gx1,gx2,[0:.01:1],Lb,Ub,runNum,"10^x","pwr10_4.txt")

PadeShamPoly2 (@ (x) tinv (0.95,x) ,gx1,gx2,[2:0.1:100],Lb,Ub,runNum,"tinv(0.95,x)","tinv1_4.txt")

PadeShamPoly2 (@ (x) tinv (0.975,x) ,gx1,gx2,[2:0.1:100],Lb,Ub,runNum,"tinv(0.975,x)","tinv2_4.txt")

PadeShamPoly2 (@ (x) log10 (gamma (x)) ,gx1,gx2,[2:0.1:100],Lb,Ub,runNum,"log10Gamma(x)","log10Gamma_4.txt")

PadeShamPoly2 (@ (x) digamma (x) ,gx1,gx2,[2:0.1:100],Lb,Ub,runNum,"digamma(x)","digamma_4.txt")

PadeShamPoly2 (@ (x) trigamma (x) ,gx1,gx2,[1:0.1:100],Lb,Ub,runNum,"trigamma(x)","trigamma_4.txt")

PadeShamPoly2 (@ (x) besselj (0,x) ,gx1,gx2,[2:0.1:30],Lb,Ub,runNum,"J0(x)","J0x_4.txt")

PadeShamPoly2 (@ (x) besselj (1,x) ,gx1,gx2,[0:0.1:30],Lb,Ub,runNum,"J1(x)","J1x_4.txt")

PadeShamPoly2 (@ (x) besselj (2,x) ,gx1,gx2,[0:0.1:30],Lb,Ub,runNum,"J2(x)","J2x_4.txt")

PadeShamPoly2 (@ (x) besselj (3,x) ,gx1,gx2,[0:0.1:30],Lb,Ub,runNum,"J3(x)","J3x_4.txt")

PadeShamPoly2 (@ (x) besselj (4,x) ,gx1,gx2,[0:0.1:30],Lb,Ub,runNum,"J4(x)","J4x_4.txt")

PadeShamPoly2 (@ (x) besselj (5,x) ,gx1,gx2,[0:0.1:30],Lb,Ub,runNum,"J5(x)","J5x_4.txt")

PadeShamPoly2 (@ (x) integral (@ (z) sin (z.^2) ,0,x) ,gx1,gx2,[1:0.01:5],Lb,Ub,runNum,"FresnelSine(x)","FresnelSine_4.txt")

PadeShamPoly2 (@ (x) integral (@ (z) cos (z.^2) ,0,x) ,gx1,gx2,[0.5:0.01:5],Lb,Ub,runNum,"FresnelCosine(x)","FresnelCosine_4.txt")

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PadeShamPoly2 (@ (x) integral (@ (z) sin (z) ./z, 0, x) , gx1, gx2, [1.3:0.1:20] , Lb, Ub, runNum, "Si (x) ", "Si_4.txt")
    PadeShamPoly2 (@ (x) 0.57721566+log (x) -integral (@ (z) (1-
cos (z)) ./z, 0, x) , gx1, gx2, [0.5:0.1:20] , Lb, Ub, runNum, "Ci (x) ", "Ci_4.txt")

PadeShamPoly2 (@ (x) asinh (x) , gx1, gx2, [0:0.1:100] , Lb, Ub, runNum, "asinh (x) ", "asinh_4.txt")

PadeShamPoly2 (@ (x) acosh (x) , gx1, gx2, [1:0.1:100] , Lb, Ub, runNum, "acosh (x) ", "acosh_4.txt")

PadeShamPoly2 (@ (x) atanh (x) , gx1, gx2, [0:0.001:0.999] , Lb, Ub, runNum, "atanh (x) ", "atanh_4.txt")
end

if selIdx==0 || selIdx==5
    lstA = [0 1.4 0 2 ];
    lstB = [0 0.5 0 2];
    lstOrder = [3 7];
    Lb = [lstA(1) lstB(1) lstA(3) lstB(3) lstOrder(1) lstOrder(1)];
    Ub = [lstA(2) lstB(2) lstA(4) lstB(4) lstOrder(2) lstOrder(2)];
    gx1 = @(i,A,B)A+B*sqrt(i);
    gx2 = @(i,A,B)A+B*i;

PadeShamPoly2 (@ (x) acos (x) , gx1, gx2, [0:.01:1] , Lb, Ub, runNum, "arccos (x) ", "arccos_5.txt")

PadeShamPoly2 (@ (x) asin (x) , gx1, gx2, [0:.01:1] , Lb, Ub, runNum, "arcsin (x) ", "arcsin_5.txt")

PadeShamPoly2 (@ (x) atan (x) , gx1, gx2, [0:.01:1] , Lb, Ub, runNum, "arctan (x) ", "arctan_5.txt")

PadeShamPoly2 (@ (x) sin (x) , gx1, gx2, [0:pi/1000:pi/2] , Lb, Ub, runNum, "sin (x) ", "sin_5.txt")

PadeShamPoly2 (@ (x) cos (x) , gx1, gx2, [0:pi/1000:pi/2] , Lb, Ub, runNum, "cos (x) ", "cos_5.txt")

PadeShamPoly2 (@ (x) tan (x) , gx1, gx2, [0:.01:1] , Lb, Ub, runNum, "tan (x) ", "tan_5.txt")

PadeShamPoly2 (@ (x) sinh (x) , gx1, gx2, [0:.01:5] , Lb, Ub, runNum, "sinh (x) ", "sinh_5.txt")

PadeShamPoly2 (@ (x) cosh (x) , gx1, gx2, [0:.01:5] , Lb, Ub, runNum, "cosh (x) ", "cosh_5.txt")

PadeShamPoly2 (@ (x) tanh (x) , gx1, gx2, [0:.01:3] , Lb, Ub, runNum, "tanh (x) ", "tanh_5.txt")

PadeShamPoly2 (@ (x) erf (x) , gx1, gx2, [0:.01:2.1] , Lb, Ub, runNum, "erf (x) ", "erf_5.txt")

PadeShamPoly2 (@ (x) exp (x) , gx1, gx2, [0:.01:2] , Lb, Ub, runNum, "exp (x) ", "exp_5.txt")

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PadeShamPoly2 (@ (x) log (x) ,gx1,gx2,[1:.01:10],Lb,Ub,runNum,"ln(x)","ln_5.txt")

PadeShamPoly2 (@ (x) log10 (x) ,gx1,gx2,[1:.01:10],Lb,Ub,runNum,"log(x)","log_5.txt")

PadeShamPoly2 (@ (x) 10.^x,gx1,gx2,[0:.01:1],Lb,Ub,runNum,"10^x","pwr10_5.txt")

PadeShamPoly2 (@ (x) tinv (0.95,x) ,gx1,gx2,[2:0.1:100],Lb,Ub,runNum,"tinv(0.95,x)","tinv1_5.txt")

PadeShamPoly2 (@ (x) tinv (0.975,x) ,gx1,gx2,[2:0.1:100],Lb,Ub,runNum,"tinv(0.975,x)","tinv2_5.txt")

PadeShamPoly2 (@ (x) log10 (gamma (x)) ,gx1,gx2,[2:0.1:100],Lb,Ub,runNum,"log10Gamma(x)","log10Gamma_5.txt")

PadeShamPoly2 (@ (x) digamma (x) ,gx1,gx2,[2:0.1:100],Lb,Ub,runNum,"digamma(x)","digamma_5.txt")

PadeShamPoly2 (@ (x) trigamma (x) ,gx1,gx2,[1:0.1:100],Lb,Ub,runNum,"trigamma(x)","trigamma_5.txt")

PadeShamPoly2 (@ (x) besselj (0,x) ,gx1,gx2,[2:0.1:30],Lb,Ub,runNum,"J0(x)","J0x_5.txt")

PadeShamPoly2 (@ (x) besselj (1,x) ,gx1,gx2,[0:0.1:30],Lb,Ub,runNum,"J1(x)","J1x_5.txt")

PadeShamPoly2 (@ (x) besselj (2,x) ,gx1,gx2,[0:0.1:30],Lb,Ub,runNum,"J2(x)","J2x_5.txt")

PadeShamPoly2 (@ (x) besselj (3,x) ,gx1,gx2,[0:0.1:30],Lb,Ub,runNum,"J3(x)","J3x_5.txt")

PadeShamPoly2 (@ (x) besselj (4,x) ,gx1,gx2,[0:0.1:30],Lb,Ub,runNum,"J4(x)","J4x_5.txt")

PadeShamPoly2 (@ (x) besselj (5,x) ,gx1,gx2,[0:0.1:30],Lb,Ub,runNum,"J5(x)","J5x_5.txt")

PadeShamPoly2 (@ (x) integral (@ (z) sin (z.^2) ,0,x) ,gx1,gx2,[1:0.01:5],Lb,Ub,runNum,"FresnelSine(x)","FresnelSine_5.txt")

PadeShamPoly2 (@ (x) integral (@ (z) cos (z.^2) ,0,x) ,gx1,gx2,[0.5:0.01:5],Lb,Ub,runNum,"FresnelCosine(x)","FresnelCosine_5.txt")

PadeShamPoly2 (@ (x) integral (@ (z) sin (z) ./z ,0,x) ,gx1,gx2,[1.3:0.1:20],Lb,Ub,runNum,"Si(x)","Si_5.txt")

PadeShamPoly2 (@ (x) 0.57721566+log (x) -integral (@ (z) (1-cos (z)) ./z ,0,x) ,gx1,gx2,[0.5:0.1:20],Lb,Ub,runNum,"Ci(x)","Ci_5.txt")

PadeShamPoly2 (@ (x) asinh (x) ,gx1,gx2,[0:0.1:100],Lb,Ub,runNum,"asinh(x)","asinh_5.txt")

PadeShamPoly2 (@ (x) acosh (x) ,gx1,gx2,[1:0.1:100],Lb,Ub,runNum,"acosh(x)","acosh_5.txt")

```

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PadeShamPoly2 (@ (x) atanh (x) , gx1 , gx2 , [0:0.001:0.999] , Lb , Ub , runNum , "atanh (x) " , "a
tanh_5.txt")
end

if selIdx==0 || selIdx==6
    lstA = [0 0.34 0 0.55];
    lstB = [0.1 0.39 0.1 0.4];
    lstC = [0 0.36 0 0.4];
    lstOrder = [3 7];
    Lb = [lstA(1) lstB(1) lstC(1) lstA(3) lstB(3) lstC(3) lstOrder(1)
lstOrder(1)];
    Ub = [lstA(2) lstB(2) lstC(2) lstA(4) lstB(4) lstC(4) lstOrder(2)
lstOrder(2)];
    gx1 = @(i,A,B,C)A+B*i+C*sqrt(i);
    gx2 = @(i,A,B,C)A+B*sqrt(i)+C*i;

PadeShamPoly3 (@ (x) acos (x) , gx1 , gx2 , [0:.01:1] , Lb , Ub , runNum , "arccos (x) " , "arccos_
6.txt")

PadeShamPoly3 (@ (x) asin (x) , gx1 , gx2 , [0:.01:1] , Lb , Ub , runNum , "arcsin (x) " , "arcsin_
6.txt")

PadeShamPoly3 (@ (x) atan (x) , gx1 , gx2 , [0:.01:1] , Lb , Ub , runNum , "arctan (x) " , "arctan_
6.txt")

PadeShamPoly3 (@ (x) sin (x) , gx1 , gx2 , [0:pi/1000:pi/2] , Lb , Ub , runNum , "sin (x) " , "sin_
6.txt")

PadeShamPoly3 (@ (x) cos (x) , gx1 , gx2 , [0:pi/1000:pi/2] , Lb , Ub , runNum , "cos (x) " , "cos_
6.txt")

PadeShamPoly3 (@ (x) tan (x) , gx1 , gx2 , [0:.01:1] , Lb , Ub , runNum , "tan (x) " , "tan_6.txt")

PadeShamPoly3 (@ (x) sinh (x) , gx1 , gx2 , [0:.01:5] , Lb , Ub , runNum , "sinh (x) " , "sinh_6.tx
t")

PadeShamPoly3 (@ (x) cosh (x) , gx1 , gx2 , [0:.01:5] , Lb , Ub , runNum , "cosh (x) " , "cosh_6.tx
t")

PadeShamPoly3 (@ (x) tanh (x) , gx1 , gx2 , [0:.01:3] , Lb , Ub , runNum , "tanh (x) " , "tanh_6.tx
t")

PadeShamPoly3 (@ (x) erf (x) , gx1 , gx2 , [0:.01:2.1] , Lb , Ub , runNum , "erf (x) " , "erf_6.txt
")

PadeShamPoly3 (@ (x) exp (x) , gx1 , gx2 , [0:.01:2] , Lb , Ub , runNum , "exp (x) " , "exp_6.txt")

PadeShamPoly3 (@ (x) log (x) , gx1 , gx2 , [1:.01:10] , Lb , Ub , runNum , "ln (x) " , "ln_6.txt")

PadeShamPoly3 (@ (x) log10 (x) , gx1 , gx2 , [1:.01:10] , Lb , Ub , runNum , "log (x) " , "log_6.tx
t")

PadeShamPoly3 (@ (x) 10.^x , gx1 , gx2 , [0:.01:1] , Lb , Ub , runNum , "10^x" , "pwr10_6.txt")

PadeShamPoly3 (@ (x) tinv (0.95 , x) , gx1 , gx2 , [2:0.1:100] , Lb , Ub , runNum , "tinv (0.95 , x)
" , "tinv1_6.txt")

```

```

PadeShamPoly3(@ (x) tinv(0.975,x),gx1,gx2,[2:0.1:100],Lb,Ub,runNum,"tinv(0.975,
x)","tinv2_6.txt")

PadeShamPoly3(@ (x) log10(gamma(x)),gx1,gx2,[2:0.1:100],Lb,Ub,runNum,"log10Gamm
a(x)","log10Gamma_6.txt")

PadeShamPoly3(@ (x) digamma(x),gx1,gx2,[2:0.1:100],Lb,Ub,runNum,"digamma(x)","d
igamma_6.txt")

PadeShamPoly3(@ (x) trigamma(x),gx1,gx2,[1:0.1:100],Lb,Ub,runNum,"trigamma(x)",
"trigamma_6.txt")

PadeShamPoly3(@ (x) besselj(0,x),gx1,gx2,[2:0.1:30],Lb,Ub,runNum,"J0(x)","J0x_6
.txt")

PadeShamPoly3(@ (x) besselj(1,x),gx1,gx2,[0:0.1:30],Lb,Ub,runNum,"J1(x)","J1x_6
.txt")

PadeShamPoly3(@ (x) besselj(2,x),gx1,gx2,[0:0.1:30],Lb,Ub,runNum,"J2(x)","J2x_6
.txt")

PadeShamPoly3(@ (x) besselj(3,x),gx1,gx2,[0:0.1:30],Lb,Ub,runNum,"J3(x)","J3x_6
.txt")

PadeShamPoly3(@ (x) besselj(4,x),gx1,gx2,[0:0.1:30],Lb,Ub,runNum,"J4(x)","J4x_6
.txt")

PadeShamPoly3(@ (x) besselj(5,x),gx1,gx2,[0:0.1:30],Lb,Ub,runNum,"J5(x)","J5x_6
.txt")

PadeShamPoly3(@ (x) integral(@(z) sin(z.^2),0,x),gx1,gx2,[1:0.01:5],Lb,Ub,runNum
,"FresnelSine(x)","FresnelSine_6.txt")

PadeShamPoly3(@ (x) integral(@(z) cos(z.^2),0,x),gx1,gx2,[0.5:0.01:5],Lb,Ub,runN
um,"FresnelCosine(x)","FresnelCosine_6.txt")

PadeShamPoly3(@ (x) integral(@(z) sin(z)./z,0,x),gx1,gx2,[1.3:0.1:20],Lb,Ub,runN
um,"Si(x)","Si_6.txt")
PadeShamPoly3(@ (x) 0.57721566+log(x)-integral(@(z) (1-
cos(z))./z,0,x),gx1,gx2,[0.5:0.1:20],Lb,Ub,runNum,"Ci(x)","Ci_6.txt")

PadeShamPoly3(@ (x) asinh(x),gx1,gx2,[0:0.1:100],Lb,Ub,runNum,"asinh(x)","asinh
_6.txt")

PadeShamPoly3(@ (x) acosh(x),gx1,gx2,[1:0.1:100],Lb,Ub,runNum,"acosh(x)","acosh
_6.txt")

PadeShamPoly3(@ (x) atanh(x),gx1,gx2,[0:0.001:0.999],Lb,Ub,runNum,"atanh(x)","a
tanh_6.txt")
end

if selIdx==0 || selIdx==7
    lstA = [0 0.5 0 2];
    lstB = [0.1 0.5 0.1 3.5];
    lstOrder = [3 7];
    Lb = [lstA(1) lstB(1) lstA(3) lstB(3) lstOrder(1) lstOrder(1)];

```

```

Ub = [1stA(2) 1stB(2) 1stA(4) 1stB(4) 1stOrder(2) 1stOrder(2)];
gx1 = @(i,A,B)A+B*i;
gx2 = @(i,A,B)A+B*log10(i)^4;

PadeShamPoly2(@(x)acos(x),gx1,gx2,[0:.01:1],Lb,Ub,runNum,"arccos(x)","arccos_
7.txt")

PadeShamPoly2(@(x)asin(x),gx1,gx2,[0:.01:1],Lb,Ub,runNum,"arcsin(x)","arcsin_
7.txt")

PadeShamPoly2(@(x)atan(x),gx1,gx2,[0:.01:1],Lb,Ub,runNum,"arctan(x)","arctan_
7.txt")

PadeShamPoly2(@(x)sin(x),gx1,gx2,[0:pi/1000:pi/2],Lb,Ub,runNum,"sin(x)","sin_
7.txt")

PadeShamPoly2(@(x)cos(x),gx1,gx2,[0:pi/1000:pi/2],Lb,Ub,runNum,"cos(x)","cos_
7.txt")

PadeShamPoly2(@(x)tan(x),gx1,gx2,[0:.01:1],Lb,Ub,runNum,"tan(x)","tan_7.txt")

PadeShamPoly2(@(x)sinh(x),gx1,gx2,[0:.01:5],Lb,Ub,runNum,"sinh(x)","sinh_7.tx
t")

PadeShamPoly2(@(x)cosh(x),gx1,gx2,[0:.01:5],Lb,Ub,runNum,"cosh(x)","cosh_7.tx
t")

PadeShamPoly2(@(x)tanh(x),gx1,gx2,[0:.01:3],Lb,Ub,runNum,"tanh(x)","tanh_7.tx
t")

PadeShamPoly2(@(x)erf(x),gx1,gx2,[0:.01:2.1],Lb,Ub,runNum,"erf(x)","erf_7.txt
")

PadeShamPoly2(@(x)exp(x),gx1,gx2,[0:.01:2],Lb,Ub,runNum,"exp(x)","exp_7.txt")

PadeShamPoly2(@(x)log(x),gx1,gx2,[1:.01:10],Lb,Ub,runNum,"ln(x)","ln_7.txt")

PadeShamPoly2(@(x)log10(x),gx1,gx2,[1:.01:10],Lb,Ub,runNum,"log(x)","log_7.tx
t")

PadeShamPoly2(@(x)10.^x,gx1,gx2,[0:.01:1],Lb,Ub,runNum,"10^x","pwr10_7.txt")

PadeShamPoly2(@(x)tinvt(0.95,x),gx1,gx2,[2:0.1:100],Lb,Ub,runNum,"tinvt(0.95,x)
","tinvt1_7.txt")

PadeShamPoly2(@(x)tinvt(0.975,x),gx1,gx2,[2:0.1:100],Lb,Ub,runNum,"tinvt(0.975,
x)","tinvt2_7.txt")

PadeShamPoly2(@(x)log10(gamma(x)),gx1,gx2,[2:0.1:100],Lb,Ub,runNum,"log10Gamm
a(x)","log10Gamma_7.txt")

PadeShamPoly2(@(x)digamma(x),gx1,gx2,[2:0.1:100],Lb,Ub,runNum,"digamma(x)","d
igamma_7.txt")

PadeShamPoly2(@(x)trigamma(x),gx1,gx2,[1:0.1:100],Lb,Ub,runNum,"trigamma(x)","
trigamma_7.txt")

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PadeShamPoly2 (@ (x) besselj (0,x) ,gx1,gx2,[2:0.1:30],Lb,Ub,runNum,"J0(x)","J0x_7
.txt")

PadeShamPoly2 (@ (x) besselj (1,x) ,gx1,gx2,[0:0.1:30],Lb,Ub,runNum,"J1(x)","J1x_7
.txt")

PadeShamPoly2 (@ (x) besselj (2,x) ,gx1,gx2,[0:0.1:30],Lb,Ub,runNum,"J2(x)","J2x_7
.txt")

PadeShamPoly2 (@ (x) besselj (3,x) ,gx1,gx2,[0:0.1:30],Lb,Ub,runNum,"J3(x)","J3x_7
.txt")

PadeShamPoly2 (@ (x) besselj (4,x) ,gx1,gx2,[0:0.1:30],Lb,Ub,runNum,"J4(x)","J4x_7
.txt")

PadeShamPoly2 (@ (x) besselj (5,x) ,gx1,gx2,[0:0.1:30],Lb,Ub,runNum,"J5(x)","J5x_7
.txt")

PadeShamPoly2 (@ (x) integral (@ (z) sin (z.^2) ,0,x) ,gx1,gx2,[1:0.01:5],Lb,Ub,runNum
,"FresnelSine(x)","FresnelSine_7.txt")

PadeShamPoly2 (@ (x) integral (@ (z) cos (z.^2) ,0,x) ,gx1,gx2,[0.5:0.01:5],Lb,Ub,runN
um,"FresnelCosine(x)","FresnelCosine_7.txt")

PadeShamPoly2 (@ (x) integral (@ (z) sin (z) ./z,0,x) ,gx1,gx2,[1.3:0.1:20],Lb,Ub,runN
um,"Si(x)","Si_7.txt")
PadeShamPoly2 (@ (x) 0.57721566+log(x)-integral (@ (z) (1-
cos(z)) ./z,0,x) ,gx1,gx2,[0.5:0.1:20],Lb,Ub,runNum,"Ci(x)","CI_7.txt")

PadeShamPoly2 (@ (x) asinh (x) ,gx1,gx2,[0:0.1:100],Lb,Ub,runNum,"asinh(x)","asinh
_7.txt")

PadeShamPoly2 (@ (x) acosh (x) ,gx1,gx2,[1:0.1:100],Lb,Ub,runNum,"acosh(x)","acosh
_7.txt")

PadeShamPoly2 (@ (x) atanh (x) ,gx1,gx2,[0:0.001:0.999],Lb,Ub,runNum,"atanh(x)","a
tanh_7.txt")
end

if selIdx==0 || selIdx==8
    lstA = [0.1 0.5 0.1 2];
    lstB = [0.1 3.5 0.1 0.5];
    lstOrder = [3 7];
    Lb = [lstA(1) lstB(1) lstA(3) lstB(3) lstOrder(1) lstOrder(1)];
    Ub = [lstA(2) lstB(2) lstA(4) lstB(4) lstOrder(2) lstOrder(2)];
    gx1 = @(i,A,B) A+B/i;
    gx2 = @(i,A,B) A+B*log10(i)^4;

PadeShamPoly2 (@ (x) acos (x) ,gx1,gx2,[0:.01:1],Lb,Ub,runNum,"arccos(x)","arccos_
8.txt")

PadeShamPoly2 (@ (x) asin (x) ,gx1,gx2,[0:.01:1],Lb,Ub,runNum,"arcsin(x)","arcsin_
8.txt")

PadeShamPoly2 (@ (x) atan (x) ,gx1,gx2,[0:.01:1],Lb,Ub,runNum,"arctan(x)","arctan_
8.txt")

```

```

PadeShamPoly2 (@ (x) sin (x) , gx1 , gx2 , [0:pi/1000:pi/2] , Lb , Ub , runNum , "sin (x)" , "sin_
8.txt")

PadeShamPoly2 (@ (x) cos (x) , gx1 , gx2 , [0:pi/1000:pi/2] , Lb , Ub , runNum , "cos (x)" , "cos_
8.txt")

PadeShamPoly2 (@ (x) tan (x) , gx1 , gx2 , [0:.01:1] , Lb , Ub , runNum , "tan (x)" , "tan_8.txt")

PadeShamPoly2 (@ (x) sinh (x) , gx1 , gx2 , [0:.01:5] , Lb , Ub , runNum , "sinh (x)" , "sinh_8.tx
t")

PadeShamPoly2 (@ (x) cosh (x) , gx1 , gx2 , [0:.01:5] , Lb , Ub , runNum , "cosh (x)" , "cosh_8.tx
t")

PadeShamPoly2 (@ (x) tanh (x) , gx1 , gx2 , [0:.01:3] , Lb , Ub , runNum , "tanh (x)" , "tanh_8.tx
t")

PadeShamPoly2 (@ (x) erf (x) , gx1 , gx2 , [0:.01:2.1] , Lb , Ub , runNum , "erf (x)" , "erf_8.txt
")

PadeShamPoly2 (@ (x) exp (x) , gx1 , gx2 , [0:.01:2] , Lb , Ub , runNum , "exp (x)" , "exp_8.txt")

PadeShamPoly2 (@ (x) log (x) , gx1 , gx2 , [1:.01:10] , Lb , Ub , runNum , "ln (x)" , "ln_8.txt")

PadeShamPoly2 (@ (x) log10 (x) , gx1 , gx2 , [1:.01:10] , Lb , Ub , runNum , "log (x)" , "log_8.tx
t")

PadeShamPoly2 (@ (x) 10.^x , gx1 , gx2 , [0:.01:1] , Lb , Ub , runNum , "10^x" , "pwr10_8.txt")

PadeShamPoly2 (@ (x) tinv (0.95 , x) , gx1 , gx2 , [2:0.1:100] , Lb , Ub , runNum , "tinv (0.95 , x)
" , "tinv1_8.txt")

PadeShamPoly2 (@ (x) tinv (0.975 , x) , gx1 , gx2 , [2:0.1:100] , Lb , Ub , runNum , "tinv (0.975 ,
x)" , "tinv2_8.txt")

PadeShamPoly2 (@ (x) log10 (gamma (x)) , gx1 , gx2 , [2:0.1:100] , Lb , Ub , runNum , "log10Gamm
a (x)" , "log10Gamma_8.txt")

PadeShamPoly2 (@ (x) digamma (x) , gx1 , gx2 , [2:0.1:100] , Lb , Ub , runNum , "digamma (x)" , "d
igamma_8.txt")

PadeShamPoly2 (@ (x) trigamma (x) , gx1 , gx2 , [1:0.1:100] , Lb , Ub , runNum , "trigamma (x)" , "
trigamma_8.txt")

PadeShamPoly2 (@ (x) besselj (0 , x) , gx1 , gx2 , [2:0.1:30] , Lb , Ub , runNum , "J0 (x)" , "J0x_8
.txt")

PadeShamPoly2 (@ (x) besselj (1 , x) , gx1 , gx2 , [0:0.1:30] , Lb , Ub , runNum , "J1 (x)" , "J1x_8
.txt")

PadeShamPoly2 (@ (x) besselj (2 , x) , gx1 , gx2 , [0:0.1:30] , Lb , Ub , runNum , "J2 (x)" , "J2x_8
.txt")

PadeShamPoly2 (@ (x) besselj (3 , x) , gx1 , gx2 , [0:0.1:30] , Lb , Ub , runNum , "J3 (x)" , "J3x_8
.txt")

```

```

PadeShamPoly2 (@ (x) besselj (4, x) , gx1, gx2, [0:0.1:30] , Lb, Ub, runNum, "J4 (x) ", "J4x_8
.txt")

PadeShamPoly2 (@ (x) besselj (5, x) , gx1, gx2, [0:0.1:30] , Lb, Ub, runNum, "J5 (x) ", "J5x_8
.txt")

PadeShamPoly2 (@ (x) integral (@ (z) sin (z.^2) , 0, x) , gx1, gx2, [1:0.01:5] , Lb, Ub, runNum
, "FresnelSine (x) ", "FresnelSine_8.txt")

PadeShamPoly2 (@ (x) integral (@ (z) cos (z.^2) , 0, x) , gx1, gx2, [0.5:0.01:5] , Lb, Ub, runN
um, "FresnelCosine (x) ", "FresnelCosine_8.txt")

PadeShamPoly2 (@ (x) integral (@ (z) sin (z) ./z, 0, x) , gx1, gx2, [1.3:0.1:20] , Lb, Ub, runN
um, "Si (x) ", "Si_8.txt")
    PadeShamPoly2 (@ (x) 0.57721566+log (x) -integral (@ (z) (1-
cos (z)) ./z, 0, x) , gx1, gx2, [0.5:0.1:20] , Lb, Ub, runNum, "Ci (x) ", "CI_8.txt")

PadeShamPoly2 (@ (x) asinh (x) , gx1, gx2, [0:0.1:100] , Lb, Ub, runNum, "asinh (x) ", "asinh
_8.txt")

PadeShamPoly2 (@ (x) acosh (x) , gx1, gx2, [1:0.1:100] , Lb, Ub, runNum, "acosh (x) ", "acosh
_8.txt")

PadeShamPoly2 (@ (x) atanh (x) , gx1, gx2, [0:0.001:0.999] , Lb, Ub, runNum, "atanh (x) ", "a
tanh_8.txt")
end

if selIdx==0 || selIdx==9
    lstA = [0 2.2 0 1.4];
    lstB = [0 0.5 0 2];
    lstOrder = [3 7];
    Lb = [lstA(1) lstB(1) lstA(3) lstB(3) lstOrder(1) lstOrder(1)];
    Ub = [lstA(2) lstB(2) lstA(4) lstB(4) lstOrder(2) lstOrder(2)];
    gx1 = @(i, A, B) A+B*log10 (i) ^4;
    gx2 = @(i, A, B) A+B*i;

PadeShamPoly2 (@ (x) acos (x) , gx1, gx2, [0:.01:1] , Lb, Ub, runNum, "arccos (x) ", "arccos_
9.txt")

PadeShamPoly2 (@ (x) asin (x) , gx1, gx2, [0:.01:1] , Lb, Ub, runNum, "arcsin (x) ", "arcsin_
9.txt")

PadeShamPoly2 (@ (x) atan (x) , gx1, gx2, [0:.01:1] , Lb, Ub, runNum, "arctan (x) ", "arctan_
9.txt")

PadeShamPoly2 (@ (x) sin (x) , gx1, gx2, [0:pi/1000:pi/2] , Lb, Ub, runNum, "sin (x) ", "sin_
9.txt")

PadeShamPoly2 (@ (x) cos (x) , gx1, gx2, [0:pi/1000:pi/2] , Lb, Ub, runNum, "cos (x) ", "cos_
9.txt")

PadeShamPoly2 (@ (x) tan (x) , gx1, gx2, [0:.01:1] , Lb, Ub, runNum, "tan (x) ", "tan_9.txt")

PadeShamPoly2 (@ (x) sinh (x) , gx1, gx2, [0:.01:5] , Lb, Ub, runNum, "sinh (x) ", "sinh_9.tx
t")

```

```
PadeShamPoly2 (@ (x) cosh (x) ,gx1,gx2,[0:.01:5],Lb,Ub,runNum,"cosh(x)","cosh_9.txt")
```

```
PadeShamPoly2 (@ (x) tanh (x) ,gx1,gx2,[0:.01:3],Lb,Ub,runNum,"tanh(x)","tanh_9.txt")
```

```
PadeShamPoly2 (@ (x) erf (x) ,gx1,gx2,[0:.01:2.1],Lb,Ub,runNum,"erf(x)","erf_9.txt")
```

```
PadeShamPoly2 (@ (x) exp (x) ,gx1,gx2,[0:.01:2],Lb,Ub,runNum,"exp(x)","exp_9.txt")
```

```
PadeShamPoly2 (@ (x) log (x) ,gx1,gx2,[1:.01:10],Lb,Ub,runNum,"ln(x)","ln_9.txt")
```

```
PadeShamPoly2 (@ (x) log10 (x) ,gx1,gx2,[1:.01:10],Lb,Ub,runNum,"log(x)","log_9.txt")
```

```
PadeShamPoly2 (@ (x) 10.^x,gx1,gx2,[0:.01:1],Lb,Ub,runNum,"10^x","pwr10_9.txt")
```

```
PadeShamPoly2 (@ (x) tinv (0.95,x) ,gx1,gx2,[2:0.1:100],Lb,Ub,runNum,"tinv(0.95,x)","tinv1_9.txt")
```

```
PadeShamPoly2 (@ (x) tinv (0.975,x) ,gx1,gx2,[2:0.1:100],Lb,Ub,runNum,"tinv(0.975,x)","tinv2_9.txt")
```

```
PadeShamPoly2 (@ (x) log10 (gamma (x)) ,gx1,gx2,[2:0.1:100],Lb,Ub,runNum,"log10Gamma(x)","log10Gamma_9.txt")
```

```
PadeShamPoly2 (@ (x) digamma (x) ,gx1,gx2,[2:0.1:100],Lb,Ub,runNum,"digamma(x)","digamma_9.txt")
```

```
PadeShamPoly2 (@ (x) trigamma (x) ,gx1,gx2,[1:0.1:100],Lb,Ub,runNum,"trigamma(x)","trigamma_9.txt")
```

```
PadeShamPoly2 (@ (x) besselj (0,x) ,gx1,gx2,[2:0.1:30],Lb,Ub,runNum,"J0(x)","J0x_9.txt")
```

```
PadeShamPoly2 (@ (x) besselj (1,x) ,gx1,gx2,[0:0.1:30],Lb,Ub,runNum,"J1(x)","J1x_9.txt")
```

```
PadeShamPoly2 (@ (x) besselj (2,x) ,gx1,gx2,[0:0.1:30],Lb,Ub,runNum,"J2(x)","J2x_9.txt")
```

```
PadeShamPoly2 (@ (x) besselj (3,x) ,gx1,gx2,[0:0.1:30],Lb,Ub,runNum,"J3(x)","J3x_9.txt")
```

```
PadeShamPoly2 (@ (x) besselj (4,x) ,gx1,gx2,[0:0.1:30],Lb,Ub,runNum,"J4(x)","J4x_9.txt")
```

```
PadeShamPoly2 (@ (x) besselj (5,x) ,gx1,gx2,[0:0.1:30],Lb,Ub,runNum,"J5(x)","J5x_9.txt")
```

```
PadeShamPoly2 (@ (x) integral (@ (z) sin (z.^2) ,0,x) ,gx1,gx2,[1:0.01:5],Lb,Ub,runNum,"FresnelSine(x)","FresnelSine_9.txt")
```

```
PadeShamPoly2 (@ (x) integral (@ (z) cos (z.^2) ,0,x) ,gx1,gx2,[0.5:0.01:5],Lb,Ub,runNum,"FresnelCosine(x)","FresnelCosine_9.txt")
```

```

PadeShamPoly2 (@ (x) integral (@ (z) sin (z) ./z, 0, x) , gx1, gx2, [1.3:0.1:20] , Lb, Ub, runNum, "Si (x) ", "Si_9.txt")
PadeShamPoly2 (@ (x) 0.57721566+log (x) -integral (@ (z) (1-cos (z)) ./z, 0, x) , gx1, gx2, [0.5:0.1:20] , Lb, Ub, runNum, "Ci (x) ", "CI_9.txt")

PadeShamPoly2 (@ (x) asinh (x) , gx1, gx2, [0:0.1:100] , Lb, Ub, runNum, "asinh (x) ", "asinh_9.txt")

PadeShamPoly2 (@ (x) acosh (x) , gx1, gx2, [1:0.1:100] , Lb, Ub, runNum, "acosh (x) ", "acosh_9.txt")

PadeShamPoly2 (@ (x) atanh (x) , gx1, gx2, [0:0.001:0.999] , Lb, Ub, runNum, "atanh (x) ", "atanh_9.txt")
end

if selIdx==0 || selIdx==10
    lstA = [0 1.4 0 2.2];
    lstB = [0.1 0.5 0.1 2];
    lstOrder = [3 7];
    Lb = [lstA(1) lstB(1) lstA(3) lstB(3) lstOrder(1) lstOrder(1)];
    Ub = [lstA(2) lstB(2) lstA(4) lstB(4) lstOrder(2) lstOrder(2)];
    gx1 = @(i,A,B) A+B*log10(i)^4;
    gx2 = @(i,A,B) A+B/i;

PadeShamPoly2 (@ (x) acos (x) , gx1, gx2, [0:.01:1] , Lb, Ub, runNum, "arccos (x) ", "arccos_10.txt")

PadeShamPoly2 (@ (x) asin (x) , gx1, gx2, [0:.01:1] , Lb, Ub, runNum, "arcsin (x) ", "arcsin_10.txt")

PadeShamPoly2 (@ (x) atan (x) , gx1, gx2, [0:.01:1] , Lb, Ub, runNum, "arctan (x) ", "arctan_10.txt")

PadeShamPoly2 (@ (x) sin (x) , gx1, gx2, [0:pi/1000:pi/2] , Lb, Ub, runNum, "sin (x) ", "sin_10.txt")

PadeShamPoly2 (@ (x) cos (x) , gx1, gx2, [0:pi/1000:pi/2] , Lb, Ub, runNum, "cos (x) ", "cos_10.txt")

PadeShamPoly2 (@ (x) tan (x) , gx1, gx2, [0:.01:1] , Lb, Ub, runNum, "tan (x) ", "tan_10.txt")
)

PadeShamPoly2 (@ (x) sinh (x) , gx1, gx2, [0:.01:5] , Lb, Ub, runNum, "sinh (x) ", "sinh_10.txt")

PadeShamPoly2 (@ (x) cosh (x) , gx1, gx2, [0:.01:5] , Lb, Ub, runNum, "cosh (x) ", "cosh_10.txt")

PadeShamPoly2 (@ (x) tanh (x) , gx1, gx2, [0:.01:3] , Lb, Ub, runNum, "tanh (x) ", "tanh_10.txt")

PadeShamPoly2 (@ (x) erf (x) , gx1, gx2, [0:.01:2.1] , Lb, Ub, runNum, "erf (x) ", "erf_10.txt")

PadeShamPoly2 (@ (x) exp (x) , gx1, gx2, [0:.01:2] , Lb, Ub, runNum, "exp (x) ", "exp_10.txt")
)

```

```

PadeShamPoly2 (@ (x) log (x) , gx1 , gx2 , [1 : .01 : 10] , Lb , Ub , runNum , "ln (x) " , "ln_10.txt")

PadeShamPoly2 (@ (x) log10 (x) , gx1 , gx2 , [1 : .01 : 10] , Lb , Ub , runNum , "log (x) " , "log_10.t
xt")

PadeShamPoly2 (@ (x) 10.^x , gx1 , gx2 , [0 : .01 : 1] , Lb , Ub , runNum , "10^x" , "pwr10_10.txt")

PadeShamPoly2 (@ (x) tinv (0.95 , x) , gx1 , gx2 , [2 : 0.1 : 100] , Lb , Ub , runNum , "tinv (0.95 , x)
" , "tinv1_10.txt")

PadeShamPoly2 (@ (x) tinv (0.975 , x) , gx1 , gx2 , [2 : 0.1 : 100] , Lb , Ub , runNum , "tinv (0.975 ,
x) " , "tinv2_10.txt")

PadeShamPoly2 (@ (x) log10 (gamma (x) ) , gx1 , gx2 , [2 : 0.1 : 100] , Lb , Ub , runNum , "log10Gamm
a (x) " , "log10Gamma_10.txt")

PadeShamPoly2 (@ (x) digamma (x) , gx1 , gx2 , [2 : 0.1 : 100] , Lb , Ub , runNum , "digamma (x) " , "d
igamma_10.txt")

PadeShamPoly2 (@ (x) trigamma (x) , gx1 , gx2 , [1 : 0.1 : 100] , Lb , Ub , runNum , "trigamma (x) " ,
"trigamma_10.txt")

PadeShamPoly2 (@ (x) besselj (0 , x) , gx1 , gx2 , [2 : 0.1 : 30] , Lb , Ub , runNum , "J0 (x) " , "J0x_1
0.txt")

PadeShamPoly2 (@ (x) besselj (1 , x) , gx1 , gx2 , [0 : 0.1 : 30] , Lb , Ub , runNum , "J1 (x) " , "J1x_1
0.txt")

PadeShamPoly2 (@ (x) besselj (2 , x) , gx1 , gx2 , [0 : 0.1 : 30] , Lb , Ub , runNum , "J2 (x) " , "J2x_1
0.txt")

PadeShamPoly2 (@ (x) besselj (3 , x) , gx1 , gx2 , [0 : 0.1 : 30] , Lb , Ub , runNum , "J3 (x) " , "J3x_1
0.txt")

PadeShamPoly2 (@ (x) besselj (4 , x) , gx1 , gx2 , [0 : 0.1 : 30] , Lb , Ub , runNum , "J4 (x) " , "J4x_1
0.txt")

PadeShamPoly2 (@ (x) besselj (5 , x) , gx1 , gx2 , [0 : 0.1 : 30] , Lb , Ub , runNum , "J5 (x) " , "J5x_1
0.txt")

PadeShamPoly2 (@ (x) integral (@ (z) sin (z.^2) , 0 , x) , gx1 , gx2 , [1 : 0.01 : 5] , Lb , Ub , runNum
, "FresnelSine (x) " , "FresnelSine_10.txt")

PadeShamPoly2 (@ (x) integral (@ (z) cos (z.^2) , 0 , x) , gx1 , gx2 , [0.5 : 0.01 : 5] , Lb , Ub , runN
um , "FresnelCosine (x) " , "FresnelCosine_10.txt")

PadeShamPoly2 (@ (x) integral (@ (z) sin (z) ./z , 0 , x) , gx1 , gx2 , [1.3 : 0.1 : 20] , Lb , Ub , runN
um , "Si (x) " , "Si_10.txt")
    PadeShamPoly2 (@ (x) 0.57721566+log (x) -integral (@ (z) (1-
cos (z) ) ./z , 0 , x) , gx1 , gx2 , [0.5 : 0.1 : 20] , Lb , Ub , runNum , "Ci (x) " , "Ci_10.txt")

PadeShamPoly2 (@ (x) asinh (x) , gx1 , gx2 , [0 : 0.1 : 100] , Lb , Ub , runNum , "asinh (x) " , "asinh
_10.txt")

PadeShamPoly2 (@ (x) acosh (x) , gx1 , gx2 , [1 : 0.1 : 100] , Lb , Ub , runNum , "acosh (x) " , "acosh
_10.txt")

```

```

PadeShamPoly2 (@(x) atanh(x),gx1,gx2,[0:0.001:0.999],Lb,Ub,runNum,"atanh(x)","a
tanh_10.txt")
end

toc;

for i=1:7
    beep;
    pause(3)
end

if bShutdown
    system('shutdown -s');
else
    fprintf("\n\nDone!\n\n");
end

```

The above listing has the following global and operational variables:

- The global variable bUseDiary is a Boolean flag used to tell the functions PadeShamPoly2() and PadeShamPoly3() whether you want to copy the screen output to diary text files.
- The global variable bDeleteIfExists is a Boolean flag used to tell the functions PadeShamPoly2() and PadeShamPoly3() whether you want to delete diary files if they exist.
- The variable selIdx allows you to select calculations for one of the seven groups (when set to the targeted group number) or all of the groups (when set to 0). I am using this scheme to reduce calculation time which can be done by working with a specific set of approximations.
- The Boolean variable bShutdown tells the Matlab script whether to shut down the computer when done.

When the script goAll is done with the calculation it executes the script playmynotes.m to play ten random sinusoidal notes. You can comment out the call to the audible script to keep the script quiet. I wrote the script for playmynotes.m based on the code I found in the fine book *Programming Fundamentals Using MATLAB* written by M. Weeks and published in 2020 by Mercury Learning. The book comes with a CD that contains all the Matlab source code and figures in that book. The source code for playmynotes.m is:

```

n = 10;
frq = 8192;
notes = 220 + fix(220*rand(n,1));

for i =1:n
    znotes(i,1:frq) = 0.5*sin(2*pi*notes(i)*(0:frq-1)/frq);

```

```

end

for i = 1:n
    sound(znotes(i,1:frq));
    pause(1)
end

```

You can edit the script in goAll.m to perform Pade-Shammas polynomial fitting for other functions you are interested in or for other Shammas polynomial orders.

GENERAL COMMENTS ON RESULTS

The next sections show the results of fitting various common functions each with a variety of Pade-Shammas polynomials. There is a summary table for each fitted function showing the Shammas polynomial parameters, the F statistic, the AICc statistic, the root mean sum of errors squared, and orders of the fitted Pade-Shammas polynomials. I use the the root mean sum of errors squared to select the best Shammas polynomial powers. I have deliberately not included the coefficient of determination and its adjusted value in the table. Why? These values are very close to 1 and we are better served to look at other statistics. I have included the values for the F and AICc statistics, even though they are infinity and minus infinity.

The quality of the curve fitting varies between very good and (honestly) somewhat disappointing. This variation shows that the process of using the Pade-Shammas polynomials works and does not, by some erroneous calculations, always produce excellent results.

 The next sections display the results of the Pade-Shammas polynomials. The regression coefficients for the $Q_n(x)$ and $D_m(x)$ Shammas polynomials appear as one long list labeled x1 and up. The results include the orders for the $Q_n(x)$ and $D_m(x)$ Shammas polynomials which should easily indicate with regression coefficients belong to $Q_n(x)$ Shammas polynomials and which one belong to $D_m(x)$ Shammas polynomials. The results also include the list of powers associated with $Q_n(x)$ and $D_m(x)$ Shammas polynomials. Here is an example. Looking at the results for the inverse hyperbolic cosine:

Fitting $\arccos(x)$ in range (0.000000, 1.000000)

Shammas polynomial power for $Q(x)$ is $A+B*i$

Shammas polynomial power for $D(x)$ is $A+B/i$

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	1.78718464339559	0	Inf	0
x1	-6.80661585506549	0	-Inf	0
x2	9.6533136054281	0	Inf	0
x3	-6.46644797115271	0	-Inf	0
x4	2.10673742781716	0	Inf	0
x5	-0.267674739189263	0	-Inf	0
x6	-0.81223151838756	0	-Inf	0
x7	58.9111167401054	0	Inf	0
x8	-564.012996898473	0	-Inf	0
x9	1755.70989114239	0	Inf	0
x10	-2150.00021174932	0	-Inf	0
x11	901.201183728077	0	Inf	0

Number of observations: 101, Error degrees of freedom: 89

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	6.03050467944024	100	0.0603050467944024		
Model	6.03050467944024	11	0.548227698130931	Inf	0
Residual	0	89	0		

for $Q(x)$ $A = 0.056552$, $B = 0.376320$

for $D(x)$ $A = 0.236217$, $B = 0.950135$

order $Q(x) = 5.000000$, order $D(x) = 6.000000$

Xmin = 0.000000 and Xmax = 1.000000

Ymin = 0.000000 and Ymax = 1.570796

List of powers for $Q(x)$: [0.432873, 0.809193, 1.185513, 1.561833, 1.938153]

List of powers for $D(x)$: [1.186352, 0.711285, 0.552929, 0.473751, 0.426244, 0.394573]

Fitting $\arccos(x)$ in range (1.000000, 2.000000)

MSS of errors squared = 1.844830e-06

Corrected MSS of errors squared = 3.689661e-06

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

First look at the orders of the $Q_n(x)$ and $D_m(x)$ Shammas polynomials. They are 5 and 6, respectively. Then look at the list of regression coefficients labeled x1 through x11. Since $Q_n(x)$ has the order of 5, its regression coefficients are the ones labeled x1 through x5. The remaining six regression coefficients are labeled x6 through x11. You can find the list of the $Q_n(x)$ and $D_m(x)$ Shammas polynomials two lines below the line showing the polynomial orders.

RESULTS FOR THE INVERSE HYPERBOLIC COSINE

Combo Powers 1

Fitting $\arccos(x)$ in range (0.000000, 1.000000)

Shammas polynomial power for $Q(x)$ is $A+B*i$

Shammas polynomial power for $D(x)$ is $A+B/i$

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	1.78718464339559	0	Inf	0
x1	-6.80661585506549	0	-Inf	0
x2	9.6533136054281	0	Inf	0
x3	-6.46644797115271	0	-Inf	0
x4	2.10673742781716	0	Inf	0
x5	-0.267674739189263	0	-Inf	0
x6	-0.81223151838756	0	-Inf	0
x7	58.9111167401054	0	Inf	0
x8	-564.012996898473	0	-Inf	0
x9	1755.70989114239	0	Inf	0
x10	-2150.00021174932	0	-Inf	0
x11	901.201183728077	0	Inf	0

Number of observations: 101, Error degrees of freedom: 89

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	6.03050467944024	100	0.0603050467944024		
Model	6.03050467944024	11	0.548227698130931	Inf	0
Residual	0	89	0		

for $Q(x)$ $A = 0.056552$, $B = 0.376320$

for $D(x)$ $A = 0.236217$, $B = 0.950135$

order $Q(x) = 5.000000$, order $D(x) = 6.000000$

Xmin = 0.000000 and Xmax = 1.000000

Ymin = 0.000000 and Ymax = 1.570796

List of powers for $Q(x)$: [0.432873, 0.809193, 1.185513, 1.561833, 1.938153]

List of powers for $D(x)$: [1.186352, 0.711285, 0.552929, 0.473751, 0.426244, 0.394573]

Fitting $\arccos(x)$ in range (1.000000, 2.000000)

MSS of errors squared = 1.844830e-06

Corrected MSS of errors squared = 3.689661e-06

R-squared = 1.00000000

R-squared Adjusted = 1.00000000
 Particle swarm AICc = -1.000000e+99
 AIC = -Inf
 AICc = -Inf

Combo Powers 2

Fitting arccos(x) in range (0.000000, 1.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B*sqrt(i)

Linear regression model:

$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.00344191386912198	0	-Inf	0
x1	0.0020780382488008	0	Inf	0
x2	-0.000384034422383167	0	-Inf	0
x3	1.79856121548491e-05	0	Inf	0
x4	4.78942431652761e-07	0	Inf	0
x5	231.905143649663	0	Inf	0
x6	-965.604692145	0	-Inf	0
x7	1161.42120716579	0	Inf	0
x8	0	0	NaN	NaN
x9	-777.252641536728	0	-Inf	0
x10	350.531847586943	0	Inf	0

Number of observations: 101, Error degrees of freedom: 91

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	6.03050467941582	100	0.0603050467941582		
Model	6.03050467941582	9	0.670056075490647	Inf	0
Residual	0	91	0		

for Q(x) A = 0.001981, B = 1.911365

for D(x) A = 0.264853, B = 0.191291

order Q(x) = 4.000000, order D(x) = 6.000000

Xmin = 0.000000 and Xmax = 1.000000

Ymin = 0.000000 and Ymax = 1.570796

List of powers for Q(x): [1.913346, 3.824710, 5.736075, 7.647439]

List of powers for D(x): [0.456144, 0.535379, 0.596179, 0.647435, 0.692593, 0.733418]

Fitting arccos(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.974457e-05

Corrected MSS of errors squared = 3.948914e-05

R-squared = 1.00000000
 R-squared Adjusted = 1.00000000
 Particle swarm AICc = -1.000000e+99
 AIC = -Inf
 AICc = -Inf

Combo Powers 3

Fitting arccos(x) in range (0.000000, 1.000000)
 Shammas polynomial power for Q(x) is $A+B\sqrt{i}+C*i$
 Shammas polynomial power for D(x) is $A+B*i+C*\sqrt{i}$

Linear regression model:

$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14}$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-20.0243721337301	0	-Inf	0
x1	645.685324069326	0	Inf	0
x2	-2214.21907786615	0	-Inf	0
x3	2286.91413871679	0	Inf	0
x4	0	0	NaN	NaN
x5	-907.058549199906	0	-Inf	0
x6	0	0	NaN	NaN
x7	208.754872129389	0	Inf	0
x8	7.54416921352212	0	Inf	0
x9	-17.5298969239089	0	-Inf	0
x10	20.383564351642	0	Inf	0
x11	-14.0941881330259	0	-Inf	0
x12	5.94293132348749	0	Inf	0
x13	-1.42045379200095	0	-Inf	0
x14	0.147706102374084	0	Inf	0

Number of observations: 101, Error degrees of freedom: 88

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	6.03050467939546	100	0.0603050467939546		
Model	6.03050467939546	12	0.502542056616289	Inf	0
Residual	0	88	0		

for Q(x) A = 0.000000, B = 0.387383, C = 0.000000

for D(x) A = 0.285967, B = 0.390000, C = 0.360000

order Q(x) = 7.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 1.000000

Ymin = 0.000000 and Ymax = 1.570796

List of powers for Q(x): [0.387383, 0.547843, 0.670968, 0.774767, 0.866215, 0.948891, 1.024920]

List of powers for D(x): [1.035967, 1.575084, 2.079506, 2.565967, 3.040952, 3.507784, 3.968438]

Fitting arccos(x) in range (1.000000, 2.000000)

MSS of errors squared = 8.796334e-07

Corrected MSS of errors squared = 2.154653e-06

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 4

Fitting arccos(x) in range (0.000000, 1.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-104.752459183043	0	-Inf	0
x1	-21034.9903921502	0	-Inf	0
x2	285150.115169067	0	Inf	0
x3	-670481.586356525	0	-Inf	0
x4	406471.390621525	0	Inf	0
x5	12099.2324518826	0	Inf	0
x6	-28424.7060481147	0	-Inf	0
x7	0	0	NaN	NaN
x8	50322.4483765903	0	Inf	0
x9	-47401.4712873717	0	-Inf	0
x10	13405.4082156314	0	Inf	0

Number of observations: 101, Error degrees of freedom: 91

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	6.0305046789929	100	0.060305046789929		
Model	6.0305046789929	9	0.670056075443655	Inf	0
Residual	0	91	0		

for Q(x) A = 0.349023, B = 0.110780

for D(x) A = 1.941409, B = 0.042538

```

order Q(x) = 4.000000, order D(x) = 6.000000
Xmin = 0.000000 and Xmax = 1.000000
Ymin = 0.000000 and Ymax = 1.570796
List of powers for Q(x): [0.459803, 0.404413, 0.385950, 0.376718]
List of powers for D(x): [1.983946, 2.026484, 2.069022, 2.111560, 2.154098, 2.196636]
Fitting arccos(x) in range (1.000000, 2.000000)
MSS of errors squared = 3.159230e-06
Corrected MSS of errors squared = 6.318460e-06
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 5

```

Fitting arccos(x) in range (0.000000, 1.000000)
Shammas polynomial power for Q(x) is A+B*sqrt(i)
Shammas polynomial power for D(x) is A+B*i

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	2.45917002510567	0	Inf	0
x1	94.1847612727789	0	Inf	0
x2	-330.983911801166	0	-Inf	0
x3	298.889075337411	0	Inf	0
x4	0	0	NaN	NaN
x5	-63.8735878000179	0	-Inf	0
x6	1.49484924669878	0	Inf	0
x7	-1.29569947906911	0	-Inf	0
x8	0.641624060101299	0	Inf	0
x9	-0.226124052387783	0	-Inf	0
x10	0.0551917776142029	0	Inf	0
x11	-0.00814677952241908	0	-Inf	0
x12	0.000551689972781628	0	Inf	0

Number of observations: 101, Error degrees of freedom: 89

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	6.03050467943075	100	0.0603050467943076		
Model	6.03050467943075	11	0.548227698130069	Inf	0
Residual	0	89	0		

```

for Q(x) A = 1.239021, B = 0.485504
for D(x) A = 0.834369, B = 1.356227
order Q(x) = 5.000000, order D(x) = 7.000000
Xmin = 0.000000 and Xmax = 1.000000
Ymin = 0.000000 and Ymax = 1.570796
List of powers for Q(x): [1.724525, 1.925628, 2.079939, 2.210029, 2.324642]
List of powers for D(x): [2.190596, 3.546824, 4.903051, 6.259278, 7.615506, 8.971733,
10.327960]
Fitting arccos(x) in range (1.000000, 2.000000)
MSS of errors squared = 7.293224e-06
Corrected MSS of errors squared = 1.458645e-05
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 6

```

Fitting arccos(x) in range (0.000000, 1.000000)
Shammas polynomial power for Q(x) is A+B*i+C*sqrt(i)
Shammas polynomial power for D(x) is A+B*sqrt(i)+C*i

```

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	2.92670883642286	0	Inf	0
x1	-12.6296775801737	0	-Inf	0
x2	19.0724627808923	0	Inf	0
x3	-13.19441214747	0	-Inf	0
x4	4.41708141435887	0	Inf	0
x5	-0.578208907319174	0	-Inf	0
x6	1.5390197316	0	Inf	0
x7	2.81142493226107	0	Inf	0
x8	-8.03902322038588	0	-Inf	0
x9	6.86430983990744	0	Inf	0
x10	-2.51661353726474	0	-Inf	0
x11	0.333905055537451	0	Inf	0

Number of observations: 101, Error degrees of freedom: 89

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

SumSq	DF	MeanSq	F	pValue

Total	6.03050467942523	100	0.0603050467942523		
Model	6.03050467942523	11	0.548227698129567	Inf	0
Residual	0	89	0		

for Q(x) A = 0.116844, B = 0.390000, C = 0.000000
 for D(x) A = 0.000000, B = 0.100000, C = 0.275551
 order Q(x) = 5.000000, order D(x) = 6.000000
 Xmin = 0.000000 and Xmax = 1.000000
 Ymin = 0.000000 and Ymax = 1.570796
 List of powers for Q(x): [0.506844, 0.896844, 1.286844, 1.676844, 2.066844]
 List of powers for D(x): [0.375551, 0.692523, 0.999857, 1.302203, 1.601360, 1.898253]
 Fitting arccos(x) in range (1.000000, 2.000000)
 MSS of errors squared = 1.287433e-06
 Corrected MSS of errors squared = 3.153554e-06
 R-squared = 1.00000000
 R-squared Adjusted = 1.00000000
 Particle swarm AICc = -1.000000e+99
 AIC = -Inf
 AICc = -Inf

Combo Powers 7

Fitting arccos(x) in range (0.000000, 1.000000)
 Shammas polynomial power for Q(x) is A+B*i
 Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	2.74068196127851e-10	0	Inf	0
x1	-6.26591541977305e-10	0	-Inf	0
x2	4.91163573108682e-10	0	Inf	0
x3	-1.10142290578121e-10	0	-Inf	0
x4	0.999599375334744	0	Inf	0
x5	0.000529149337178329	0	Inf	0
x6	-0.000165355966709569	0	-Inf	0
x7	4.25654295737421e-05	0	Inf	0
x8	-5.73414903444941e-06	0	-Inf	0

Number of observations: 101, Error degrees of freedom: 92

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	6.03050467942532	100	0.0603050467942532		

Model	6.03050467942532	8	0.753813084928166	Inf	0
Residual	0	92	0		

```

for Q(x) A = 0.347882, B = 0.500000
for D(x) A = 0.000000, B = 0.249282
order Q(x) = 3.000000, order D(x) = 5.000000
Xmin = 0.000000 and Xmax = 1.000000
Ymin = 0.000000 and Ymax = 1.570796
List of powers for Q(x): [0.847882, 1.347882, 1.847882]
List of powers for D(x): [0.000000, 0.002047, 0.012918, 0.032753, 0.059501]
Fitting arccos(x) in range (1.000000, 2.000000)
MSS of errors squared = 6.173678e-04
Corrected MSS of errors squared = 1.234736e-03
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 8

```

Fitting arccos(x) in range (0.000000, 1.000000)
Shammas polynomial power for Q(x) is A+B/i
Shammas polynomial power for D(x) is A+B*log10(i)^4

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.406208148080047	0	Inf	0
x1	0.000663051591820872	0	Inf	0
x2	-0.199128370103149	0	-Inf	0
x3	3.93956056390281	0	Inf	0
x4	-17.0002552518939	0	-Inf	0
x5	18.4803009269846	0	Inf	0
x6	6.78341161930091	0	Inf	0
x7	-12.4101645258799	0	-Inf	0
x8	58.5950785625056	0	Inf	0
x9	0	0	NaN	NaN
x10	-87.8959330037354	0	-Inf	0
x11	30.3005563602318	0	Inf	0

Number of observations: 101, Error degrees of freedom: 90

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

SumSq	DF	MeanSq	F	pValue
-------	----	--------	---	--------

Total	6.03050467942589	100	0.0603050467942589		
Model	6.03050467942589	10	0.603050467942589	Inf	0
Residual	0	90	0		

```

for Q(x) A = 0.108874, B = 2.320730
for D(x) A = 0.147859, B = 0.255635
order Q(x) = 7.000000, order D(x) = 4.000000
Xmin = 0.000000 and Xmax = 1.000000
Ymin = 0.000000 and Ymax = 1.570796
List of powers for Q(x): [2.429604, 1.269239, 0.882451, 0.689056, 0.573020, 0.495662,
0.440407]
List of powers for D(x): [0.147859, 0.149958, 0.161106, 0.181446]
Fitting arccos(x) in range (1.000000, 2.000000)
MSS of errors squared = 5.676412e-06
Corrected MSS of errors squared = 1.135282e-05
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 9

```

Fitting arccos(x) in range (0.000000, 1.000000)
Shammas polynomial power for Q(x) is A+B*log10(i)^4
Shammas polynomial power for D(x) is A+B*i

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	2.42725122974367	0	Inf	0
x1	20461.4612636582	0	Inf	0
x2	0	0	NaN	NaN
x3	-33242.0488173047	0	-Inf	0
x4	0	0	NaN	NaN
x5	27759.5672437456	0	Inf	0
x6	-19120.3531324796	0	-Inf	0
x7	4139.28991821164	0	Inf	0
x8	2.5359427698876	0	Inf	0
x9	-2.6960124995062	0	-Inf	0
x10	1.26580379453711	0	Inf	0
x11	-0.313438681451887	0	-Inf	0
x12	0.0358410751753927	0	Inf	0

Number of observations: 101, Error degrees of freedom: 90

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	6.03050467939134	100	0.0603050467939134		
Model	6.03050467939134	10	0.603050467939134	Inf	0
Residual	0	90	0		

for Q(x) A = 1.452690, B = 0.500000

for D(x) A = 0.668176, B = 0.766200

order Q(x) = 7.000000, order D(x) = 5.000000

Xmin = 0.000000 and Xmax = 1.000000

Ymin = 0.000000 and Ymax = 1.570796

List of powers for Q(x): [1.452690, 1.456796, 1.478601, 1.518384, 1.572035, 1.636017, 1.707724]

List of powers for D(x): [1.434376, 2.200577, 2.966777, 3.732977, 4.499178]

Fitting arccos(x) in range (1.000000, 2.000000)

MSS of errors squared = 5.053811e-06

Corrected MSS of errors squared = 1.010762e-05

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 10

Fitting arccos(x) in range (0.000000, 1.000000)

Shammas polynomial power for Q(x) is A+B*log10(i)^4

Shammas polynomial power for D(x) is A+B/i

Linear regression model:

$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	3.18070625802966	0	Inf	0
x1	-17720.2019475356	0	-Inf	0
x2	0	0	NaN	NaN
x3	35999.6931659775	0	Inf	0
x4	-23358.2202985217	0	-Inf	0
x5	5075.62034003838	0	Inf	0
x6	-0.150997462482178	0	-Inf	0
x7	33.0748013108299	0	Inf	0
x8	-541.38019713694	0	-Inf	0
x9	2235.78332770241	0	Inf	0
x10	-3231.55410319585	0	-Inf	0
x11	1505.19118567605	0	Inf	0

Number of observations: 101, Error degrees of freedom: 90

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	6.03050467942014	100	0.0603050467942014		
Model	6.03050467942014	10	0.603050467942014	Inf	0
Residual	0	90	0		

for $Q(x)$ $A = 1.400000$, $B = 0.500000$

for $D(x)$ $A = 1.343447$, $B = 2.000000$

order $Q(x) = 5.000000$, order $D(x) = 6.000000$

$X_{min} = 0.000000$ and $X_{max} = 1.000000$

$Y_{min} = 0.000000$ and $Y_{max} = 1.570796$

List of powers for $Q(x)$: [1.400000, 1.404106, 1.425911, 1.465695, 1.519345]

List of powers for $D(x)$: [3.343447, 2.343447, 2.010113, 1.843447, 1.743447, 1.676780]

Fitting $\arccos(x)$ in range (1.000000, 2.000000)

MSS of errors squared = 8.633311e-06

Corrected MSS of errors squared = 1.726662e-05

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Comments

The following table shows the summary results for the ten models that fit the inverse hyperbolic cosine function. The results are sorted in ascending order using the MSSE values.

<i>Power $Q(x)$</i>	<i>Power $D(x)$</i>	<i>MSSE</i>	<i>Q/D Orders</i>
$A+B*i$	$A+B/i$	8.18486000E-06	6, 6
$A+B*i+C*\sqrt{i}$	$A+B*\sqrt{i}+C*i$	1.28407100E-05	7, 6
$A+B*i$	$A+B*\sqrt{i}$	1.35260900E-05	6, 6
$A+B/i$	$A+B*i$	1.62865900E-05	5, 6
$A+B/i$	$A+B*\log_{10}(i)^4$	1.64928700E-05	7, 7
$A+B*\log_{10}(i)^4$	$A+B/i$	2.80437300E-05	7, 3
$A+B*i$	$A+B*\log_{10}(i)^4$	4.06603400E-05	4, 3
$A+B*\sqrt{i}+C*i$	$A+B*i+C*\sqrt{i}$	4.63663000E-05	3, 4
$A+B*\sqrt{i}$	$A+B*i$	6.00782300E-04	7, 7
$A+B*\log_{10}(i)^4$	$A+B*i$	1.25311900E-03	5, 7

RESULTS FOR THE ARC COSINE

Combo Powers 1

Fitting $\arccos(x)$ in range (0.000000, 1.000000)

Shammas polynomial power for $Q(x)$ is $A+B*i$

Shammas polynomial power for $D(x)$ is $A+B/i$

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	1.78718464339559	0	Inf	0
x1	-6.80661585506549	0	-Inf	0
x2	9.6533136054281	0	Inf	0
x3	-6.46644797115271	0	-Inf	0
x4	2.10673742781716	0	Inf	0
x5	-0.267674739189263	0	-Inf	0
x6	-0.81223151838756	0	-Inf	0
x7	58.9111167401054	0	Inf	0
x8	-564.012996898473	0	-Inf	0
x9	1755.70989114239	0	Inf	0
x10	-2150.00021174932	0	-Inf	0
x11	901.201183728077	0	Inf	0

Number of observations: 101, Error degrees of freedom: 89

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	6.03050467944024	100	0.0603050467944024		
Model	6.03050467944024	11	0.548227698130931	Inf	0
Residual	0	89	0		

for $Q(x)$ $A = 0.056552$, $B = 0.376320$

for $D(x)$ $A = 0.236217$, $B = 0.950135$

order $Q(x) = 5.000000$, order $D(x) = 6.000000$

Xmin = 0.000000 and Xmax = 1.000000

Ymin = 0.000000 and Ymax = 1.570796

List of powers for $Q(x)$: [0.432873, 0.809193, 1.185513, 1.561833, 1.938153]

List of powers for $D(x)$: [1.186352, 0.711285, 0.552929, 0.473751, 0.426244, 0.394573]

Fitting $\arccos(x)$ in range (1.000000, 2.000000)

MSS of errors squared = 1.844830e-06

Corrected MSS of errors squared = 3.689661e-06

R-squared = 1.00000000

R-squared Adjusted = 1.00000000
 Particle swarm AICc = -1.000000e+99
 AIC = -Inf
 AICc = -Inf

Combo Powers 2

Fitting arccos(x) in range (0.000000, 1.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B*sqrt(i)

Linear regression model:

$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.00344191386912198	0	-Inf	0
x1	0.0020780382488008	0	Inf	0
x2	-0.000384034422383167	0	-Inf	0
x3	1.79856121548491e-05	0	Inf	0
x4	4.78942431652761e-07	0	Inf	0
x5	231.905143649663	0	Inf	0
x6	-965.604692145	0	-Inf	0
x7	1161.42120716579	0	Inf	0
x8	0	0	NaN	NaN
x9	-777.252641536728	0	-Inf	0
x10	350.531847586943	0	Inf	0

Number of observations: 101, Error degrees of freedom: 91

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	6.03050467941582	100	0.0603050467941582		
Model	6.03050467941582	9	0.670056075490647	Inf	0
Residual	0	91	0		

for Q(x) A = 0.001981, B = 1.911365

for D(x) A = 0.264853, B = 0.191291

order Q(x) = 4.000000, order D(x) = 6.000000

Xmin = 0.000000 and Xmax = 1.000000

Ymin = 0.000000 and Ymax = 1.570796

List of powers for Q(x): [1.913346, 3.824710, 5.736075, 7.647439]

List of powers for D(x): [0.456144, 0.535379, 0.596179, 0.647435, 0.692593, 0.733418]

Fitting arccos(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.974457e-05

Corrected MSS of errors squared = 3.948914e-05

R-squared = 1.00000000
 R-squared Adjusted = 1.00000000
 Particle swarm AICc = -1.000000e+99
 AIC = -Inf
 AICc = -Inf

Combo Powers 3

Fitting arccos(x) in range (0.000000, 1.000000)
 Shammas polynomial power for Q(x) is $A+B\sqrt{i}+C*i$
 Shammas polynomial power for D(x) is $A+B*i+C*\sqrt{i}$

Linear regression model:

$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14}$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-20.0243721337301	0	-Inf	0
x1	645.685324069326	0	Inf	0
x2	-2214.21907786615	0	-Inf	0
x3	2286.91413871679	0	Inf	0
x4	0	0	NaN	NaN
x5	-907.058549199906	0	-Inf	0
x6	0	0	NaN	NaN
x7	208.754872129389	0	Inf	0
x8	7.54416921352212	0	Inf	0
x9	-17.5298969239089	0	-Inf	0
x10	20.383564351642	0	Inf	0
x11	-14.0941881330259	0	-Inf	0
x12	5.94293132348749	0	Inf	0
x13	-1.42045379200095	0	-Inf	0
x14	0.147706102374084	0	Inf	0

Number of observations: 101, Error degrees of freedom: 88

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	6.03050467939546	100	0.0603050467939546		
Model	6.03050467939546	12	0.502542056616289	Inf	0
Residual	0	88	0		

for Q(x) A = 0.000000, B = 0.387383, C = 0.000000

for D(x) A = 0.285967, B = 0.390000, C = 0.360000

order Q(x) = 7.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 1.000000

Ymin = 0.000000 and Ymax = 1.570796

List of powers for Q(x): [0.387383, 0.547843, 0.670968, 0.774767, 0.866215, 0.948891, 1.024920]

List of powers for D(x): [1.035967, 1.575084, 2.079506, 2.565967, 3.040952, 3.507784, 3.968438]

Fitting arccos(x) in range (1.000000, 2.000000)

MSS of errors squared = 8.796334e-07

Corrected MSS of errors squared = 2.154653e-06

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 4

Fitting arccos(x) in range (0.000000, 1.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-104.752459183043	0	-Inf	0
x1	-21034.9903921502	0	-Inf	0
x2	285150.115169067	0	Inf	0
x3	-670481.586356525	0	-Inf	0
x4	406471.390621525	0	Inf	0
x5	12099.2324518826	0	Inf	0
x6	-28424.7060481147	0	-Inf	0
x7	0	0	NaN	NaN
x8	50322.4483765903	0	Inf	0
x9	-47401.4712873717	0	-Inf	0
x10	13405.4082156314	0	Inf	0

Number of observations: 101, Error degrees of freedom: 91

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	6.0305046789929	100	0.060305046789929		
Model	6.0305046789929	9	0.670056075443655	Inf	0
Residual	0	91	0		

for Q(x) A = 0.349023, B = 0.110780

for D(x) A = 1.941409, B = 0.042538

```

order Q(x) = 4.000000, order D(x) = 6.000000
Xmin = 0.000000 and Xmax = 1.000000
Ymin = 0.000000 and Ymax = 1.570796
List of powers for Q(x): [0.459803, 0.404413, 0.385950, 0.376718]
List of powers for D(x): [1.983946, 2.026484, 2.069022, 2.111560, 2.154098, 2.196636]
Fitting arccos(x) in range (1.000000, 2.000000)
MSS of errors squared = 3.159230e-06
Corrected MSS of errors squared = 6.318460e-06
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 5

```

Fitting arccos(x) in range (0.000000, 1.000000)
Shammas polynomial power for Q(x) is A+B*sqrt(i)
Shammas polynomial power for D(x) is A+B*i

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	2.45917002510567	0	Inf	0
x1	94.1847612727789	0	Inf	0
x2	-330.983911801166	0	-Inf	0
x3	298.889075337411	0	Inf	0
x4	0	0	NaN	NaN
x5	-63.8735878000179	0	-Inf	0
x6	1.49484924669878	0	Inf	0
x7	-1.29569947906911	0	-Inf	0
x8	0.641624060101299	0	Inf	0
x9	-0.226124052387783	0	-Inf	0
x10	0.0551917776142029	0	Inf	0
x11	-0.00814677952241908	0	-Inf	0
x12	0.000551689972781628	0	Inf	0

Number of observations: 101, Error degrees of freedom: 89

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	6.03050467943075	100	0.0603050467943076		
Model	6.03050467943075	11	0.548227698130069	Inf	0
Residual	0	89	0		

```

for Q(x) A = 1.239021, B = 0.485504
for D(x) A = 0.834369, B = 1.356227
order Q(x) = 5.000000, order D(x) = 7.000000
Xmin = 0.000000 and Xmax = 1.000000
Ymin = 0.000000 and Ymax = 1.570796
List of powers for Q(x): [1.724525, 1.925628, 2.079939, 2.210029, 2.324642]
List of powers for D(x): [2.190596, 3.546824, 4.903051, 6.259278, 7.615506, 8.971733,
10.327960]
Fitting arccos(x) in range (1.000000, 2.000000)
MSS of errors squared = 7.293224e-06
Corrected MSS of errors squared = 1.458645e-05
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 6

```

Fitting arccos(x) in range (0.000000, 1.000000)
Shammas polynomial power for Q(x) is A+B*i+C*sqrt(i)
Shammas polynomial power for D(x) is A+B*sqrt(i)+C*i

```

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	2.92670883642286	0	Inf	0
x1	-12.6296775801737	0	-Inf	0
x2	19.0724627808923	0	Inf	0
x3	-13.19441214747	0	-Inf	0
x4	4.41708141435887	0	Inf	0
x5	-0.578208907319174	0	-Inf	0
x6	1.5390197316	0	Inf	0
x7	2.81142493226107	0	Inf	0
x8	-8.03902322038588	0	-Inf	0
x9	6.86430983990744	0	Inf	0
x10	-2.51661353726474	0	-Inf	0
x11	0.333905055537451	0	Inf	0

Number of observations: 101, Error degrees of freedom: 89

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

SumSq	DF	MeanSq	F	pValue

Total	6.03050467942523	100	0.0603050467942523		
Model	6.03050467942523	11	0.548227698129567	Inf	0
Residual	0	89	0		

for Q(x) A = 0.116844, B = 0.390000, C = 0.000000
 for D(x) A = 0.000000, B = 0.100000, C = 0.275551
 order Q(x) = 5.000000, order D(x) = 6.000000
 Xmin = 0.000000 and Xmax = 1.000000
 Ymin = 0.000000 and Ymax = 1.570796
 List of powers for Q(x): [0.506844, 0.896844, 1.286844, 1.676844, 2.066844]
 List of powers for D(x): [0.375551, 0.692523, 0.999857, 1.302203, 1.601360, 1.898253]
 Fitting arccos(x) in range (1.000000, 2.000000)
 MSS of errors squared = 1.287433e-06
 Corrected MSS of errors squared = 3.153554e-06
 R-squared = 1.00000000
 R-squared Adjusted = 1.00000000
 Particle swarm AICc = -1.000000e+99
 AIC = -Inf
 AICc = -Inf

Combo Powers 7

Fitting arccos(x) in range (0.000000, 1.000000)
 Shammas polynomial power for Q(x) is A+B*i
 Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	2.74068196127851e-10	0	Inf	0
x1	-6.26591541977305e-10	0	-Inf	0
x2	4.91163573108682e-10	0	Inf	0
x3	-1.10142290578121e-10	0	-Inf	0
x4	0.999599375334744	0	Inf	0
x5	0.000529149337178329	0	Inf	0
x6	-0.000165355966709569	0	-Inf	0
x7	4.25654295737421e-05	0	Inf	0
x8	-5.73414903444941e-06	0	-Inf	0

Number of observations: 101, Error degrees of freedom: 92

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	6.03050467942532	100	0.0603050467942532		

Model	6.03050467942532	8	0.753813084928166	Inf	0
Residual	0	92	0		

```

for Q(x) A = 0.347882, B = 0.500000
for D(x) A = 0.000000, B = 0.249282
order Q(x) = 3.000000, order D(x) = 5.000000
Xmin = 0.000000 and Xmax = 1.000000
Ymin = 0.000000 and Ymax = 1.570796
List of powers for Q(x): [0.847882, 1.347882, 1.847882]
List of powers for D(x): [0.000000, 0.002047, 0.012918, 0.032753, 0.059501]
Fitting arccos(x) in range (1.000000, 2.000000)
MSS of errors squared = 6.173678e-04
Corrected MSS of errors squared = 1.234736e-03
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 8

```

Fitting arccos(x) in range (0.000000, 1.000000)
Shammas polynomial power for Q(x) is A+B/i
Shammas polynomial power for D(x) is A+B*log10(i)^4

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.406208148080047	0	Inf	0
x1	0.000663051591820872	0	Inf	0
x2	-0.199128370103149	0	-Inf	0
x3	3.93956056390281	0	Inf	0
x4	-17.0002552518939	0	-Inf	0
x5	18.4803009269846	0	Inf	0
x6	6.78341161930091	0	Inf	0
x7	-12.4101645258799	0	-Inf	0
x8	58.5950785625056	0	Inf	0
x9	0	0	NaN	NaN
x10	-87.8959330037354	0	-Inf	0
x11	30.3005563602318	0	Inf	0

Number of observations: 101, Error degrees of freedom: 90

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

SumSq	DF	MeanSq	F	pValue
-------	----	--------	---	--------

Total	6.03050467942589	100	0.0603050467942589		
Model	6.03050467942589	10	0.603050467942589	Inf	0
Residual	0	90	0		

```

for Q(x) A = 0.108874, B = 2.320730
for D(x) A = 0.147859, B = 0.255635
order Q(x) = 7.000000, order D(x) = 4.000000
Xmin = 0.000000 and Xmax = 1.000000
Ymin = 0.000000 and Ymax = 1.570796
List of powers for Q(x): [2.429604, 1.269239, 0.882451, 0.689056, 0.573020, 0.495662,
0.440407]
List of powers for D(x): [0.147859, 0.149958, 0.161106, 0.181446]
Fitting arccos(x) in range (1.000000, 2.000000)
MSS of errors squared = 5.676412e-06
Corrected MSS of errors squared = 1.135282e-05
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 9

```

Fitting arccos(x) in range (0.000000, 1.000000)
Shammas polynomial power for Q(x) is A+B*log10(i)^4
Shammas polynomial power for D(x) is A+B*i

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	2.42725122974367	0	Inf	0
x1	20461.4612636582	0	Inf	0
x2	0	0	NaN	NaN
x3	-33242.0488173047	0	-Inf	0
x4	0	0	NaN	NaN
x5	27759.5672437456	0	Inf	0
x6	-19120.3531324796	0	-Inf	0
x7	4139.28991821164	0	Inf	0
x8	2.5359427698876	0	Inf	0
x9	-2.6960124995062	0	-Inf	0
x10	1.26580379453711	0	Inf	0
x11	-0.313438681451887	0	-Inf	0
x12	0.0358410751753927	0	Inf	0

Number of observations: 101, Error degrees of freedom: 90

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	6.03050467939134	100	0.0603050467939134		
Model	6.03050467939134	10	0.603050467939134	Inf	0
Residual	0	90	0		

for Q(x) A = 1.452690, B = 0.500000

for D(x) A = 0.668176, B = 0.766200

order Q(x) = 7.000000, order D(x) = 5.000000

Xmin = 0.000000 and Xmax = 1.000000

Ymin = 0.000000 and Ymax = 1.570796

List of powers for Q(x): [1.452690, 1.456796, 1.478601, 1.518384, 1.572035, 1.636017, 1.707724]

List of powers for D(x): [1.434376, 2.200577, 2.966777, 3.732977, 4.499178]

Fitting arccos(x) in range (1.000000, 2.000000)

MSS of errors squared = 5.053811e-06

Corrected MSS of errors squared = 1.010762e-05

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 10

Fitting arccos(x) in range (0.000000, 1.000000)

Shammas polynomial power for Q(x) is A+B*log10(i)^4

Shammas polynomial power for D(x) is A+B/i

Linear regression model:

$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	3.18070625802966	0	Inf	0
x1	-17720.2019475356	0	-Inf	0
x2	0	0	NaN	NaN
x3	35999.6931659775	0	Inf	0
x4	-23358.2202985217	0	-Inf	0
x5	5075.62034003838	0	Inf	0
x6	-0.150997462482178	0	-Inf	0
x7	33.0748013108299	0	Inf	0
x8	-541.38019713694	0	-Inf	0
x9	2235.78332770241	0	Inf	0
x10	-3231.55410319585	0	-Inf	0
x11	1505.19118567605	0	Inf	0

Number of observations: 101, Error degrees of freedom: 90

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	6.03050467942014	100	0.0603050467942014		
Model	6.03050467942014	10	0.603050467942014	Inf	0
Residual	0	90	0		

for Q(x) A = 1.400000, B = 0.500000

for D(x) A = 1.343447, B = 2.000000

order Q(x) = 5.000000, order D(x) = 6.000000

Xmin = 0.000000 and Xmax = 1.000000

Ymin = 0.000000 and Ymax = 1.570796

List of powers for Q(x): [1.400000, 1.404106, 1.425911, 1.465695, 1.519345]

List of powers for D(x): [3.343447, 2.343447, 2.010113, 1.843447, 1.743447, 1.676780]

Fitting arccos(x) in range (1.000000, 2.000000)

MSS of errors squared = 8.633311e-06

Corrected MSS of errors squared = 1.726662e-05

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Comments

The following table shows the summary results for the ten models that fit the arc cosine function. The results are sorted in ascending order using the MSSE values.

<i>Power Q(x)</i>	<i>Power D(x)</i>	<i>MSSE</i>	<i>Q/D Orders</i>
$A+B*\sqrt{i}+C*i$	$A+B*i+C*\sqrt{i}$	8.79633400E-07	7, 7
$A+B*i+C*\sqrt{i}$	$A+B*\sqrt{i}+C*i$	1.28743300E-06	5, 6
$A+B*i$	$A+B/i$	1.84483000E-06	5, 6
$A+B/i$	$A+B*i$	3.15923000E-06	4, 6
$A+B*\log_{10}(i)^4$	$A+B*i$	5.05381100E-06	7, 5
$A+B/i$	$A+B*\log_{10}(i)^4$	5.67641200E-06	7, 4
$A+B*\sqrt{i}$	$A+B*i$	7.29322400E-06	5, 7
$A+B*\log_{10}(i)^4$	$A+B/i$	8.63331100E-06	5, 6
$A+B*i$	$A+B*\sqrt{i}$	1.97445700E-05	4, 6
$A+B*i$	$A+B*\log_{10}(i)^4$	6.17367800E-04	3, 5

RESULTS FOR THE ARC SINE

Combo Powers 1

Fitting $\arcsin(x)$ in range (0.000000, 1.000000)

Shammas polynomial power for Q(x) is $A+B*i$

Shammas polynomial power for D(x) is $A+B/i$

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.155104591598829	0	Inf	0
x1	0.792580326996735	0	Inf	0
x2	-1.9985315976736	0	-Inf	0
x3	1.03082286632676	0	Inf	0
x4	0.667891929397875	0	Inf	0
x5	-0.920662682653841	0	-Inf	0
x6	0.385445203129974	0	Inf	0
x7	-0.0622751748219482	0	-Inf	0
x8	0.107722876362513	0	Inf	0
x9	-1.85974735879219	0	-Inf	0
x10	2.70164902276283	0	Inf	0

Number of observations: 101, Error degrees of freedom: 90

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	6.03050467942531	100	0.0603050467942531		
Model	6.03050467942531	10	0.603050467942531	Inf	0
Residual	0	90	0		

```

for Q(x) A = 0.393592, B = 0.500000
for D(x) A = 0.000000, B = 2.000000
order Q(x) = 7.000000, order D(x) = 3.000000
Xmin = 0.000000 and Xmax = 1.000000
Ymin = 0.000000 and Ymax = 1.570796
List of powers for Q(x): [0.893592, 1.393592, 1.893592, 2.393592, 2.893592, 3.393592,
3.893592]
List of powers for D(x): [2.000000, 1.000000, 0.666667]
Fitting arcsin(x) in range (1.000000, 2.000000)
MSS of errors squared = 4.454714e-06
Corrected MSS of errors squared = 8.909428e-06
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 2

```

Fitting arcsin(x) in range (0.000000, 1.000000)
Shammas polynomial power for Q(x) is A+B*i
Shammas polynomial power for D(x) is A+B*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-316.096663768558	0	-Inf	0
x1	5468.23699888159	0	Inf	0
x2	-17986.2775280837	0	-Inf	0
x3	27166.5258353754	0	Inf	0
x4	-22036.6881715953	0	-Inf	0
x5	9327.53399096007	0	Inf	0
x6	-1623.01005911952	0	-Inf	0
x7	26.696521673497	0	Inf	0
x8	-71.2302902142634	0	-Inf	0
x9	65.7911116440932	0	Inf	0
x10	-20.4817457881904	0	-Inf	0

Number of observations: 101, Error degrees of freedom: 90

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	6.03050467919624	100	0.0603050467919624		

Model	6.03050467919624	10	0.603050467919624	Inf	0
Residual	0	90	0		

```

for Q(x) A = 0.165254, B = 0.209403
for D(x) A = 1.753459, B = 0.710906
order Q(x) = 6.000000, order D(x) = 4.000000
Xmin = 0.000000 and Xmax = 1.000000
Ymin = 0.000000 and Ymax = 1.570796
List of powers for Q(x): [0.374657, 0.584060, 0.793463, 1.002866, 1.212269, 1.421672]
List of powers for D(x): [2.464364, 2.758831, 2.984784, 3.175270]
Fitting arcsin(x) in range (1.000000, 2.000000)
MSS of errors squared = 7.785457e-05
Corrected MSS of errors squared = 1.557091e-04
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 3

```

Fitting arcsin(x) in range (0.000000, 1.000000)
Shammas polynomial power for Q(x) is A+B*sqrt(i)+C*i
Shammas polynomial power for D(x) is A+B*i+C*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.155153929064789	0	Inf	0
x1	-1.9951057430529	0	-Inf	0
x2	15.26619323609	0	Inf	0
x3	-38.4645824677163	0	-Inf	0
x4	42.9336183370067	0	Inf	0
x5	-22.2833492872169	0	-Inf	0
x6	4.41214090495314	0	Inf	0
x7	1.391988837828	0	Inf	0
x8	3.00218943919776	0	Inf	0
x9	-7.84841133415597	0	-Inf	0
x10	6.41779463710883	0	Inf	0
x11	-2.29027339888061	0	-Inf	0
x12	0.302642909775046	0	Inf	0

Number of observations: 101, Error degrees of freedom: 88

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

SumSq	DF	MeanSq	F	pValue
-------	----	--------	---	--------

Total	6.03050467942423	100	0.0603050467942423		
Model	6.03050467942423	12	0.502542056618686	Inf	0
Residual	0	88	0		

for Q(x) A = 0.124675, B = 0.280979, C = 0.234849
 for D(x) A = 0.031437, B = 0.367847, C = 0.217766
 order Q(x) = 6.000000, order D(x) = 6.000000
 Xmin = 0.000000 and Xmax = 1.000000
 Ymin = 0.000000 and Ymax = 1.570796
 List of powers for Q(x): [0.640503, 0.991737, 1.315892, 1.626029, 1.927208, 2.222024]
 List of powers for D(x): [0.617050, 1.075098, 1.512159, 1.938356, 2.357611, 2.771934]
 Fitting arcsin(x) in range (1.000000, 2.000000)
 MSS of errors squared = 6.900648e-07
 Corrected MSS of errors squared = 1.690307e-06
 R-squared = 1.00000000
 R-squared Adjusted = 1.00000000
 Particle swarm AICc = -1.000000e+99
 AIC = -Inf
 AICc = -Inf

Combo Powers 4

Fitting arcsin(x) in range (0.000000, 1.000000)
 Shammas polynomial power for Q(x) is A+B/i
 Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.0093025140740655	0	Inf	0
x1	-0.129408287690096	0	-Inf	0
x2	1.75605341851454	0	Inf	0
x3	-4.06183412772373	0	-Inf	0
x4	2.42610590316923	0	Inf	0
x5	140.687073037416	0	Inf	0
x6	-535.400071028559	0	-Inf	0
x7	843.176320432808	0	Inf	0
x8	-681.182085767203	0	-Inf	0
x9	280.788865914731	0	Inf	0
x10	-47.0703220095301	0	-Inf	0

Number of observations: 101, Error degrees of freedom: 90

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	6.03050467942823	100	0.0603050467942823		
Model	6.03050467942823	10	0.603050467942823	Inf	0
Residual	0	90	0		

for Q(x) A = 0.141382, B = 0.415496
 for D(x) A = 0.306499, B = 0.080607
 order Q(x) = 4.000000, order D(x) = 6.000000
 Xmin = 0.000000 and Xmax = 1.000000
 Ymin = 0.000000 and Ymax = 1.570796
 List of powers for Q(x): [0.556878, 0.349130, 0.279880, 0.245256]
 List of powers for D(x): [0.387106, 0.467714, 0.548321, 0.628929, 0.709536, 0.790144]
 Fitting arcsin(x) in range (1.000000, 2.000000)
 MSS of errors squared = 4.135899e-06
 Corrected MSS of errors squared = 8.271799e-06
 R-squared = 1.00000000
 R-squared Adjusted = 1.00000000
 Particle swarm AICc = -1.000000e+99
 AIC = -Inf
 AICc = -Inf

Combo Powers 5

Fitting arcsin(x) in range (0.000000, 1.000000)
 Shammas polynomial power for Q(x) is A+B*sqrt(i)
 Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.122526465274246	0	Inf	0
x1	-1.53637625908017	0	-Inf	0
x2	24.2408319249948	0	Inf	0
x3	-68.4699180126643	0	-Inf	0
x4	69.409922577163	0	Inf	0
x5	-23.7332878847797	0	-Inf	0
x6	6.11561173351167	0	Inf	0
x7	-7.76556260051111	0	-Inf	0
x8	2.61625208278327	0	Inf	0

Number of observations: 101, Error degrees of freedom: 92

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

SumSq	DF	MeanSq	F	pValue
-------	----	--------	---	--------

Total	6.0305046794248	100	0.060305046794248		
Model	6.0305046794248	8	0.7538130849281	Inf	0
Residual	0	92	0		

```

for Q(x) A = 0.423285, B = 0.445449
for D(x) A = 0.440899, B = 0.206532
order Q(x) = 5.000000, order D(x) = 3.000000
Xmin = 0.000000 and Xmax = 1.000000
Ymin = 0.000000 and Ymax = 1.570796
List of powers for Q(x): [0.868734, 1.053245, 1.194826, 1.314184, 1.419340]
List of powers for D(x): [0.647431, 0.853963, 1.060495]
Fitting arcsin(x) in range (1.000000, 2.000000)
MSS of errors squared = 2.381114e-05
Corrected MSS of errors squared = 4.762228e-05
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 6

```

Fitting arcsin(x) in range (0.000000, 1.000000)
Shammas polynomial power for Q(x) is A+B*i+C*sqrt(i)
Shammas polynomial power for D(x) is A+B*sqrt(i)+C*i

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.0951644320878801	0	Inf	0
x1	-0.323247190064438	0	-Inf	0
x2	0.332148939213132	0	Inf	0
x3	-0.101897243209315	0	-Inf	0
x4	58.7118082060612	0	Inf	0
x5	-300.222400226751	0	-Inf	0
x6	666.964525802957	0	Inf	0
x7	-800.628898040261	0	-Inf	0
x8	543.760912299428	0	Inf	0
x9	-197.518451238876	0	-Inf	0
x10	29.9303342594845	0	Inf	0

Number of observations: 101, Error degrees of freedom: 90

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	6.03050467941885	100	0.0603050467941885		
Model	6.03050467941885	10	0.603050467941885	Inf	0
Residual	0	90	0		

for Q(x) A = 0.098418, B = 0.222362, C = 0.197025
 for D(x) A = 0.235704, B = 0.332108, C = 0.065543
 order Q(x) = 3.000000, order D(x) = 7.000000
 Xmin = 0.000000 and Xmax = 1.000000
 Ymin = 0.000000 and Ymax = 1.570796
 List of powers for Q(x): [0.517805, 0.821777, 1.106761]
 List of powers for D(x): [0.633354, 0.836460, 1.007559, 1.162089, 1.306032, 1.442453, 1.573176]
 Fitting arcsin(x) in range (1.000000, 2.000000)
 MSS of errors squared = 4.866569e-06
 Corrected MSS of errors squared = 1.192061e-05
 R-squared = 1.00000000
 R-squared Adjusted = 1.00000000
 Particle swarm AICc = -1.000000e+99
 AIC = -Inf
 AICc = -Inf

Combo Powers 7

Fitting arcsin(x) in range (0.000000, 1.000000)
 Shammas polynomial power for Q(x) is A+B*i
 Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00841639104793857	0	Inf	0
x1	-0.0208745763754811	0	-Inf	0
x2	0.0172242983520441	0	Inf	0
x3	-0.00472737931240668	0	-Inf	0
x4	12775.4414602807	0	Inf	0
x5	0	0	NaN	NaN
x6	-20526.9182608153	0	-Inf	0
x7	0	0	NaN	NaN
x8	16682.2342649052	0	Inf	0
x9	-11396.4112637618	0	-Inf	0
x10	2466.6537606576	0	Inf	0

Number of observations: 101, Error degrees of freedom: 92

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	6.03050467949475	100	0.0603050467949475		
Model	6.03050467949475	8	0.753813084936844	Inf	0
Residual	0	92	0		

for Q(x) A = 0.000000, B = 0.201350

for D(x) A = 0.224223, B = 0.100000

order Q(x) = 3.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 1.000000

Ymin = 0.000000 and Ymax = 1.570796

List of powers for Q(x): [0.201350, 0.402700, 0.604050]

List of powers for D(x): [0.224223, 0.225044, 0.229405, 0.237361, 0.248092, 0.260888, 0.275229]

Fitting arcsin(x) in range (1.000000, 2.000000)

MSS of errors squared = 3.552285e-05

Corrected MSS of errors squared = 7.104570e-05

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 8

Fitting arcsin(x) in range (0.000000, 1.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.0284958139646195	0	-Inf	0
x1	-1.33393447861336	0	-Inf	0
x2	15.6596781224476	0	Inf	0
x3	-24.9607446159004	0	-Inf	0
x4	0	0	NaN	NaN
x5	0	0	NaN	NaN
x6	10.663676062484	0	Inf	0
x7	27.6781644241676	0	Inf	0
x8	0	0	NaN	NaN
x9	0	0	NaN	NaN
x10	-71.0198699336445	0	-Inf	0
x11	58.0581442138091	0	Inf	0

x12 -13.7166179807239 0 -Inf 0

Number of observations: 101, Error degrees of freedom: 92

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	6.03050467942533	100	0.0603050467942533		
Model	6.03050467942533	8	0.753813084928166	Inf	0
Residual	0	92	0		

for Q(x) A = 0.293335, B = 0.190787

for D(x) A = 0.171994, B = 0.340801

order Q(x) = 6.000000, order D(x) = 6.000000

Xmin = 0.000000 and Xmax = 1.000000

Ymin = 0.000000 and Ymax = 1.570796

List of powers for Q(x): [0.484122, 0.388729, 0.356931, 0.341032, 0.331493, 0.325133]

List of powers for D(x): [0.171994, 0.174792, 0.189655, 0.216771, 0.253339, 0.296949]

Fitting arcsin(x) in range (1.000000, 2.000000)

MSS of errors squared = 2.112682e-05

Corrected MSS of errors squared = 4.225364e-05

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 9

Fitting arcsin(x) in range (0.000000, 1.000000)

Shammas polynomial power for Q(x) is A+B*log10(i)^4

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-8.82491067715182	0	-Inf	0
x1	405.525339456773	0	Inf	0
x2	0	0	NaN	NaN
x3	0	0	NaN	NaN
x4	-795.601077142714	0	-Inf	0
x5	399.095099103134	0	Inf	0
x6	7.01193263965501	0	Inf	0
x7	-15.4193721968094	0	-Inf	0
x8	16.1868406080263	0	Inf	0

x9	-9.9823437278285	0	-Inf	0
x10	3.7156007690354	0	Inf	0
x11	-0.777840592421326	0	-Inf	0
x12	0.0707317295788661	0	Inf	0

Number of observations: 101, Error degrees of freedom: 90

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	6.03050467941841	100	0.0603050467941841		
Model	6.03050467941841	10	0.603050467941841	Inf	0
Residual	0	90	0		

for Q(x) A = 0.440438, B = 0.468652

for D(x) A = 1.400000, B = 0.798537

order Q(x) = 5.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 1.000000

Ymin = 0.000000 and Ymax = 1.570796

List of powers for Q(x): [0.440438, 0.444286, 0.464724, 0.502014, 0.552300]

List of powers for D(x): [2.198537, 2.997075, 3.795612, 4.594150, 5.392687, 6.191225, 6.989762]

Fitting arcsin(x) in range (1.000000, 2.000000)

MSS of errors squared = 3.174352e-06

Corrected MSS of errors squared = 6.348704e-06

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 10

Fitting arcsin(x) in range (0.000000, 1.000000)

Shammas polynomial power for Q(x) is A+B*log10(i)^4

Shammas polynomial power for D(x) is A+B/i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12 + x13 + x14$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0	0	NaN	NaN
x1	1638.36987937215	0	Inf	0
x2	0	0	NaN	NaN
x3	-4616.4058999314	0	-Inf	0
x4	5371.64792189777	0	Inf	0

x5	-3424.28428420176	0	-Inf	0
x6	1219.71249910095	0	Inf	0
x7	-189.038714943316	0	-Inf	0
x8	-165.174024384218	0	-Inf	0
x9	1868.96921809976	0	Inf	0
x10	-2782.96866732553	0	-Inf	0
x11	0	0	NaN	NaN
x12	0	0	NaN	NaN
x13	0	0	NaN	NaN
x14	1080.17207231546	0	Inf	0

Number of observations: 101, Error degrees of freedom: 91

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	6.03050467940557	100	0.0603050467940557		
Model	6.03050467940557	9	0.670056075489508	Inf	0
Residual	0	91	0		

for Q(x) A = 0.000000, B = 0.500000

for D(x) A = 0.409582, B = 0.100000

order Q(x) = 7.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 1.000000

Ymin = 0.000000 and Ymax = 1.570796

List of powers for Q(x): [0.000000, 0.004106, 0.025911, 0.065695, 0.119345, 0.183327, 0.255034]

List of powers for D(x): [0.509582, 0.459582, 0.442915, 0.434582, 0.429582, 0.426249, 0.423868]

Fitting arcsin(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.038542e-05

Corrected MSS of errors squared = 2.077084e-05

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Comments

The following table shows the summary results for the ten models that fit the arc sine function. The results are sorted in ascending order using the MSSE values.

<i>Power Q(x)</i>	<i>Power D(x)</i>	<i>MSSE</i>	<i>Q/D Orders</i>
$A+B*\sqrt{i}+C*i$	$A+B*i+C*\sqrt{i}$	6.90064800E-07	6, 6
$A+B*\log_{10}(i)^4$	$A+B*i$	3.17435200E-06	5, 7
$A+B/i$	$A+B*i$	4.13589900E-06	4, 6
$A+B*i$	$A+B/i$	4.45471400E-06	7, 3
$A+B*i+C*\sqrt{i}$	$A+B*\sqrt{i}+C*i$	4.86656900E-06	3, 7
$A+B*\log_{10}(i)^4$	$A+B/i$	1.03854200E-05	7, 7
$A+B/i$	$A+B*\log_{10}(i)^4$	2.11268200E-05	6, 6
$A+B*\sqrt{i}$	$A+B*i$	2.38111400E-05	5, 3
$A+B*i$	$A+B*\log_{10}(i)^4$	3.55228500E-05	3, 7
$A+B*i$	$A+B*\sqrt{i}$	7.78545700E-05	6, 4

RESULTS FOR THE ARC TANGENT

Combo Powers 1

Fitting $\arctan(x)$ in range (0.000000, 1.000000)

Shammas polynomial power for Q(x) is $A+B*i$

Shammas polynomial power for D(x) is $A+B/i$

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.445892260833855	0	Inf	0
x1	-2.12182204512986	0	-Inf	0
x2	4.12264586618821	0	Inf	0
x3	-3.55838942921666	0	-Inf	0
x4	1.19803458817063	0	Inf	0
x5	-1.47471972727663	0	-Inf	0
x6	26.7708793233276	0	Inf	0
x7	-75.3528851000624	0	-Inf	0
x8	50.970364264888	0	Inf	0

Number of observations: 101, Error degrees of freedom: 92

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	8.80077198802868	100	0.0880077198802868		
Model	8.80077198802868	8	1.10009649850359	Inf	0
Residual	0	92	0		

for Q(x) A = 0.006623, B = 0.410740

```

for D(x) A = 0.492764, B = 0.940364
order Q(x) = 4.000000, order D(x) = 4.000000
Xmin = 0.000000 and Xmax = 1.000000
Ymin = 0.000000 and Ymax = 0.785398
List of powers for Q(x): [0.417364, 0.828104, 1.238844, 1.649584]
List of powers for D(x): [1.433128, 0.962946, 0.806219, 0.727855]
Fitting arctan(x) in range (1.000000, 2.000000)
MSS of errors squared = 5.934888e-10
Corrected MSS of errors squared = 1.186978e-09
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 2

```

Fitting arctan(x) in range (0.000000, 1.000000)
Shammas polynomial power for Q(x) is A+B*i
Shammas polynomial power for D(x) is A+B*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.123876104382319	0	Inf	0
x1	-1.19882500935982	0	-Inf	0
x2	0.220605540005999	0	Inf	0
x3	-0.167953078265528	0	-Inf	0
x4	0.0359531264805268	0	Inf	0
x5	-0.00468183762264202	0	-Inf	0
x6	0.000356520543279228	0	Inf	0
x7	-1.22036903991303e-05	0	-Inf	0
x8	16.1525437319606	0	Inf	0
x9	-27.5419023938588	0	-Inf	0
x10	13.3800395155452	0	Inf	0

Number of observations: 101, Error degrees of freedom: 90

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	8.80077198802909	100	0.0880077198802909		
Model	8.80077198802909	10	0.880077198802909	Inf	0
Residual	0	90	0		

```

for Q(x) A = 0.119852, B = 2.000000
for D(x) A = 0.388268, B = 1.000000
order Q(x) = 7.000000, order D(x) = 3.000000
Xmin = 0.000000 and Xmax = 1.000000
Ymin = 0.000000 and Ymax = 0.785398
List of powers for Q(x): [2.119852, 4.119852, 6.119852, 8.119852, 10.119852,
12.119852, 14.119852]
List of powers for D(x): [1.388268, 1.802481, 2.120319]
Fitting arctan(x) in range (1.000000, 2.000000)
MSS of errors squared = 3.800329e-10
Corrected MSS of errors squared = 7.600658e-10
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 3

```

Fitting arctan(x) in range (0.000000, 1.000000)
Shammas polynomial power for Q(x) is A+B*sqrt(i)+C*i
Shammas polynomial power for D(x) is A+B*i+C*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.162841847984507	0	-Inf	0
x1	1.46410362699407	0	Inf	0
x2	-2.24355635928432	0	-Inf	0
x3	0.989325393517932	0	Inf	0
x4	6.66529925117279	0	Inf	0
x5	-15.6644190773943	0	-Inf	0
x6	18.8073669601644	0	Inf	0
x7	-13.2411658514719	0	-Inf	0
x8	5.54516565729264	0	Inf	0
x9	-1.28753220946879	0	-Inf	0
x10	0.12825445646326	0	Inf	0

Number of observations: 101, Error degrees of freedom: 90

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	8.80077198802921	100	0.0880077198802921		
Model	8.80077198802921	10	0.880077198802921	Inf	0

```

Residual          0      90      0

for Q(x) A = 0.088338, B = 0.195147, C = 0.192693
for D(x) A = 0.107764, B = 0.377230, C = 0.338232
order Q(x) = 3.000000, order D(x) = 7.000000
Xmin = 0.000000 and Xmax = 1.000000
Ymin = 0.000000 and Ymax = 0.785398
List of powers for Q(x): [0.476178, 0.749704, 1.004422]
List of powers for D(x): [0.823226, 1.340555, 1.825288, 2.293146, 2.750222, 3.199637,
3.643249]
Fitting arctan(x) in range (1.000000, 2.000000)
MSS of errors squared = 1.279671e-12
Corrected MSS of errors squared = 3.134541e-12
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 4

```

Fitting arctan(x) in range (0.000000, 1.000000)
Shammas polynomial power for Q(x) is A+B/i
Shammas polynomial power for D(x) is A+B*i

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-2.94507022614328	0	-Inf	0
x1	43.7551175138434	0	Inf	0
x2	-122.709235112857	0	-Inf	0
x3	0	0	NaN	NaN
x4	0	0	NaN	NaN
x5	0	0	NaN	NaN
x6	0	0	NaN	NaN
x7	82.1188418369785	0	Inf	0
x8	2.03655092377652	0	Inf	0
x9	-2.0109267895811	0	-Inf	0
x10	0.9733224404893	0	Inf	0
x11	-0.243655447634162	0	-Inf	0
x12	0.0250548425817294	0	Inf	0

Number of observations: 101, Error degrees of freedom: 92

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

SumSq	DF	MeanSq	F	pValue
-------	----	--------	---	--------

Total	8.80077198802733	100	0.0880077198802733		
Model	8.80077198802733	8	1.10009649850342	Inf	0
Residual	0	92	0		

```

for Q(x) A = 0.345043, B = 0.168053
for D(x) A = 0.138451, B = 0.771767
order Q(x) = 7.000000, order D(x) = 5.000000
Xmin = 0.000000 and Xmax = 1.000000
Ymin = 0.000000 and Ymax = 0.785398
List of powers for Q(x): [0.513096, 0.429069, 0.401061, 0.387056, 0.378654, 0.373052,
0.369050]
List of powers for D(x): [0.910218, 1.681985, 2.453752, 3.225519, 3.997286]
Fitting arctan(x) in range (1.000000, 2.000000)
MSS of errors squared = 3.612021e-09
Corrected MSS of errors squared = 7.224042e-09
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 5

```

Fitting arctan(x) in range (0.000000, 1.000000)
Shammas polynomial power for Q(x) is A+B*sqrt(i)
Shammas polynomial power for D(x) is A+B*i

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.485712689539938	0	-Inf	0
x1	279.651429810022	0	Inf	0
x2	-624.802458490856	0	-Inf	0
x3	345.956060472903	0	Inf	0
x4	1.45908560681395	0	Inf	0
x5	-1.13482013353087	0	-Inf	0
x6	0.442534885461541	0	Inf	0
x7	-0.0945678356922677	0	-Inf	0
x8	0.0084483480547867	0	Inf	0

Number of observations: 101, Error degrees of freedom: 92

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

SumSq	DF	MeanSq	F	pValue
-------	----	--------	---	--------

Total	8.80077198803471	100	0.0880077198803471		
Model	8.80077198803471	8	1.10009649850434	Inf	0
Residual	0	92	0		

```

for Q(x) A = 0.765027, B = 0.080091
for D(x) A = 0.000000, B = 0.940605
order Q(x) = 3.000000, order D(x) = 5.000000
Xmin = 0.000000 and Xmax = 1.000000
Ymin = 0.000000 and Ymax = 0.785398
List of powers for Q(x): [0.845118, 0.878292, 0.903748]
List of powers for D(x): [0.940605, 1.881210, 2.821816, 3.762421, 4.703026]
Fitting arctan(x) in range (1.000000, 2.000000)
MSS of errors squared = 3.098586e-09
Corrected MSS of errors squared = 6.197173e-09
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 6

```

Fitting arctan(x) in range (0.000000, 1.000000)
Shammas polynomial power for Q(x) is A+B*i+C*sqrt(i)
Shammas polynomial power for D(x) is A+B*sqrt(i)+C*i

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.165219282408609	0	-Inf	0
x1	1.98925876059503	0	Inf	0
x2	-5.67465323118929	0	-Inf	0
x3	7.88219799347698	0	Inf	0
x4	-5.87575090935296	0	-Inf	0
x5	2.18691856668181	0	Inf	0
x6	-0.271427832159987	0	-Inf	0
x7	11.0384233644191	0	Inf	0
x8	-16.9520196137623	0	-Inf	0
x9	0	0	NaN	NaN
x10	12.5454220929485	0	Inf	0
x11	-5.70314990925576	0	-Inf	0

Number of observations: 101, Error degrees of freedom: 90

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	8.80077198802917	100	0.0880077198802917		
Model	8.80077198802917	10	0.880077198802917	Inf	0
Residual	0	90	0		

for Q(x) A = 0.068155, B = 0.372385, C = 0.189872

for D(x) A = 0.353046, B = 0.270693, C = 0.118375

order Q(x) = 6.000000, order D(x) = 5.000000

Xmin = 0.000000 and Xmax = 1.000000

Ymin = 0.000000 and Ymax = 0.785398

List of powers for Q(x): [0.630412, 1.081445, 1.514178, 1.937439, 2.354647, 2.767555]

List of powers for D(x): [0.742115, 0.972615, 1.177026, 1.367933, 1.550211]

Fitting arctan(x) in range (1.000000, 2.000000)

MSS of errors squared = 3.289756e-12

Corrected MSS of errors squared = 8.058224e-12

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 7

Fitting arctan(x) in range (0.000000, 1.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.45935752307657	0	-Inf	0
x1	20.3200787609706	0	Inf	0
x2	-69.8260703284428	0	-Inf	0
x3	101.124708536242	0	Inf	0
x4	-73.7310448592401	0	-Inf	0
x5	25.6898088967176	0	Inf	0
x6	-3.03538441695893	0	-Inf	0
x7	19.0614046436815	0	Inf	0
x8	0	0	NaN	NaN
x9	-30.9572170630975	0	-Inf	0
x10	15.5239720460253	0	Inf	0
x11	-2.7108986928403	0	-Inf	0

Number of observations: 101, Error degrees of freedom: 90

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	8.80077198802836	100	0.0880077198802836		
Model	8.80077198802836	10	0.880077198802836	Inf	0
Residual	0	90	0		

for Q(x) A = 0.480991, B = 0.267168

for D(x) A = 0.802732, B = 3.398424

order Q(x) = 6.000000, order D(x) = 5.000000

Xmin = 0.000000 and Xmax = 1.000000

Ymin = 0.000000 and Ymax = 0.785398

List of powers for Q(x): [0.748159, 1.015328, 1.282496, 1.549665, 1.816833, 2.084001]

List of powers for D(x): [0.802732, 0.830639, 0.978845, 1.249247, 1.613901]

Fitting arctan(x) in range (1.000000, 2.000000)

MSS of errors squared = 7.523587e-12

Corrected MSS of errors squared = 1.504717e-11

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 8

Fitting arctan(x) in range (0.000000, 1.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12}$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	3.09343179213206	0	Inf	0
x1	4.6407913663464	0	Inf	0
x2	-57.2871985382875	0	-Inf	0
x3	99.7460973723337	0	Inf	0
x4	0	0	NaN	NaN
x5	0	0	NaN	NaN
x6	-50.1023169524498	0	-Inf	0
x7	24795.6138812981	0	Inf	0
x8	0	0	NaN	NaN
x9	-58248.8209358242	0	-Inf	0
x10	50037.5679634882	0	Inf	0
x11	-19711.0378385325	0	-Inf	0

```
x12          3127.58612453037    0    Inf    0
```

Number of observations: 101, Error degrees of freedom: 91

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	8.80077198778127	100	0.0880077198778127		
Model	8.80077198778127	9	0.977863554197919	Inf	0
Residual	0	91	0		

for Q(x) A = 0.209913, B = 0.771673

for D(x) A = 1.241104, B = 0.471804

order Q(x) = 6.000000, order D(x) = 6.000000

Xmin = 0.000000 and Xmax = 1.000000

Ymin = 0.000000 and Ymax = 0.785398

List of powers for Q(x): [0.981586, 0.595750, 0.467137, 0.402831, 0.364248, 0.338525]

List of powers for D(x): [1.241104, 1.244978, 1.265554, 1.303094, 1.353719, 1.414093]

Fitting arctan(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.300598e-10

Corrected MSS of errors squared = 2.601196e-10

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 9

Fitting arctan(x) in range (0.000000, 1.000000)

Shammas polynomial power for Q(x) is A+B*log10(i)^4

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

```
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11
```

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-18.9168818810562	0	-Inf	0
x1	1027.79977619072	0	Inf	0
x2	0	0	NaN	NaN
x3	0	0	NaN	NaN
x4	0	0	NaN	NaN
x5	-2630.43197485937	0	-Inf	0
x6	1621.84993934744	0	Inf	0
x7	1.61843273972505	0	Inf	0
x8	-1.39822928539732	0	-Inf	0

x9	0.601476667587934	0	Inf	0
x10	-0.135110963549739	0	-Inf	0
x11	0.0125720120613554	0	Inf	0

Number of observations: 101, Error degrees of freedom: 92

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	8.8007719880488	100	0.088007719880488		
Model	8.8007719880488	8	1.1000964985061	Inf	0
Residual	0	92	0		

for Q(x) A = 0.222697, B = 0.123460

for D(x) A = 0.097571, B = 0.900719

order Q(x) = 6.000000, order D(x) = 5.000000

Xmin = 0.000000 and Xmax = 1.000000

Ymin = 0.000000 and Ymax = 0.785398

List of powers for Q(x): [0.222697, 0.223711, 0.229095, 0.238918, 0.252165, 0.267964]

List of powers for D(x): [0.998290, 1.899010, 2.799729, 3.700448, 4.601168]

Fitting arctan(x) in range (1.000000, 2.000000)

MSS of errors squared = 4.447702e-09

Corrected MSS of errors squared = 8.895405e-09

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 10

Fitting arctan(x) in range (0.000000, 1.000000)

Shammas polynomial power for Q(x) is A+B*log10(i)^4

Shammas polynomial power for D(x) is A+B/i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-1742.81299397204	0	-Inf	0
x1	0	0	NaN	NaN
x2	2071.91758837424	0	Inf	0
x3	-328.871052166302	0	-Inf	0
x4	-0.45952876224799	0	-Inf	0
x5	115.913692838364	0	Inf	0
x6	-1771.56835464585	0	-Inf	0

x7	6927.3166587614	0	Inf	0
x8	-9632.56245029152	0	-Inf	0
x9	4362.12643984888	0	Inf	0

Number of observations: 101, Error degrees of freedom: 92

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	8.80077198806196	100	0.0880077198806196		
Model	8.80077198806196	8	1.10009649850774	Inf	0
Residual	0	92	0		

for Q(x) A = 0.000000, B = 0.469100

for D(x) A = 2.200000, B = 2.000000

order Q(x) = 3.000000, order D(x) = 6.000000

Xmin = 0.000000 and Xmax = 1.000000

Ymin = 0.000000 and Ymax = 0.785398

List of powers for Q(x): [0.000000, 0.003852, 0.024310]

List of powers for D(x): [4.200000, 3.200000, 2.866667, 2.700000, 2.600000, 2.533333]

Fitting arctan(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.883579e-09

Corrected MSS of errors squared = 3.767157e-09

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Comments

The following table shows the summary results for the ten models that fit the arc tangent function. The results are sorted in ascending order using the MSSE values.

<i>Power Q(x)</i>	<i>Power D(x)</i>	<i>MSSE</i>	<i>Q/D Orders</i>
$A+B*\sqrt{i}+C*i$	$A+B*i+C*\sqrt{i}$	1.27967100E-12	3, 7
$A+B*i+C*\sqrt{i}$	$A+B*\sqrt{i}+C*i$	3.28975600E-12	6, 5
$A+B*i$	$A+B*\log_{10}(i)^4$	7.52358700E-12	6, 5
$A+B/i$	$A+B*\log_{10}(i)^4$	1.30059800E-10	6, 6
$A+B*i$	$A+B*\sqrt{i}$	3.80032900E-10	7, 3
$A+B*i$	$A+B/i$	5.93488800E-10	4, 4
$A+B*\log_{10}(i)^4$	$A+B/i$	1.88357900E-09	3, 6
$A+B*\sqrt{i}$	$A+B*i$	3.09858600E-09	3, 5
$A+B/i$	$A+B*i$	3.61202100E-09	7, 5
$A+B*\log_{10}(i)^4$	$A+B*i$	4.44770200E-09	6, 5

RESULTS FOR INVERSE HYPERBOLIC SINE

Combo Powers 1

Fitting $\text{asinh}(x)$ in range (0.000000, 100.000000)

Shammas polynomial power for $Q(x)$ is $A+B*i$

Shammas polynomial power for $D(x)$ is $A+B/i$

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-2.14451765727953	0	-Inf	0
x1	36.2738069177188	0	Inf	0
x2	-117.218884160774	0	-Inf	0
x3	172.124947129582	0	Inf	0
x4	-139.461787961169	0	-Inf	0
x5	66.1114392187673	0	Inf	0
x6	-17.7231434107921	0	-Inf	0
x7	2.03814675004937	0	Inf	0
x8	0.952727415789328	0	Inf	0
x9	-6.52579442665919	0	-Inf	0
x10	6.57306014488637	0	Inf	0

Number of observations: 1001, Error degrees of freedom: 990

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	32.9032466460461	1000	0.0329032466460461		
Model	32.9032466460461	10	3.29032466460461	Inf	0
Residual	0	990	0		

```

for Q(x) A = 0.218373, B = 0.407323
for D(x) A = 0.192453, B = 1.609663
order Q(x) = 7.000000, order D(x) = 3.000000
Xmin = 0.000000 and Xmax = 100.000000
Ymin = 0.000000 and Ymax = 5.298342
List of powers for Q(x): [0.625696, 1.033019, 1.440342, 1.847665, 2.254988, 2.662311,
3.069634]
List of powers for D(x): [1.802116, 0.997285, 0.729008]
Fitting asinh(x) in range (1.000000, 2.000000)
MSS of errors squared = 7.190311e-05
Corrected MSS of errors squared = 1.438062e-04
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 2

```

Fitting asinh(x) in range (0.000000, 100.000000)
Shammas polynomial power for Q(x) is A+B*i
Shammas polynomial power for D(x) is A+B*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.0450357791079649	0	-Inf	0
x1	0.174520424233481	0	Inf	0
x2	-0.232075754279639	0	-Inf	0
x3	0.123012797720244	0	Inf	0
x4	-0.0221716902615048	0	-Inf	0
x5	0.00173057213956074	0	Inf	0
x6	2185.69218855001	0	Inf	0
x7	-7289.13145887479	0	-Inf	0
x8	6543.36137307767	0	Inf	0
x9	0	0	NaN	NaN
x10	-1438.92208329759	0	-Inf	0

Number of observations: 1001, Error degrees of freedom: 991

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	32.9032466456161	1000	0.0329032466456161		

Model	32.9032466456161	9	3.65591629395735	Inf	0
Residual	0	991	0		

```

for Q(x) A = 0.000000, B = 1.082981
for D(x) A = 0.679197, B = 0.100000
order Q(x) = 5.000000, order D(x) = 5.000000
Xmin = 0.000000 and Xmax = 100.000000
Ymin = 0.000000 and Ymax = 5.298342
List of powers for Q(x): [1.082981, 2.165962, 3.248943, 4.331924, 5.414905]
List of powers for D(x): [0.779197, 0.820618, 0.852402, 0.879197, 0.902804]
Fitting asinh(x) in range (1.000000, 2.000000)
MSS of errors squared = 4.705004e-05
Corrected MSS of errors squared = 9.410007e-05
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 3

```

Fitting asinh(x) in range (0.000000, 100.000000)
Shammas polynomial power for Q(x) is A+B*sqrt(i)+C*i
Shammas polynomial power for D(x) is A+B*i+C*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.967695187708087	0	Inf	0
x1	-22.8470224891173	0	-Inf	0
x2	69.1371128679986	0	Inf	0
x3	-88.7264485842763	0	-Inf	0
x4	59.9785338913139	0	Inf	0
x5	-23.3168124140627	0	-Inf	0
x6	5.34534651507902	0	Inf	0
x7	-0.538404560319292	0	-Inf	0
x8	6.61503597152494	0	Inf	0
x9	-13.0282778335308	0	-Inf	0
x10	10.4107909396978	0	Inf	0
x11	-2.99754949087565	0	-Inf	0

Number of observations: 1001, Error degrees of freedom: 989

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

SumSq	DF	MeanSq	F	pValue
-------	----	--------	---	--------

Total	32.9032466460472	1000	0.0329032466460472		
Model	32.9032466460472	11	2.99120424054975	Inf	0
Residual	0	989	0		

for Q(x) A = 0.478606, B = 0.279151, C = 0.292276
 for D(x) A = 0.003078, B = 0.341862, C = 0.285987
 order Q(x) = 7.000000, order D(x) = 4.000000
 Xmin = 0.000000 and Xmax = 100.000000
 Ymin = 0.000000 and Ymax = 5.298342
 List of powers for Q(x): [1.050033, 1.457938, 1.838939, 2.206014, 2.564189, 2.916042, 3.263105]
 List of powers for D(x): [0.630927, 1.091249, 1.524008, 1.942501]
 Fitting asinh(x) in range (1.000000, 2.000000)
 MSS of errors squared = 3.296642e-06
 Corrected MSS of errors squared = 8.075090e-06
 R-squared = 1.00000000
 R-squared Adjusted = 1.00000000
 Particle swarm AICc = -1.000000e+99
 AIC = -Inf
 AICc = -Inf

Combo Powers 4

Fitting asinh(x) in range (0.000000, 100.000000)
 Shammas polynomial power for Q(x) is A+B/i
 Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-15.0101988826474	0	-Inf	0
x1	67.2991628023773	0	Inf	0
x2	-3468.49688258463	0	-Inf	0
x3	13470.4984301405	0	Inf	0
x4	0	0	NaN	NaN
x5	-24504.5764764475	0	-Inf	0
x6	0	0	NaN	NaN
x7	14450.2859721491	0	Inf	0
x8	26.1508918800314	0	Inf	0
x9	-54.147586807736	0	-Inf	0
x10	37.2232380715636	0	Inf	0
x11	-8.22655035165104	0	-Inf	0

Number of observations: 1001, Error degrees of freedom: 991
 R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	32.9032466470381	1000	0.0329032466470381		
Model	32.9032466470381	9	3.65591629411535	Inf	0
Residual	0	991	0		

for Q(x) A = 0.432714, B = 0.485734

for D(x) A = 0.593967, B = 0.175626

order Q(x) = 7.000000, order D(x) = 4.000000

Xmin = 0.000000 and Xmax = 100.000000

Ymin = 0.000000 and Ymax = 5.298342

List of powers for Q(x): [0.918448, 0.675581, 0.594625, 0.554147, 0.529861, 0.513669, 0.502104]

List of powers for D(x): [0.769593, 0.945219, 1.120845, 1.296471]

Fitting asinh(x) in range (1.000000, 2.000000)

MSS of errors squared = 5.452112e-05

Corrected MSS of errors squared = 1.090422e-04

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 5

Fitting asinh(x) in range (0.000000, 100.000000)

Shammas polynomial power for Q(x) is A+B*sqrt(i)

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.023123281136356	0	Inf	0
x1	3.54361833385025	0	Inf	0
x2	-26.316382107417	0	-Inf	0
x3	56.4642395681896	0	Inf	0
x4	-48.006917029666	0	-Inf	0
x5	14.2923181871983	0	Inf	0
x6	1812.93274704239	0	Inf	0
x7	-5728.95945715177	0	-Inf	0
x8	5464.49488984737	0	Inf	0
x9	0	0	NaN	NaN
x10	-2524.59352923432	0	-Inf	0
x11	977.125349261071	0	Inf	0

Number of observations: 1001, Error degrees of freedom: 990

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	32.9032466457454	1000	0.0329032466457454		
Model	32.9032466457454	10	3.29032466457454	Inf	0
Residual	0	990	0		

for Q(x) A = 1.233009, B = 0.404928

for D(x) A = 0.621781, B = 0.053063

order Q(x) = 5.000000, order D(x) = 6.000000

Xmin = 0.000000 and Xmax = 100.000000

Ymin = 0.000000 and Ymax = 5.298342

List of powers for Q(x): [1.637937, 1.805664, 1.934365, 2.042865, 2.138456]

List of powers for D(x): [0.674844, 0.727906, 0.780969, 0.834032, 0.887095, 0.940158]

Fitting asinh(x) in range (1.000000, 2.000000)

MSS of errors squared = 5.348970e-05

Corrected MSS of errors squared = 1.069794e-04

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 6

Fitting asinh(x) in range (0.000000, 100.000000)

Shammas polynomial power for Q(x) is A+B*i+C*sqrt(i)

Shammas polynomial power for D(x) is A+B*sqrt(i)+C*i

Linear regression model:

$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-1.0233987395316	0	-Inf	0
x1	10.9512147068091	0	Inf	0
x2	-27.3095460238455	0	-Inf	0
x3	29.0410543073934	0	Inf	0
x4	-14.7372297930287	0	-Inf	0
x5	3.42262493122086	0	Inf	0
x6	-0.344719771200192	0	-Inf	0
x7	81.9475297395425	0	Inf	0
x8	-374.677421167112	0	-Inf	0
x9	640.716640812662	0	Inf	0
x10	-480.525999540931	0	-Inf	0

x11 133.539250535222 0 Inf 0

Number of observations: 1001, Error degrees of freedom: 989

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	32.9032466462866	1000	0.0329032466462866		
Model	32.9032466462866	11	2.99120424057151	Inf	0
Residual	0	989	0		

for Q(x) A = 0.181281, B = 0.390000, C = 0.301090

for D(x) A = 0.177914, B = 0.333694, C = 0.088756

order Q(x) = 6.000000, order D(x) = 5.000000

Xmin = 0.000000 and Xmax = 100.000000

Ymin = 0.000000 and Ymax = 5.298342

List of powers for Q(x): [0.872371, 1.387087, 1.872785, 2.343462, 2.804539, 3.258799]

List of powers for D(x): [0.600364, 0.827341, 1.022158, 1.200328, 1.367858]

Fitting asinh(x) in range (1.000000, 2.000000)

MSS of errors squared = 8.141111e-06

Corrected MSS of errors squared = 1.994157e-05

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 7

Fitting asinh(x) in range (0.000000, 100.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	1.68982976722729e-05	0	Inf	0
x1	-5.31624864697074e-05	0	-Inf	0
x2	-6.96635078026792e-06	0	-Inf	0
x3	0.000107422925938688	0	Inf	0
x4	-7.80239834642407e-05	0	-Inf	0
x5	1.38316281006349e-05	0	Inf	0
x6	5.3950332669694	0	Inf	0
x7	0	0	NaN	NaN
x8	-7.61099192467554	0	-Inf	0

x9	4.29979653536749	0	Inf	0
x10	-1.24543852797149	0	-Inf	0
x11	0.161600650278824	0	Inf	0

Number of observations: 1001, Error degrees of freedom: 990

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	32.9032466460555	1000	0.0329032466460555		
Model	32.9032466460555	10	3.29032466460555	Inf	0
Residual	0	990	0		

for Q(x) A = 0.261145, B = 0.360304

for D(x) A = 0.050102, B = 0.742318

order Q(x) = 5.000000, order D(x) = 6.000000

Xmin = 0.000000 and Xmax = 100.000000

Ymin = 0.000000 and Ymax = 5.298342

List of powers for Q(x): [0.621449, 0.981753, 1.342057, 1.702361, 2.062665]

List of powers for D(x): [0.050102, 0.056198, 0.088570, 0.147634, 0.227286, 0.322275]

Fitting asinh(x) in range (1.000000, 2.000000)

MSS of errors squared = 5.096778e-05

Corrected MSS of errors squared = 1.019356e-04

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 8

Fitting asinh(x) in range (0.000000, 100.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00374109339115809	0	Inf	0
x1	-0.336150155920557	0	-Inf	0
x2	1.46302884668906	0	Inf	0
x3	-1.13061977403754	0	-Inf	0
x4	15.2113481734674	0	Inf	0
x5	0	0	NaN	NaN
x6	0	0	NaN	NaN

x7	-47.8467155065369	0	-Inf	0
x8	54.2512417963746	0	Inf	0
x9	-25.1357533959588	0	-Inf	0
x10	4.51987892250811	0	Inf	0

Number of observations: 1001, Error degrees of freedom: 992

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	32.9032466460573	1000	0.0329032466460573		
Model	32.9032466460573	8	4.11290583075716	Inf	0
Residual	0	992	0		

for Q(x) A = 0.494204, B = 0.100000

for D(x) A = 0.100000, B = 0.403265

order Q(x) = 3.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 100.000000

Ymin = 0.000000 and Ymax = 5.298342

List of powers for Q(x): [0.594204, 0.544204, 0.527537]

List of powers for D(x): [0.100000, 0.103312, 0.120898, 0.152985, 0.196255, 0.247858, 0.305693]

Fitting asinh(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.430254e-04

Corrected MSS of errors squared = 2.860508e-04

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 9

Fitting asinh(x) in range (0.000000, 100.000000)

Shammas polynomial power for Q(x) is A+B*log10(i)^4

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12 + x13

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.783714813005078	0	Inf	0
x1	-80.516707553197	0	-Inf	0
x2	0	0	NaN	NaN
x3	0	0	NaN	NaN
x4	0	0	NaN	NaN

x5	212.433349945279	0	Inf	0
x6	-132.700354681013	0	-Inf	0
x7	195.510588479864	0	Inf	0
x8	-835.811670821581	0	-Inf	0
x9	1545.18492157775	0	Inf	0
x10	-1568.36139412823	0	-Inf	0
x11	915.752560800032	0	Inf	0
x12	-290.074741876711	0	-Inf	0
x13	38.7997334306039	0	Inf	0

Number of observations: 1001, Error degrees of freedom: 990

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	32.9032466458121	1000	0.0329032466458121		
Model	32.9032466458121	10	3.29032466458121	Inf	0
Residual	0	990	0		

for Q(x) A = 0.945767, B = 0.361377

for D(x) A = 0.872576, B = 0.213233

order Q(x) = 6.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 100.000000

Ymin = 0.000000 and Ymax = 5.298342

List of powers for Q(x): [0.945767, 0.948735, 0.964495, 0.993248, 1.032025, 1.078268]

List of powers for D(x): [1.085808, 1.299041, 1.512274, 1.725506, 1.938739, 2.151972, 2.365204]

Fitting asinh(x) in range (1.000000, 2.000000)

MSS of errors squared = 3.462925e-05

Corrected MSS of errors squared = 6.925851e-05

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 10

Fitting asinh(x) in range (0.000000, 100.000000)

Shammas polynomial power for Q(x) is A+B*log10(i)^4

Shammas polynomial power for D(x) is A+B/i

Linear regression model:

$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$

Estimated Coefficients:

Estimate	SE	tStat	pValue
----------	----	-------	--------

(Intercept)	0.14013428002441	0	Inf	0
x1	-134.377182059412	0	-Inf	0
x2	0	0	NaN	NaN
x3	0	0	NaN	NaN
x4	291.810784995966	0	Inf	0
x5	-157.57373704284	0	-Inf	0
x6	-0.0642475632462283	0	-Inf	0
x7	12.0413363583929	0	Inf	0
x8	-176.231567608282	0	-Inf	0
x9	697.622875422299	0	Inf	0
x10	-992.868079122183	0	-Inf	0
x11	460.4996823385	0	Inf	0

Number of observations: 1001, Error degrees of freedom: 991

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	32.9032466459683	1000	0.0329032466459683		
Model	32.9032466459683	9	3.65591629399647	Inf	0
Residual	0	991	0		

for Q(x) A = 0.549236, B = 0.105511

for D(x) A = 0.213628, B = 1.040109

order Q(x) = 5.000000, order D(x) = 6.000000

Xmin = 0.000000 and Xmax = 100.000000

Ymin = 0.000000 and Ymax = 5.298342

List of powers for Q(x): [0.549236, 0.550103, 0.554704, 0.563099, 0.574421]

List of powers for D(x): [1.253737, 0.733682, 0.560331, 0.473655, 0.421650, 0.386979]

Fitting asinh(x) in range (1.000000, 2.000000)

MSS of errors squared = 2.633384e-04

Corrected MSS of errors squared = 5.266769e-04

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Comments

The following table shows the summary results for the ten models that fit the inverse hyperbolic sine function. The results are sorted in ascending order using the MSSE values.

<i>Power Q(x)</i>	<i>Power D(x)</i>	<i>MSSE</i>	<i>Q/D Orders</i>
$A+B*\sqrt{i}+C*i$	$A+B*i+C*\sqrt{i}$	3.29664200E-06	7, 4
$A+B*i+C*\sqrt{i}$	$A+B*\sqrt{i}+C*i$	8.14111100E-06	6, 5
$A+B*\log_{10}(i)^4$	$A+B*i$	3.46292500E-05	6, 7
$A+B*i$	$A+B*\sqrt{i}$	4.70500400E-05	5, 5
$A+B*i$	$A+B*\log_{10}(i)^4$	5.09677800E-05	5, 6
$A+B*\sqrt{i}$	$A+B*i$	5.34897000E-05	5, 6
$A+B/i$	$A+B*i$	5.45211200E-05	7, 4
$A+B*i$	$A+B/i$	7.19031100E-05	7, 3
$A+B/i$	$A+B*\log_{10}(i)^4$	1.43025400E-04	3, 7
$A+B*\log_{10}(i)^4$	$A+B/i$	2.63338400E-04	5, 6

RESULTS FOR INVERSE HYPERBOLIC TANGENT

Combo Powers 1

Fitting $\operatorname{atanh}(x)$ in range (0.000000, 0.999000)

Shammas polynomial power for Q(x) is $A+B*i$

Shammas polynomial power for D(x) is $A+B/i$

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.406353937369108	0	Inf	0
x1	-1.50875926477524	0	-Inf	0
x2	1.68871976556263	0	Inf	0
x3	-0.629289513196044	0	-Inf	0
x4	0.0635585632917016	0	Inf	0
x5	0.531586996305683	0	Inf	0
x6	-5.4124879501537	0	-Inf	0
x7	6.73028802139563	0	Inf	0
x8	-0.869970534684305	0	-Inf	0

Number of observations: 1000, Error degrees of freedom: 991

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	23.1321964724073	999	0.0231553518242316		
Model	23.1321964724073	8	2.89152455905092	Inf	0
Residual	0	991	0		

for Q(x) A = 0.377843, B = 0.359662

```

for D(x) A = 0.203803, B = 0.894132
order Q(x) = 4.000000, order D(x) = 4.000000
Xmin = 0.000000 and Xmax = 0.999000
Ymin = 0.000000 and Ymax = 3.800201
List of powers for Q(x): [0.737505, 1.097168, 1.456830, 1.816493]
List of powers for D(x): [1.097935, 0.650869, 0.501847, 0.427336]
Fitting atanh(x) in range (1.000000, 2.000000)
MSS of errors squared = 3.784132e-05
Corrected MSS of errors squared = 7.568264e-05
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 2

```

Fitting atanh(x) in range (0.000000, 0.999000)
Shammas polynomial power for Q(x) is A+B*i
Shammas polynomial power for D(x) is A+B*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.270645592458558	0	Inf	0
x1	2.40988126015471	0	Inf	0
x2	-4.94897746703267	0	-Inf	0
x3	2.10694778808598	0	Inf	0
x4	1.15219421447138	0	Inf	0
x5	-1.06896233020222	0	-Inf	0
x6	0.21297883948716	0	Inf	0
x7	18.7542834167773	0	Inf	0
x8	-47.1075417681732	0	-Inf	0
x9	41.9536114919628	0	Inf	0
x10	-12.7350610577924	0	-Inf	0

Number of observations: 1000, Error degrees of freedom: 989

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	23.1321964724119	999	0.0231553518242362		
Model	23.1321964724119	10	2.31321964724119	Inf	0
Residual	0	989	0		

```

for Q(x) A = 0.013446, B = 0.689062
for D(x) A = 1.257288, B = 0.924750
order Q(x) = 6.000000, order D(x) = 4.000000
Xmin = 0.000000 and Xmax = 0.999000
Ymin = 0.000000 and Ymax = 3.800201
List of powers for Q(x): [0.702508, 1.391570, 2.080633, 2.769695, 3.458757, 4.147820]
List of powers for D(x): [2.182038, 2.565082, 2.859001, 3.106787]
Fitting atanh(x) in range (1.000000, 2.000000)
MSS of errors squared = 2.576124e-05
Corrected MSS of errors squared = 5.152247e-05
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 3

```

Fitting atanh(x) in range (0.000000, 0.999000)
Shammas polynomial power for Q(x) is A+B*sqrt(i)+C*i
Shammas polynomial power for D(x) is A+B*i+C*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.399760149772137	0	Inf	0
x1	-2.20182410508741	0	-Inf	0
x2	3.56008917431021	0	Inf	0
x3	-2.40917832710349	0	-Inf	0
x4	0.75800350343304	0	Inf	0
x5	-0.0917439393859492	0	-Inf	0
x6	2.7577402313989	0	Inf	0
x7	-1.18389020232405	0	-Inf	0
x8	-2.3332798381026	0	-Inf	0
x9	2.35073114718947	0	Inf	0
x10	-0.606407793002654	0	-Inf	0

Number of observations: 1000, Error degrees of freedom: 989

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	23.1321964724077	999	0.023155351824232		
Model	23.1321964724077	10	2.31321964724077	Inf	0
Residual	0	989	0		

```

for Q(x) A = 0.371835, B = 0.199761, C = 0.360517
for D(x) A = 0.005979, B = 0.228029, C = 0.273980
order Q(x) = 5.000000, order D(x) = 5.000000
Xmin = 0.000000 and Xmax = 0.999000
Ymin = 0.000000 and Ymax = 3.800201
List of powers for Q(x): [0.932112, 1.375373, 1.799381, 2.213424, 2.621098]
List of powers for D(x): [0.507988, 0.849503, 1.164614, 1.466056, 1.758763]
Fitting atanh(x) in range (1.000000, 2.000000)
MSS of errors squared = 1.387804e-05
Corrected MSS of errors squared = 3.399411e-05
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 4

```

Fitting atanh(x) in range (0.000000, 0.999000)
Shammas polynomial power for Q(x) is A+B/i
Shammas polynomial power for D(x) is A+B*i

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-23.258744549089	0	-Inf	0
x1	-2046.83280826925	0	-Inf	0
x2	25444.0372419789	0	Inf	0
x3	-45429.1270842868	0	-Inf	0
x4	0	0	NaN	NaN
x5	22055.2123420064	0	Inf	0
x6	11231.2513215863	0	Inf	0
x7	-23789.9419341926	0	-Inf	0
x8	0	0	NaN	NaN
x9	23464.2209761065	0	Inf	0
x10	0	0	NaN	NaN
x11	-18784.1145638436	0	-Inf	0
x12	7879.55325344881	0	Inf	0

Number of observations: 1000, Error degrees of freedom: 990

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

SumSq	DF	MeanSq	F	pValue

Total	23.1321964684173	999	0.0231553518202375		
Model	23.1321964684173	9	2.57024405204636	Inf	0
Residual	0	990	0		

```

for Q(x) A = 0.304791, B = 0.126692
for D(x) A = 1.339794, B = 0.030098
order Q(x) = 5.000000, order D(x) = 7.000000
Xmin = 0.000000 and Xmax = 0.999000
Ymin = 0.000000 and Ymax = 3.800201
List of powers for Q(x): [0.431484, 0.368137, 0.347022, 0.336464, 0.330130]
List of powers for D(x): [1.369893, 1.399991, 1.430090, 1.460188, 1.490287, 1.520385,
1.550484]
Fitting atanh(x) in range (1.000000, 2.000000)
MSS of errors squared = 2.593434e-05
Corrected MSS of errors squared = 5.186868e-05
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 5

```

Fitting atanh(x) in range (0.000000, 0.999000)
Shammas polynomial power for Q(x) is A+B*sqrt(i)
Shammas polynomial power for D(x) is A+B*i

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.154172135691383	0	Inf	0
x1	1170.70786165124	0	Inf	0
x2	0	0	NaN	NaN
x3	-4890.68125405751	0	-Inf	0
x4	0	0	NaN	NaN
x5	6214.44317444712	0	Inf	0
x6	0	0	NaN	NaN
x7	-2494.46659734955	0	-Inf	0
x8	-47.3571712673885	0	-Inf	0
x9	169.15782828226	0	Inf	0
x10	-213.865479541217	0	-Inf	0
x11	115.630309136569	0	Inf	0
x12	-22.7228434624871	0	-Inf	0

Number of observations: 1000, Error degrees of freedom: 990
R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	23.1321964724534	999	0.0231553518242777		
Model	23.1321964724534	9	2.57024405249482	Inf	0
Residual	0	990	0		

for Q(x) A = 1.068381, B = 0.104338

for D(x) A = 0.973203, B = 0.209428

order Q(x) = 7.000000, order D(x) = 5.000000

Xmin = 0.000000 and Xmax = 0.999000

Ymin = 0.000000 and Ymax = 3.800201

List of powers for Q(x): [1.172719, 1.215937, 1.249100, 1.277057, 1.301688, 1.323956, 1.344434]

List of powers for D(x): [1.182631, 1.392059, 1.601487, 1.810915, 2.020343]

Fitting atanh(x) in range (1.000000, 2.000000)

MSS of errors squared = 2.498098e-05

Corrected MSS of errors squared = 4.996196e-05

R-squared = 1.000000000

R-squared Adjusted = 1.000000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 6

Fitting atanh(x) in range (0.000000, 0.999000)

Shammas polynomial power for Q(x) is $A+B*i+C*\sqrt{i}$

Shammas polynomial power for D(x) is $A+B*\sqrt{i}+C*i$

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.0819752462859763	0	-Inf	0
x1	16.0270906107929	0	Inf	0
x2	-44.4612523953817	0	-Inf	0
x3	43.9707237651727	0	Inf	0
x4	-17.4996711525533	0	-Inf	0
x5	2.09668826315225	0	Inf	0
x6	3.92132868856024	0	Inf	0
x7	-4.8681403100212	0	-Inf	0
x8	2.24996325291963	0	Inf	0
x9	-0.354755490099881	0	-Inf	0

Number of observations: 1000, Error degrees of freedom: 990

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	23.1321964724062	999	0.0231553518242305		
Model	23.1321964724062	9	2.57024405248958	Inf	0
Residual	0	990	0		

for Q(x) A = 0.323968, B = 0.191054, C = 0.102660

for D(x) A = 0.320174, B = 0.186642, C = 0.365860

order Q(x) = 5.000000, order D(x) = 4.000000

Xmin = 0.000000 and Xmax = 0.999000

Ymin = 0.000000 and Ymax = 3.800201

List of powers for Q(x): [0.617682, 0.851260, 1.074943, 1.293505, 1.508794]

List of powers for D(x): [0.872676, 1.315845, 1.741027, 2.156897]

Fitting atanh(x) in range (1.000000, 2.000000)

MSS of errors squared = 2.245080e-05

Corrected MSS of errors squared = 5.499302e-05

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 7

Fitting atanh(x) in range (0.000000, 0.999000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.390858792578101	0	Inf	0
x1	1.85785521774987	0	Inf	0
x2	-5.74159223434831	0	-Inf	0
x3	4.83032977338845	0	Inf	0
x4	-1.28019267056756	0	-Inf	0
x5	57.7214440247408	0	Inf	0
x6	0	0	NaN	NaN
x7	-58.6659703772942	0	-Inf	0
x8	-15.1936598855809	0	-Inf	0
x9	17.0809274315898	0	Inf	0

Number of observations: 1000, Error degrees of freedom: 991

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	23.1321964724106	999	0.0231553518242348		
Model	23.1321964724106	8	2.89152455905133	Inf	0
Residual	0	991	0		

for Q(x) A = 0.212391, B = 0.343269

for D(x) A = 1.008043, B = 0.809702

order Q(x) = 4.000000, order D(x) = 5.000000

Xmin = 0.000000 and Xmax = 0.999000

Ymin = 0.000000 and Ymax = 3.800201

List of powers for Q(x): [0.555660, 0.898929, 1.242199, 1.585468]

List of powers for D(x): [1.008043, 1.014692, 1.050004, 1.114429, 1.201311]

Fitting atanh(x) in range (1.000000, 2.000000)

MSS of errors squared = 3.474282e-05

Corrected MSS of errors squared = 6.948564e-05

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 8

Fitting atanh(x) in range (0.000000, 0.999000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12 + x13$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	5.06651103730426	0	Inf	0
x1	0.608504683457534	0	Inf	0
x2	-139.926209465709	0	-Inf	0
x3	1926.25801346214	0	Inf	0
x4	-7169.31206142662	0	-Inf	0
x5	9731.42243240709	0	Inf	0
x6	-4354.00804137849	0	-Inf	0
x7	10703.3341240862	0	Inf	0
x8	0	0	NaN	NaN
x9	0	0	NaN	NaN
x10	0	0	NaN	NaN
x11	-54821.140607231	0	-Inf	0
x12	65743.8615854074	0	Inf	0

x13 -21625.1642515671 0 -Inf 0

Number of observations: 1000, Error degrees of freedom: 989

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	23.1321964854153	999	0.0231553518372526		
Model	23.1321964854153	10	2.31321964854153	Inf	0
Residual	0	989	0		

for Q(x) A = 0.434903, B = 1.808814

for D(x) A = 1.953886, B = 0.135499

order Q(x) = 6.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 0.999000

Ymin = 0.000000 and Ymax = 3.800201

List of powers for Q(x): [2.243717, 1.339310, 1.037841, 0.887107, 0.796666, 0.736372]

List of powers for D(x): [1.953886, 1.954999, 1.960908, 1.971689, 1.986229, 2.003568, 2.023000]

Fitting atanh(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.550985e-05

Corrected MSS of errors squared = 3.101970e-05

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 9

Fitting atanh(x) in range (0.000000, 0.999000)

Shammas polynomial power for Q(x) is A+B*log10(i)^4

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	3.54930884861891e-08	0	Inf	0
x1	-3.55291869914857e-06	0	-Inf	0
x2	0	0	NaN	NaN
x3	0	0	NaN	NaN
x4	1.05652591689728e-05	0	Inf	0
x5	-9.57227624180603e-06	0	-Inf	0
x6	2.52444267975523e-06	0	Inf	0
x7	1	0	Inf	0

x8	0	0	NaN	NaN
x9	0	0	NaN	NaN
x10	0	0	NaN	NaN

Number of observations: 1000, Error degrees of freedom: 994

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	23.1321964724076	999	0.0231553518242318		
Model	23.1321964724076	5	4.62643929448152	Inf	0
Residual	0	994	0		

for Q(x) A = 0.071118, B = 0.087194

for D(x) A = 0.000000, B = 0.000000

order Q(x) = 6.000000, order D(x) = 4.000000

Xmin = 0.000000 and Xmax = 0.999000

Ymin = 0.000000 and Ymax = 3.800201

List of powers for Q(x): [0.071118, 0.071834, 0.075636, 0.082574, 0.091930, 0.103088]

List of powers for D(x): [0.000000, 0.000000, 0.000000, 0.000000]

Fitting atanh(x) in range (1.000000, 2.000000)

MSS of errors squared = 4.338416e-03

Corrected MSS of errors squared = 8.676832e-03

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 10

Fitting atanh(x) in range (0.000000, 0.999000)

Shammas polynomial power for Q(x) is A+B*log10(i)^4

Shammas polynomial power for D(x) is A+B/i

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	694.029916475126	0	Inf	0
x1	0	0	NaN	NaN
x2	-824.229395929715	0	-Inf	0
x3	130.200239498458	0	Inf	0
x4	0.0246023395629602	0	Inf	0
x5	-10.9996797540031	0	-Inf	0
x6	305.753352152817	0	Inf	0

x7	-2156.11769099158	0	-Inf	0
x8	5752.98578756156	0	Inf	0
x9	-6400.76003385399	0	-Inf	0
x10	2510.11290250015	0	Inf	0

Number of observations: 1000, Error degrees of freedom: 990

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	23.1321964708848	999	0.0231553518227075		
Model	23.1321964708848	9	2.57024405232053	Inf	0
Residual	0	990	0		

for Q(x) A = 0.000000, B = 0.100000

for D(x) A = 0.393616, B = 2.000000

order Q(x) = 3.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 0.999000

Ymin = 0.000000 and Ymax = 3.800201

List of powers for Q(x): [0.000000, 0.000821, 0.005182]

List of powers for D(x): [2.393616, 1.393616, 1.060283, 0.893616, 0.793616, 0.726950, 0.679331]

Fitting atanh(x) in range (1.000000, 2.000000)

MSS of errors squared = 3.017821e-05

Corrected MSS of errors squared = 6.035643e-05

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Comments

The following table shows the summary results for the ten models that fit the inverse hyperbolic tangent function. The results are sorted in ascending order using the MSSE values.

<i>Power Q(x)</i>	<i>Power D(x)</i>	<i>MSSE</i>	<i>Q/D Orders</i>
$A+B*\sqrt{i}+C*i$	$A+B*i+C*\sqrt{i}$	1.38780400E-05	5, 5
$A+B/i$	$A+B*\log_{10}(i)^4$	1.55098500E-05	6, 7
$A+B*i+C*\sqrt{i}$	$A+B*\sqrt{i}+C*i$	2.24508000E-05	5, 4
$A+B*\sqrt{i}$	$A+B*i$	2.49809800E-05	7, 5
$A+B*i$	$A+B*\sqrt{i}$	2.57612400E-05	6, 4
$A+B/i$	$A+B*i$	2.59343400E-05	5, 7
$A+B*\log_{10}(i)^4$	$A+B/i$	3.01782100E-05	3, 7
$A+B*i$	$A+B*\log_{10}(i)^4$	3.47428200E-05	4, 5
$A+B*i$	$A+B/i$	3.78413200E-05	4, 4
$A+B*\log_{10}(i)^4$	$A+B*i$	4.33841600E-03	6, 4

RESULTS FOR THE COSINE INTEGRAL

Combo Powers 1

Fitting $Ci(x)$ in range (0.500000, 20.000000)

Shammas polynomial power for $Q(x)$ is $A+B*i$

Shammas polynomial power for $D(x)$ is $A+B/i$

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-7.60940244115039e-08	0	-Inf	0
x1	2.68258261039838e-07	0	Inf	0
x2	-3.53408230405702e-07	0	-Inf	0
x3	2.06339784025425e-07	0	Inf	0
x4	-4.50704235779868e-08	0	-Inf	0
x5	-0.00676697968839331	0	-Inf	0
x6	1.20230133367886	0	Inf	0
x7	-18.9033062626707	0	-Inf	0
x8	81.0451086152781	0	Inf	0
x9	-124.950079046857	0	-Inf	0
x10	62.6127423402367	0	Inf	0

Number of observations: 196, Error degrees of freedom: 185

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	8.91942317983705	195	0.0457406316914721		
Model	8.91942317983705	10	0.891942317983705	Inf	0
Residual	0	185	0		

```

for Q(x) A = 0.000000, B = 0.500000
for D(x) A = 0.000000, B = 0.575345
order Q(x) = 4.000000, order D(x) = 6.000000
Xmin = 0.500000 and Xmax = 20.000000
Ymin = -0.198391 and Ymax = 0.471733
List of powers for Q(x): [0.500000, 1.000000, 1.500000, 2.000000]
List of powers for D(x): [0.575345, 0.287673, 0.191782, 0.143836, 0.115069, 0.095891]
Fitting Ci(x) in range (1.000000, 2.000000)
MSS of errors squared = 1.675476e-01
Corrected MSS of errors squared = 3.350953e-01
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 2

```

Fitting Ci(x) in range (0.500000, 20.000000)
Shammas polynomial power for Q(x) is A+B*i
Shammas polynomial power for D(x) is A+B*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	5.93557572362477e-10	0	Inf	0
x1	-1.71365394662884e-09	0	-Inf	0
x2	1.57415169024943e-09	0	Inf	0
x3	-4.27531584547055e-10	0	-Inf	0
x4	47.6733266704446	0	Inf	0
x5	-275.030905799577	0	-Inf	0
x6	646.282584515218	0	Inf	0
x7	-750.40233429865	0	-Inf	0
x8	429.713905706261	0	Inf	0
x9	-97.2365767937163	0	-Inf	0

Number of observations: 196, Error degrees of freedom: 186

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	8.91942317982826	195	0.045740631691427		
Model	8.91942317982826	9	0.991047019980917	Inf	0
Residual	0	186	0		

```

for Q(x) A = 0.500000, B = 0.883679
for D(x) A = 0.000000, B = 0.100000
order Q(x) = 3.000000, order D(x) = 6.000000
Xmin = 0.500000 and Xmax = 20.000000
Ymin = -0.198391 and Ymax = 0.471733
List of powers for Q(x): [1.383679, 2.267358, 3.151037]
List of powers for D(x): [0.100000, 0.141421, 0.173205, 0.200000, 0.223607, 0.244949]
Fitting Ci(x) in range (1.000000, 2.000000)
MSS of errors squared = 2.595621e-02
Corrected MSS of errors squared = 5.191243e-02
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 3

```

Fitting Ci(x) in range (0.500000, 20.000000)
Shammas polynomial power for Q(x) is A+B*sqrt(i)+C*i
Shammas polynomial power for D(x) is A+B*i+C*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-1.86124089998194e-05	0	-Inf	0
x1	0.00010374058683364	0	Inf	0
x2	-0.000131252352979312	0	-Inf	0
x3	4.62395715054113e-05	0	Inf	0
x4	7.99743128408037	0	Inf	0
x5	-19.4584849273279	0	-Inf	0
x6	21.4670858982511	0	Inf	0
x7	-11.3495237290221	0	-Inf	0
x8	2.3434913588007	0	Inf	0

Number of observations: 196, Error degrees of freedom: 187

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	8.91942317983569	195	0.0457406316914651		
Model	8.91942317983569	8	1.11492789747946	Inf	0
Residual	0	187	0		

```

for Q(x) A = 0.541062, B = 0.400000, C = 0.400000
for D(x) A = 0.000000, B = 0.100000, C = 0.098375
order Q(x) = 3.000000, order D(x) = 5.000000
Xmin = 0.500000 and Xmax = 20.000000
Ymin = -0.198391 and Ymax = 0.471733
List of powers for Q(x): [1.341062, 1.906747, 2.433882]
List of powers for D(x): [0.198375, 0.339123, 0.470390, 0.596750, 0.719973]
Fitting Ci(x) in range (1.000000, 2.000000)
MSS of errors squared = 1.233734e-01
Corrected MSS of errors squared = 3.022020e-01
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 4

```

Fitting Ci(x) in range (0.500000, 20.000000)
Shammas polynomial power for Q(x) is A+B/i
Shammas polynomial power for D(x) is A+B*i

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.000108544402652295	0	-Inf	0
x1	0.000730102890771419	0	Inf	0
x2	-0.00389286466429082	0	-Inf	0
x3	0.00327142794461519	0	Inf	0
x4	2788.37394429027	0	Inf	0
x5	-14369.9797284107	0	-Inf	0
x6	31197.6248901545	0	Inf	0
x7	-36448.6238244648	0	-Inf	0
x8	24132.4828588099	0	Inf	0
x9	-8575.41737578515	0	-Inf	0
x10	1276.53923529747	0	Inf	0

Number of observations: 196, Error degrees of freedom: 185

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	8.9194231797493	195	0.045740631691022		
Model	8.9194231797493	10	0.89194231797493	Inf	0
Residual	0	185	0		

```

for Q(x) A = 0.327907, B = 0.284718
for D(x) A = 0.832482, B = 0.110681
order Q(x) = 3.000000, order D(x) = 7.000000
Xmin = 0.500000 and Xmax = 20.000000
Ymin = -0.198391 and Ymax = 0.471733
List of powers for Q(x): [0.612625, 0.470266, 0.422813]
List of powers for D(x): [0.943163, 1.053844, 1.164525, 1.275205, 1.385886, 1.496567,
1.607247]
Fitting Ci(x) in range (1.000000, 2.000000)
MSS of errors squared = 1.251381e-02
Corrected MSS of errors squared = 2.502763e-02
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 5

```

Fitting Ci(x) in range (0.500000, 20.000000)
Shammas polynomial power for Q(x) is A+B*sqrt(i)
Shammas polynomial power for D(x) is A+B*i

```

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00642628704622528	0	Inf	0
x1	-0.347647054538342	0	-Inf	0
x2	1.47307940668159	0	Inf	0
x3	-2.38343446464273	0	-Inf	0
x4	1.7072235392348	0	Inf	0
x5	-0.455647585486546	0	-Inf	0
x6	2.97870661530154	0	Inf	0
x7	-2.95752189986639	0	-Inf	0
x8	0.97881519612954	0	Inf	0

Number of observations: 196, Error degrees of freedom: 187

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	8.91942317983623	195	0.0457406316914678		
Model	8.91942317983623	8	1.11492789747953	Inf	0
Residual	0	187	0		

```

for Q(x) A = 0.231060, B = 0.500000
for D(x) A = 0.000000, B = 0.025913
order Q(x) = 5.000000, order D(x) = 3.000000
Xmin = 0.500000 and Xmax = 20.000000
Ymin = -0.198391 and Ymax = 0.471733
List of powers for Q(x): [0.731060, 0.938167, 1.097085, 1.231060, 1.349094]
List of powers for D(x): [0.025913, 0.051825, 0.077738]
Fitting Ci(x) in range (1.000000, 2.000000)
MSS of errors squared = 1.333929e-01
Corrected MSS of errors squared = 2.667857e-01
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 6

```

Fitting Ci(x) in range (0.500000, 20.000000)
Shammas polynomial power for Q(x) is A+B*i+C*sqrt(i)
Shammas polynomial power for D(x) is A+B*sqrt(i)+C*i

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.0130258169588526	0	-Inf	0
x1	0.0879714481518106	0	Inf	0
x2	-0.190143362698627	0	-Inf	0
x3	0.187836596956006	0	Inf	0
x4	-0.089208989192349	0	-Inf	0
x5	0.0165706252551328	0	Inf	0
x6	209.089317659929	0	Inf	0
x7	-1266.16184498669	0	-Inf	0
x8	3159.17920270166	0	Inf	0
x9	-4155.13969422487	0	-Inf	0
x10	3048.00126802144	0	Inf	0
x11	-1185.04292395183	0	-Inf	0
x12	191.074674312528	0	Inf	0

Number of observations: 196, Error degrees of freedom: 183

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

SumSq	DF	MeanSq	F	pValue

Total	8.91942317978855	195	0.0457406316912233		
Model	8.91942317978855	12	0.743285264982379	Inf	0
Residual	0	183	0		

```

for Q(x) A = 0.000000, B = 0.390000, C = 0.360000
for D(x) A = 0.550000, B = 0.400000, C = 0.100726
order Q(x) = 5.000000, order D(x) = 7.000000
Xmin = 0.500000 and Xmax = 20.000000
Ymin = -0.198391 and Ymax = 0.471733
List of powers for Q(x): [0.750000, 1.289117, 1.793538, 2.280000, 2.754984]
List of powers for D(x): [1.050726, 1.317137, 1.544998, 1.752904, 1.948057, 2.134151,
2.313382]
Fitting Ci(x) in range (1.000000, 2.000000)
MSS of errors squared = 1.162708e-01
Corrected MSS of errors squared = 2.848042e-01
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 7

```

Fitting Ci(x) in range (0.500000, 20.000000)
Shammas polynomial power for Q(x) is A+B*i
Shammas polynomial power for D(x) is A+B*log10(i)^4

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	1.60278907604882e-09	0	Inf	0
x1	-3.96485959857353e-07	0	-Inf	0
x2	1.36647414521737e-06	0	Inf	0
x3	-1.50544973798582e-06	0	-Inf	0
x4	0	0	NaN	NaN
x5	1.2228057693986e-06	0	Inf	0
x6	-8.9529818537514e-07	0	-Inf	0
x7	2.06351177636896e-07	0	Inf	0
x8	1.00000011380591	0	Inf	0
x9	-1.51471716101589e-07	0	-Inf	0
x10	4.97785548260545e-08	0	Inf	0
x11	-1.48502632381543e-08	0	-Inf	0
x12	3.03471222268562e-09	0	Inf	0
x13	-2.97199887390862e-10	0	-Inf	0

Number of observations: 196, Error degrees of freedom: 183

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	8.91942317983629	195	0.0457406316914682		
Model	8.91942317983629	12	0.743285264986358	Inf	0
Residual	0	183	0		

for Q(x) A = 0.424932, B = 0.100000

for D(x) A = 0.000000, B = 3.492416

order Q(x) = 7.000000, order D(x) = 6.000000

Xmin = 0.500000 and Xmax = 20.000000

Ymin = -0.198391 and Ymax = 0.471733

List of powers for Q(x): [0.524932, 0.624932, 0.724932, 0.824932, 0.924932, 1.024932, 1.124932]

List of powers for D(x): [0.000000, 0.028679, 0.180984, 0.458865, 0.833605, 1.280507]

Fitting Ci(x) in range (1.000000, 2.000000)

MSS of errors squared = 4.459596e-02

Corrected MSS of errors squared = 8.919192e-02

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 8

Fitting Ci(x) in range (0.500000, 20.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14}$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-2.14813888597007e-06	0	-Inf	0
x1	4.53924402630709e-08	0	Inf	0
x2	-1.87679084446454e-05	0	-Inf	0
x3	0.000496807176512648	0	Inf	0
x4	-0.00341280817117285	0	-Inf	0
x5	0.00897259905093076	0	Inf	0
x6	-0.00989687829985957	0	-Inf	0
x7	0.00386115090826229	0	Inf	0
x8	47.0448467880923	0	Inf	0
x9	0	0	NaN	NaN
x10	-101.375207259845	0	-Inf	0
x11	84.6788552783609	0	Inf	0

x12	-37.8522149934007	0	-Inf	0
x13	9.58709514519	0	Inf	0
x14	-1.08337495840657	0	-Inf	0

Number of observations: 196, Error degrees of freedom: 182

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	8.91942317983374	195	0.0457406316914551		
Model	8.91942317983374	13	0.686109475371826	Inf	0
Residual	0	182	0		

for Q(x) A = 0.497141, B = 3.500000

for D(x) A = 0.100000, B = 0.500000

order Q(x) = 7.000000, order D(x) = 7.000000

Xmin = 0.500000 and Xmax = 20.000000

Ymin = -0.198391 and Ymax = 0.471733

List of powers for Q(x): [3.997141, 2.247141, 1.663808, 1.372141, 1.197141, 1.080474, 0.997141]

List of powers for D(x): [0.100000, 0.104106, 0.125911, 0.165695, 0.219345, 0.283327, 0.355034]

Fitting Ci(x) in range (1.000000, 2.000000)

MSS of errors squared = 6.950822e-02

Corrected MSS of errors squared = 1.390164e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 9

Fitting Ci(x) in range (0.500000, 20.000000)

Shammas polynomial power for Q(x) is $A+B*\log_{10}(i)^4$

Shammas polynomial power for D(x) is $A+B*i$

Linear regression model:

$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	3.86546667774576e-14	0	Inf	0
x1	-1.08528099234789e-07	0	-Inf	0
x2	1.37281835628108e-07	0	Inf	0
x3	-3.33159215830913e-08	0	-Inf	0
x4	4.5621447507907e-09	0	Inf	0

x5	1	0	Inf	0
x6	0	0	NaN	NaN
x7	0	0	NaN	NaN
x8	0	0	NaN	NaN

Number of observations: 196, Error degrees of freedom: 190

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	8.9194231798363	195	0.0457406316914682		
Model	8.9194231798363	5	1.78388463596726	Inf	0
Residual	0	190	0		

for Q(x) A = 2.074069, B = 0.348646

for D(x) A = 0.000000, B = 0.000000

order Q(x) = 4.000000, order D(x) = 4.000000

Xmin = 0.500000 and Xmax = 20.000000

Ymin = -0.198391 and Ymax = 0.471733

List of powers for Q(x): [2.074069, 2.076932, 2.092137, 2.119878]

List of powers for D(x): [0.000000, 0.000000, 0.000000, 0.000000]

Fitting Ci(x) in range (1.000000, 2.000000)

MSS of errors squared = 2.238553e-02

Corrected MSS of errors squared = 4.477106e-02

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 10

Fitting Ci(x) in range (0.500000, 20.000000)

Shammas polynomial power for Q(x) is A+B*log10(i)^4

Shammas polynomial power for D(x) is A+B/i

Linear regression model:

$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-9.11026194485154e-06	0	-Inf	0
x1	0.137907509974698	0	Inf	0
x2	-0.163315429128539	0	-Inf	0
x3	0.0254170514888443	0	Inf	0
x4	0.0221419723177146	0	Inf	0
x5	-10.5420317377464	0	-Inf	0

x6	297.265312038423	0	Inf	0
x7	-2107.66474029927	0	-Inf	0
x8	5638.38405545771	0	Inf	0
x9	-6282.42597551951	0	-Inf	0
x10	2465.9612380684	0	Inf	0

Number of observations: 196, Error degrees of freedom: 185

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	8.91942317983653	195	0.0457406316914694		
Model	8.91942317983653	10	0.891942317983653	Inf	0
Residual	0	185	0		

for Q(x) A = 0.610975, B = 0.234797

for D(x) A = 0.366502, B = 1.922585

order Q(x) = 3.000000, order D(x) = 7.000000

Xmin = 0.500000 and Xmax = 20.000000

Ymin = -0.198391 and Ymax = 0.471733

List of powers for Q(x): [0.610975, 0.612903, 0.623142]

List of powers for D(x): [2.289087, 1.327795, 1.007364, 0.847148, 0.751019, 0.686933, 0.641157]

Fitting Ci(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.175684e-02

Corrected MSS of errors squared = 2.351367e-02

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Comments

The following table shows the summary results for the ten models that fit the cosine integral. The results are sorted in ascending order using the MSSE values.

<i>Power Q(x)</i>	<i>Power D(x)</i>	<i>MSSE</i>	<i>Q/D Orders</i>
A+B*log10(i)^4	A+B/i	1.17568400E-02	3, 7
A+B/i	A+B*i	1.25138100E-02	3, 7
A+B*log10(i)^4	A+B*i	2.23855300E-02	4, 4
A+B*i	A+B*sqrt(i)	2.59562100E-02	3, 6
A+B*i	A+B*log10(i)^4	4.45959600E-02	7, 6
A+B/i	A+B*log10(i)^4	6.95082200E-02	7, 7
A+B*i+C*sqrt(i)	A+B*sqrt(i)+C*i	1.16270800E-01	5, 7
A+B*sqrt(i)+C*i	A+B*i+C*sqrt(i)	1.23373400E-01	3, 5
A+B*sqrt(i)	A+B*i	1.33392900E-01	5, 3
A+B*i	A+B/i	1.67547600E-01	4, 6

RESULTS FOR THE COSINE FUNCTION

Combo Powers 1

Fitting cos(x) in range (0.000000, 1.570796)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B/i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-1.25864029651642	0	-Inf	0
x1	18.7763773372301	0	Inf	0
x2	-57.4379355020938	0	-Inf	0
x3	88.9793879605054	0	Inf	0
x4	-76.443045773264	0	-Inf	0
x5	33.7692443005392	0	Inf	0
x6	-5.92578112920122	0	-Inf	0
x7	2.64881703648668	0	Inf	0
x8	-16.7190850419344	0	-Inf	0
x9	14.8404645564668	0	Inf	0

Number of observations: 501, Error degrees of freedom: 491

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	47.6265936125762	500	0.0952531872251524		
Model	47.6265936125762	9	5.29184373473069	Inf	0
Residual	0	491	0		

```

for Q(x) A = 0.064034, B = 0.403843
for D(x) A = 0.843339, B = 0.488280
order Q(x) = 6.000000, order D(x) = 3.000000
Xmin = 0.000000 and Xmax = 1.570796
Ymin = 0.000000 and Ymax = 1.000000
List of powers for Q(x): [0.467877, 0.871719, 1.275562, 1.679405, 2.083247, 2.487090]
List of powers for D(x): [1.331619, 1.087479, 1.006099]
Fitting cos(x) in range (1.000000, 2.000000)
MSS of errors squared = 7.550777e-11
Corrected MSS of errors squared = 1.510155e-10
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 2

```

Fitting cos(x) in range (0.000000, 1.570796)
Shammas polynomial power for Q(x) is A+B*i
Shammas polynomial power for D(x) is A+B*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	1.22144650675538	0	Inf	0
x1	5.99459352654346	0	Inf	0
x2	-2.29288202625886	0	-Inf	0
x3	0.21348637405101	0	Inf	0
x4	-61.9379228369219	0	-Inf	0
x5	234.094457920939	0	Inf	0
x6	-346.64218414226	0	-Inf	0
x7	231.013035240572	0	Inf	0
x8	-58.0957082918142	0	-Inf	0

Number of observations: 501, Error degrees of freedom: 492

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	47.6265936128376	500	0.0952531872256753		
Model	47.6265936128376	8	5.95332420160471	Inf	0
Residual	0	492	0		

```

for Q(x) A = 0.389530, B = 1.533495

```

```

for D(x) A = 1.951051, B = 0.864458
order Q(x) = 3.000000, order D(x) = 5.000000
Xmin = 0.000000 and Xmax = 1.570796
Ymin = 0.000000 and Ymax = 1.000000
List of powers for Q(x): [1.923025, 3.456521, 4.990016]
List of powers for D(x): [2.815509, 3.173580, 3.448337, 3.679968, 3.884039]
Fitting cos(x) in range (1.000000, 2.000000)
MSS of errors squared = 4.670557e-10
Corrected MSS of errors squared = 9.341113e-10
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 3

```

Fitting cos(x) in range (0.000000, 1.570796)
Shammas polynomial power for Q(x) is A+B*sqrt(i)+C*i
Shammas polynomial power for D(x) is A+B*i+C*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.754712604729573	0	Inf	0
x1	-8.57897944695669	0	-Inf	0
x2	22.6793844404531	0	Inf	0
x3	-24.8709839606965	0	-Inf	0
x4	12.5409701885671	0	Inf	0
x5	-2.38189221416888	0	-Inf	0
x6	5.41575112014965	0	Inf	0
x7	-5.92121249309532	0	-Inf	0
x8	0	0	NaN	NaN
x9	1.36942168135781	0	Inf	0
x10	0.531906147225184	0	Inf	0
x11	-0.467472261602765	0	-Inf	0

Number of observations: 501, Error degrees of freedom: 490

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	47.6265936125824	500	0.0952531872251647		
Model	47.6265936125824	10	4.76265936125824	Inf	0
Residual	0	490	0		

```

for Q(x) A = 0.445929, B = 0.102740, C = 0.231976
for D(x) A = 0.261724, B = 0.134281, C = 0.099588
order Q(x) = 5.000000, order D(x) = 6.000000
Xmin = 0.000000 and Xmax = 1.570796
Ymin = 0.000000 and Ymax = 1.000000
List of powers for Q(x): [0.780644, 1.055176, 1.319806, 1.579311, 1.835540]
List of powers for D(x): [0.495593, 0.671125, 0.837059, 0.998024, 1.155815, 1.311350]
Fitting cos(x) in range (1.000000, 2.000000)
MSS of errors squared = 6.466678e-13
Corrected MSS of errors squared = 1.584006e-12
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 4

```

Fitting cos(x) in range (0.000000, 1.570796)
Shammas polynomial power for Q(x) is A+B/i
Shammas polynomial power for D(x) is A+B*i

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-5.10393784324744	0	-Inf	0
x1	18.7903765762544	0	Inf	0
x2	-79.191411382349	0	-Inf	0
x3	0	0	NaN	NaN
x4	65.8297831291961	0	Inf	0
x5	6.87026060588451	0	Inf	0
x6	-14.5477298919443	0	-Inf	0
x7	14.2660284253365	0	Inf	0
x8	-7.72439153042372	0	-Inf	0
x9	2.25285064697551	0	Inf	0
x10	-0.279423496762955	0	-Inf	0

Number of observations: 501, Error degrees of freedom: 491

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	47.6265936125735	500	0.0952531872251469		
Model	47.6265936125735	9	5.29184373473038	Inf	0

Model	47.6265936126095	9	5.29184373473439	Inf	0
Residual	0	491	0		

```

for Q(x) A = 0.905518, B = 0.482809
for D(x) A = 1.013900, B = 0.523274
order Q(x) = 6.000000, order D(x) = 4.000000
Xmin = 0.000000 and Xmax = 1.570796
Ymin = 0.000000 and Ymax = 1.000000
List of powers for Q(x): [1.388327, 1.588313, 1.741768, 1.871136, 1.985112, 2.088154]
List of powers for D(x): [1.537174, 2.060448, 2.583721, 3.106995]
Fitting cos(x) in range (1.000000, 2.000000)
MSS of errors squared = 2.956566e-12
Corrected MSS of errors squared = 5.913132e-12
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 6

```

Fitting cos(x) in range (0.000000, 1.570796)
Shammas polynomial power for Q(x) is A+B*i+C*sqrt(i)
Shammas polynomial power for D(x) is A+B*sqrt(i)+C*i

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.989237207540115	0	Inf	0
x1	-0.312525179840013	0	-Inf	0
x2	0.104044491168401	0	Inf	0
x3	1.56939738663947	0	Inf	0
x4	-3.79531710730322	0	-Inf	0
x5	2.66802747704951	0	Inf	0
x6	-0.590931466992163	0	-Inf	0
x7	2.34059995129358	0	Inf	0
x8	-3.02658243757916	0	-Inf	0
x9	1.90082220388582	0	Inf	0
x10	-0.61322271075926	0	-Inf	0
x11	0.0824165890275928	0	Inf	0

Number of observations: 501, Error degrees of freedom: 489

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

SumSq	DF	MeanSq	F	pValue
-------	----	--------	---	--------

Total	47.6265936125819	500	0.0952531872251638		
Model	47.6265936125819	11	4.32969032841653	Inf	0
Residual	0	489	0		

for Q(x) A = 0.153960, B = 0.355075, C = 0.338069
 for D(x) A = 0.224563, B = 0.383983, C = 0.364428
 order Q(x) = 6.000000, order D(x) = 5.000000
 Xmin = 0.000000 and Xmax = 1.570796
 Ymin = 0.000000 and Ymax = 1.000000
 List of powers for Q(x): [0.847104, 1.342211, 1.804736, 2.250396, 2.685278, 3.112504]
 List of powers for D(x): [0.972974, 1.496453, 1.982925, 2.450241, 2.905314]
 Fitting cos(x) in range (1.000000, 2.000000)
 MSS of errors squared = 3.327676e-14
 Corrected MSS of errors squared = 8.151109e-14
 R-squared = 1.00000000
 R-squared Adjusted = 1.00000000
 Particle swarm AICc = -1.000000e+99
 AIC = -Inf
 AICc = -Inf

Combo Powers 7

Fitting cos(x) in range (0.000000, 1.570796)
 Shammas polynomial power for Q(x) is A+B*i
 Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	3.93180517438785	0	Inf	0
x1	-24.1504834908901	0	-Inf	0
x2	31.4624149944934	0	Inf	0
x3	0	0	NaN	NaN
x4	-19.6371952513863	0	-Inf	0
x5	8.4987880758648	0	Inf	0
x6	166.389732424394	0	Inf	0
x7	0	0	NaN	NaN
x8	-338.963261217054	0	-Inf	0
x9	237.401522361718	0	Inf	0
x10	-72.6367449883273	0	-Inf	0
x11	8.75608666806063	0	Inf	0

Number of observations: 501, Error degrees of freedom: 491

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	47.6265936125874	500	0.0952531872251748		
Model	47.6265936125874	9	5.29184373473193	Inf	0
Residual	0	491	0		

```

for Q(x) A = 0.123987, B = 0.123165
for D(x) A = 0.848418, B = 1.464922
order Q(x) = 5.000000, order D(x) = 6.000000
Xmin = 0.000000 and Xmax = 1.570796
Ymin = 0.000000 and Ymax = 1.000000
List of powers for Q(x): [0.247152, 0.370317, 0.493483, 0.616648, 0.739814]
List of powers for D(x): [0.848418, 0.860447, 0.924333, 1.040893, 1.198080, 1.385537]
Fitting cos(x) in range (1.000000, 2.000000)
MSS of errors squared = 1.772050e-11
Corrected MSS of errors squared = 3.544099e-11
R-squared = 1.000000000
R-squared Adjusted = 1.000000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 8

```

Fitting cos(x) in range (0.000000, 1.570796)
Shammas polynomial power for Q(x) is A+B/i
Shammas polynomial power for D(x) is A+B*log10(i)^4

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.000572618288324292	0	-Inf	0
x1	5.49619252224766e-06	0	Inf	0
x2	2.2914655870459e-05	0	Inf	0
x3	-0.00119150214296533	0	-Inf	0
x4	0.00161013147647944	0	Inf	0
x5	24.9171774093215	0	Inf	0
x6	0	0	NaN	NaN
x7	0	0	NaN	NaN
x8	0	0	NaN	NaN
x9	-98.9104055318872	0	-Inf	0
x10	106.763421273527	0	Inf	0
x11	-31.7701303619107	0	-Inf	0

Number of observations: 501, Error degrees of freedom: 492

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	47.6265936125567	500	0.0952531872251133		
Model	47.6265936125567	8	5.95332420156958	Inf	0
Residual	0	492	0		

for Q(x) A = 0.500000, B = 3.500000

for D(x) A = 0.100000, B = 0.149789

order Q(x) = 4.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 1.570796

Ymin = 0.000000 and Ymax = 1.000000

List of powers for Q(x): [4.000000, 2.250000, 1.666667, 1.375000]

List of powers for D(x): [0.100000, 0.101230, 0.107762, 0.119681, 0.135753, 0.154921, 0.176403]

Fitting cos(x) in range (1.000000, 2.000000)

MSS of errors squared = 2.884520e-08

Corrected MSS of errors squared = 5.769041e-08

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 9

Fitting cos(x) in range (0.000000, 1.570796)

Shammas polynomial power for Q(x) is A+B*log10(i)^4

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.737594988391787	0	Inf	0
x1	1037.12658152846	0	Inf	0
x2	0	0	NaN	NaN
x3	-1688.01667343977	0	-Inf	0
x4	650.90158803719	0	Inf	0
x5	1.45843075199937	0	Inf	0
x6	-1.3477909745093	0	-Inf	0
x7	0.678041018025714	0	Inf	0
x8	-0.186691769098642	0	-Inf	0
x9	0.0234653678928562	0	Inf	0

Number of observations: 501, Error degrees of freedom: 492

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	47.6265936126244	500	0.0952531872252488		
Model	47.6265936126244	8	5.95332420157805	Inf	0
Residual	0	492	0		

for Q(x) A = 1.305465, B = 0.419539

for D(x) A = 0.388522, B = 0.952958

order Q(x) = 4.000000, order D(x) = 5.000000

Xmin = 0.000000 and Xmax = 1.570796

Ymin = 0.000000 and Ymax = 1.000000

List of powers for Q(x): [1.305465, 1.308910, 1.327206, 1.360588]

List of powers for D(x): [1.341480, 2.294438, 3.247395, 4.200353, 5.153311]

Fitting cos(x) in range (1.000000, 2.000000)

MSS of errors squared = 6.060260e-09

Corrected MSS of errors squared = 1.212052e-08

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 10

Fitting cos(x) in range (0.000000, 1.570796)

Shammas polynomial power for Q(x) is A+B*log10(i)^4

Shammas polynomial power for D(x) is A+B/i

Linear regression model:

$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14}$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-136.07803072703	0	-Inf	0
x1	0	0	NaN	NaN
x2	0	0	NaN	NaN
x3	0	0	NaN	NaN
x4	0	0	NaN	NaN
x5	819.813777509187	0	Inf	0
x6	-1051.04814172298	0	-Inf	0
x7	367.484331453532	0	Inf	0
x8	2.17162099965676	0	Inf	0
x9	-108.154579725274	0	-Inf	0
x10	440.100614158171	0	Inf	0
x11	0	0	NaN	NaN

x12	-841.797717819421	0	-Inf	0
x13	0	0	NaN	NaN
x14	508.594094125131	0	Inf	0

Number of observations: 501, Error degrees of freedom: 492

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	47.6265936126396	500	0.0952531872252793		
Model	47.6265936126396	8	5.95332420157995	Inf	0
Residual	0	492	0		

for Q(x) A = 0.003510, B = 0.340888

for D(x) A = 1.220175, B = 1.068953

order Q(x) = 7.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 1.570796

Ymin = 0.000000 and Ymax = 1.000000

List of powers for Q(x): [0.003510, 0.006309, 0.021176, 0.048299, 0.084876, 0.128498, 0.177386]

List of powers for D(x): [2.289128, 1.754651, 1.576492, 1.487413, 1.433965, 1.398333, 1.372882]

Fitting cos(x) in range (1.000000, 2.000000)

MSS of errors squared = 3.262457e-09

Corrected MSS of errors squared = 6.524914e-09

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Comments

The following table shows the summary results for the ten models that fit the cosine function. The results are sorted in ascending order using the MSSE values.

<i>Power Q(x)</i>	<i>Power D(x)</i>	<i>MSSE</i>	<i>Q/D Orders</i>
$A+B*i+C*\sqrt{i}$	$A+B*\sqrt{i}+C*i$	3.32767600E-14	6, 5
$A+B*\sqrt{i}+C*i$	$A+B*i+C*\sqrt{i}$	6.46667800E-13	5, 6
$A+B*\sqrt{i}$	$A+B*i$	2.95656600E-12	6, 4
$A+B*i$	$A+B*\log_{10}(i)^4$	1.77205000E-11	5, 6
$A+B*i$	$A+B/i$	7.55077700E-11	6, 3
$A+B/i$	$A+B*i$	2.00006000E-10	4, 6
$A+B*i$	$A+B*\sqrt{i}$	4.67055700E-10	3, 5
$A+B*\log_{10}(i)^4$	$A+B/i$	3.26245700E-09	7, 7
$A+B*\log_{10}(i)^4$	$A+B*i$	6.06026000E-09	4, 5
$A+B/i$	$A+B*\log_{10}(i)^4$	2.88452000E-08	4, 7

RESULTS FOR THE HYPERBOLIC COSINE FUNCTION

Combo Powers 1

Fitting $\cosh(x)$ in range (0.000000, 5.000000)

Shammas polynomial power for Q(x) is $A+B*i$

Shammas polynomial power for D(x) is $A+B/i$

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	1.69810295986938	0	Inf	0
x1	-3.65464408386004	0	-Inf	0
x2	0.0656287570095125	0	Inf	0
x3	3.45911571835912	0	Inf	0
x4	-2.00149147965577	0	-Inf	0
x5	0.265422937608027	0	Inf	0
x6	-0.468889592700088	0	-Inf	0
x7	16.4431298799492	0	Inf	0
x8	-54.81387490937	0	-Inf	0
x9	40.0074997733007	0	Inf	0

Number of observations: 501, Error degrees of freedom: 491

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	31.2187326156277	500	0.0624374652312554		
Model	31.2187326156277	9	3.46874806840308	Inf	0
Residual	0	491	0		

```

for Q(x) A = 0.018139, B = 0.490446
for D(x) A = 0.789580, B = 1.885187
order Q(x) = 5.000000, order D(x) = 4.000000
Xmin = 0.000000 and Xmax = 5.000000
Ymin = 1.000000 and Ymax = 74.209949
List of powers for Q(x): [0.508585, 0.999031, 1.489478, 1.979924, 2.470370]
List of powers for D(x): [2.674767, 1.732174, 1.417976, 1.260877]
Fitting cosh(x) in range (1.000000, 2.000000)
MSS of errors squared = 3.850692e-09
Corrected MSS of errors squared = 7.701384e-09
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 2

```

Fitting cosh(x) in range (0.000000, 5.000000)
Shammas polynomial power for Q(x) is A+B*i
Shammas polynomial power for D(x) is A+B*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	2.05838975704338	0	Inf	0
x1	-2.36796657057005	0	-Inf	0
x2	2.08162193405346	0	Inf	0
x3	-0.847004489069138	0	-Inf	0
x4	0.214969167642856	0	Inf	0
x5	-0.0352400350293151	0	-Inf	0
x6	0.00378387284472431	0	Inf	0
x7	-260.031342204523	0	-Inf	0
x8	1004.939632484	0	Inf	0
x9	-1083.14120726034	0	-Inf	0
x10	0	0	NaN	NaN
x11	437.118578631176	0	Inf	0
x12	0	0	NaN	NaN
x13	-98.9942152930768	0	-Inf	0

Number of observations: 501, Error degrees of freedom: 489

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

SumSq	DF	MeanSq	F	pValue

Total	31.218732615803	500	0.0624374652316061		
Model	31.218732615803	11	2.83806660143664	Inf	0
Residual	0	489	0		

```

for Q(x) A = 0.250509, B = 1.737580
for D(x) A = 1.372526, B = 0.728404
order Q(x) = 6.000000, order D(x) = 7.000000
Xmin = 0.000000 and Xmax = 5.000000
Ymin = 1.000000 and Ymax = 74.209949
List of powers for Q(x): [1.988089, 3.725670, 5.463250, 7.200830, 8.938411, 10.675991]
List of powers for D(x): [2.100931, 2.402645, 2.634159, 2.829335, 3.001288, 3.156745,
3.299703]
Fitting cosh(x) in range (1.000000, 2.000000)
MSS of errors squared = 4.817278e-11
Corrected MSS of errors squared = 9.634556e-11
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 3

```

Fitting cosh(x) in range (0.000000, 5.000000)
Shammas polynomial power for Q(x) is A+B*sqrt(i)+C*i
Shammas polynomial power for D(x) is A+B*i+C*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.172206735321211	0	Inf	0
x1	-0.462174436319307	0	-Inf	0
x2	0	0	NaN	NaN
x3	1.889285667748	0	Inf	0
x4	-3.5333017962385	0	-Inf	0
x5	2.97383771299297	0	Inf	0
x6	-1.23522232751895	0	-Inf	0
x7	0.206925847902002	0	Inf	0
x8	15.8893743822059	0	Inf	0
x9	-26.6934279318371	0	-Inf	0
x10	0	0	NaN	NaN
x11	20.0570796317725	0	Inf	0
x12	0	0	NaN	NaN
x13	-13.6235834486456	0	-Inf	0
x14	5.35899996261831	0	Inf	0

Number of observations: 501, Error degrees of freedom: 489

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	31.2187326156274	500	0.0624374652312548		
Model	31.2187326156274	11	2.83806660142067	Inf	0
Residual	0	489	0		

for Q(x) A = 0.000000, B = 0.100000, C = 0.400000

for D(x) A = 0.204007, B = 0.123313, C = 0.000000

order Q(x) = 7.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 5.000000

Ymin = 1.000000 and Ymax = 74.209949

List of powers for Q(x): [0.500000, 0.941421, 1.373205, 1.800000, 2.223607, 2.644949, 3.064575]

List of powers for D(x): [0.327320, 0.450633, 0.573946, 0.697259, 0.820572, 0.943885, 1.067198]

Fitting cosh(x) in range (1.000000, 2.000000)

MSS of errors squared = 3.880142e-11

Corrected MSS of errors squared = 9.504369e-11

R-squared = 1.000000000

R-squared Adjusted = 1.000000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 4

Fitting cosh(x) in range (0.000000, 5.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	8.90027613575414	0	Inf	0
x1	98.3257624966607	0	Inf	0
x2	-1418.5592494845	0	-Inf	0
x3	2696.52585771257	0	Inf	0
x4	0	0	NaN	NaN
x5	-1385.09487152208	0	-Inf	0
x6	6.47956631227636	0	Inf	0
x7	-13.568788301224	0	-Inf	0
x8	13.4645598096816	0	Inf	0
x9	-7.32402897079258	0	-Inf	0

x10	2.09905371584395	0	Inf	0
x11	-0.248137904270104	0	-Inf	0

Number of observations: 501, Error degrees of freedom: 490

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	31.218732615628	500	0.0624374652312559		
Model	31.218732615628	10	3.1218732615628	Inf	0
Residual	0	490	0		

for Q(x) A = 0.481793, B = 0.373780

for D(x) A = 0.537257, B = 0.553071

order Q(x) = 5.000000, order D(x) = 6.000000

Xmin = 0.000000 and Xmax = 5.000000

Ymin = 1.000000 and Ymax = 74.209949

List of powers for Q(x): [0.855573, 0.668683, 0.606386, 0.575238, 0.556549]

List of powers for D(x): [1.090328, 1.643399, 2.196471, 2.749542, 3.302613, 3.855684]

Fitting cosh(x) in range (1.000000, 2.000000)

MSS of errors squared = 2.453496e-11

Corrected MSS of errors squared = 4.906992e-11

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 5

Fitting cosh(x) in range (0.000000, 5.000000)

Shammas polynomial power for Q(x) is A+B*sqrt(i)

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12 + x13 + x14$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	1.53705354211231	0	Inf	0
x1	-79.6230581546005	0	-Inf	0
x2	164.359168191927	0	Inf	0
x3	0	0	NaN	NaN
x4	-114.346075632501	0	-Inf	0
x5	0	0	NaN	NaN
x6	0	0	NaN	NaN
x7	29.047082318078	0	Inf	0

x8	-0.238912745042439	0	-Inf	0
x9	0.433069076228719	0	Inf	0
x10	-0.219940826478742	0	-Inf	0
x11	0.0611691192573972	0	Inf	0
x12	-0.0105998945876797	0	-Inf	0
x13	0.00109691882200982	0	Inf	0
x14	-5.19217906915934e-05	0	-Inf	0

Number of observations: 501, Error degrees of freedom: 489

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	31.2187326156356	500	0.0624374652312711		
Model	31.2187326156356	11	2.83806660142141	Inf	0
Residual	0	489	0		

for Q(x) A = 1.400000, B = 0.500000

for D(x) A = 1.012256, B = 1.659644

order Q(x) = 7.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 5.000000

Ymin = 1.000000 and Ymax = 74.209949

List of powers for Q(x): [1.900000, 2.107107, 2.266025, 2.400000, 2.518034, 2.624745, 2.722876]

List of powers for D(x): [2.671899, 4.331543, 5.991187, 7.650830, 9.310474, 10.970118, 12.629761]

Fitting cosh(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.242797e-10

Corrected MSS of errors squared = 2.485595e-10

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 6

Fitting cosh(x) in range (0.000000, 5.000000)

Shammas polynomial power for Q(x) is A+B*i+C*sqrt(i)

Shammas polynomial power for D(x) is A+B*sqrt(i)+C*i

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	1.40265236596829	0	Inf	0

x1	-7.66891114996069	0	-Inf	0
x2	10.1052058013441	0	Inf	0
x3	-3.8651597874213	0	-Inf	0
x4	12.8062888589918	0	Inf	0
x5	-34.4941771137314	0	-Inf	0
x6	44.0825486965317	0	Inf	0
x7	-32.238577851803	0	-Inf	0
x8	13.2414737631334	0	Inf	0
x9	-2.37134358566568	0	-Inf	0

Number of observations: 501, Error degrees of freedom: 491

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	31.2187326156292	500	0.0624374652312584		
Model	31.2187326156292	9	3.46874806840325	Inf	0
Residual	0	491	0		

for Q(x) A = 0.052139, B = 0.123988, C = 0.167805

for D(x) A = 0.202372, B = 0.164579, C = 0.207809

order Q(x) = 3.000000, order D(x) = 6.000000

Xmin = 0.000000 and Xmax = 5.000000

Ymin = 1.000000 and Ymax = 74.209949

List of powers for Q(x): [0.343931, 0.537426, 0.714749]

List of powers for D(x): [0.574760, 0.850740, 1.110858, 1.362765, 1.609425, 1.852359]

Fitting cosh(x) in range (1.000000, 2.000000)

MSS of errors squared = 5.445209e-09

Corrected MSS of errors squared = 1.333798e-08

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 7

Fitting cosh(x) in range (0.000000, 5.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	9.80549483713426e-08	0	Inf	0

x1	-8.2849681285813e-07	0	-Inf	0
x2	1.66789882966883e-06	0	Inf	0
x3	-1.45562591247211e-06	0	-Inf	0
x4	6.04470779162916e-07	0	Inf	0
x5	-1.00024873577289e-07	0	-Inf	0
x6	0.999982229038082	0	Inf	0
x7	0	0	NaN	NaN
x8	0	0	NaN	NaN
x9	8.02156954237067e-05	0	Inf	0
x10	-0.000109136827454577	0	-Inf	0
x11	5.81248471448726e-05	0	Inf	0
x12	-1.14190301602735e-05	0	-Inf	0

Number of observations: 501, Error degrees of freedom: 490

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	31.2187326156271	500	0.0624374652312542		
Model	31.2187326156271	10	3.12187326156271	Inf	0
Residual	0	490	0		

for Q(x) A = 0.000000, B = 0.448205

for D(x) A = 0.000000, B = 1.124064

order Q(x) = 5.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 5.000000

Ymin = 1.000000 and Ymax = 74.209949

List of powers for Q(x): [0.448205, 0.896410, 1.344616, 1.792821, 2.241026]

List of powers for D(x): [0.000000, 0.009231, 0.058251, 0.147690, 0.268303, 0.412142, 0.573350]

Fitting cosh(x) in range (1.000000, 2.000000)

MSS of errors squared = 5.236440e-09

Corrected MSS of errors squared = 1.047288e-08

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 8

Fitting cosh(x) in range (0.000000, 5.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12 + x13 + x14$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.769732522872825	0	Inf	0
x1	-0.0203559538192121	0	-Inf	0
x2	2.37340961864582	0	Inf	0
x3	-18.0727366047959	0	-Inf	0
x4	26.5015366454271	0	Inf	0
x5	0	0	NaN	NaN
x6	0	0	NaN	NaN
x7	-11.5624250289374	0	-Inf	0
x8	89.7028462843379	0	Inf	0
x9	0	0	NaN	NaN
x10	0	0	NaN	NaN
x11	0	0	NaN	NaN
x12	-389.913215655887	0	-Inf	0
x13	432.021415743877	0	Inf	0
x14	-130.800207573003	0	-Inf	0

Number of observations: 501, Error degrees of freedom: 491

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	31.218732615626	500	0.0624374652312519		
Model	31.218732615626	9	3.46874806840288	Inf	0
Residual	0	491	0		

for Q(x) A = 0.239890, B = 1.756489

for D(x) A = 0.581924, B = 0.436845

order Q(x) = 7.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 5.000000

Ymin = 1.000000 and Ymax = 74.209949

List of powers for Q(x): [1.996378, 1.118134, 0.825386, 0.679012, 0.591188, 0.532638, 0.490817]

List of powers for D(x): [0.581924, 0.585511, 0.604562, 0.639321, 0.686195, 0.742095, 0.804745]

Fitting cosh(x) in range (1.000000, 2.000000)

MSS of errors squared = 4.415755e-09

Corrected MSS of errors squared = 8.831511e-09

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 9

Fitting cosh(x) in range (0.000000, 5.000000)

Shammas polynomial power for $Q(x)$ is $A+B*\log_{10}(i)^4$

Shammas polynomial power for $D(x)$ is $A+B*i$

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0	0	NaN	NaN
x1	220.984497428449	0	Inf	0
x2	0	0	NaN	NaN
x3	0	0	NaN	NaN
x4	-737.171422264258	0	-Inf	0
x5	722.958536241963	0	Inf	0
x6	-206.762732177237	0	-Inf	0
x7	894.40961593367	0	Inf	0
x8	-3606.85263866812	0	-Inf	0
x9	5875.1066548184	0	Inf	0
x10	-4826.5045259019	0	-Inf	0
x11	1998.04389909918	0	Inf	0
x12	-333.21188451364	0	-Inf	0

Number of observations: 501, Error degrees of freedom: 491

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	31.2187326152927	500	0.0624374652305853		
Model	31.2187326152927	9	3.46874806836585	Inf	0
Residual	0	491	0		

for $Q(x)$ $A = 0.000000$, $B = 0.500000$

for $D(x)$ $A = 1.352630$, $B = 0.142480$

order $Q(x) = 6.000000$, order $D(x) = 6.000000$

Xmin = 0.000000 and Xmax = 5.000000

Ymin = 1.000000 and Ymax = 74.209949

List of powers for $Q(x)$: [0.000000, 0.004106, 0.025911, 0.065695, 0.119345, 0.183327]

List of powers for $D(x)$: [1.495110, 1.637589, 1.780069, 1.922549, 2.065028, 2.207508]

Fitting cosh(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.894453e-07

Corrected MSS of errors squared = 3.788907e-07

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 10

Fitting $\cosh(x)$ in range (0.000000, 5.000000)

Shammas polynomial power for $Q(x)$ is $A+B*\log_{10}(i)^4$

Shammas polynomial power for $D(x)$ is $A+B/i$

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.00265721297344983	0	-Inf	0
x1	-0.403440241754443	0	-Inf	0
x2	0	0	NaN	NaN
x3	0.68749131633825	0	Inf	0
x4	-0.28233845118247	0	-Inf	0
x5	-0.00589469366420566	0	-Inf	0
x6	0.953731136047132	0	Inf	0
x7	-13.8309353959632	0	-Inf	0
x8	52.6138969928236	0	Inf	0
x9	-60.9345939134351	0	-Inf	0
x10	0	0	NaN	NaN
x11	22.2047404636504	0	Inf	0

Number of observations: 501, Error degrees of freedom: 491

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	31.2187326156287	500	0.0624374652312573		
Model	31.2187326156287	9	3.46874806840318	Inf	0
Residual	0	491	0		

for $Q(x)$ $A = 1.216827$, $B = 0.493284$

for $D(x)$ $A = 0.000000$, $B = 2.000000$

order $Q(x) = 4.000000$, order $D(x) = 7.000000$

Xmin = 0.000000 and Xmax = 5.000000

Ymin = 1.000000 and Ymax = 74.209949

List of powers for $Q(x)$: [1.216827, 1.220878, 1.242390, 1.281640]

List of powers for $D(x)$: [2.000000, 1.000000, 0.666667, 0.500000, 0.400000, 0.333333, 0.285714]

Fitting $\cosh(x)$ in range (1.000000, 2.000000)

MSS of errors squared = 3.845795e-09

Corrected MSS of errors squared = 7.691590e-09

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

```
Particle swarm AICc = -1.000000e+99  
AIC = -Inf  
AICc = -Inf
```

Comments

The following table shows the summary results for the ten models that fit the hyperbolic cosine function. The results are sorted in ascending order using the MSSE values.

<i>Power Q(x)</i>	<i>Power D(x)</i>	<i>MSSE</i>	<i>Q/D Orders</i>
A+B/i	A+B*i	2.45349600E-11	5, 6
A+B*sqrt(i)+C*i	A+B*i+C*sqrt(i)	3.88014200E-11	7, 7
A+B*i	A+B*sqrt(i)	4.81727800E-11	6, 7
A+B*sqrt(i)	A+B*i	1.24279700E-10	7, 7
A+B*log10(i)^4	A+B/i	3.84579500E-09	4, 7
A+B*i	A+B/i	3.85069200E-09	5, 4
A+B/i	A+B*log10(i)^4	4.41575500E-09	7, 7
A+B*i	A+B*log10(i)^4	5.23644000E-09	5, 7
A+B*i+C*sqrt(i)	A+B*sqrt(i)+C*i	5.44520900E-09	3, 6
A+B*log10(i)^4	A+B*i	1.89445300E-07	6, 6

RESULTS FOR THE DIGAMMA FUNCTION

Combo Powers 1

Fitting digamma(x) in range (2.000000, 100.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B/i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.519544003403964	0	-Inf	0
x1	61.8266831319209	0	Inf	0
x2	-183.035144030482	0	-Inf	0
x3	183.719980892858	0	Inf	0
x4	-61.9922052029121	0	-Inf	0
x5	-633.682268407853	0	-Inf	0
x6	9124.59160068784	0	Inf	0
x7	-16801.0737421226	0	-Inf	0
x8	0	0	NaN	NaN
x9	8311.16463898504	0	Inf	0

Number of observations: 981, Error degrees of freedom: 972

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	41.0603113045434	980	0.0418982768413708		
Model	41.0603113045434	8	5.13253891306792	Inf	0
Residual	0	972	0		

```

for Q(x) A = 0.348137, B = 0.185956
for D(x) A = 0.962416, B = 0.100000
order Q(x) = 4.000000, order D(x) = 5.000000
Xmin = 2.000000 and Xmax = 100.000000
Ymin = 0.422784 and Ymax = 4.600162
List of powers for Q(x): [0.534093, 0.720050, 0.906006, 1.091963]
List of powers for D(x): [1.062416, 1.012416, 0.995749, 0.987416, 0.982416]
Fitting digamma(x) in range (1.000000, 2.000000)
MSS of errors squared = 4.645257e-07
Corrected MSS of errors squared = 9.290514e-07
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 2

```

Fitting digamma(x) in range (2.000000, 100.000000)
Shammas polynomial power for Q(x) is A+B*i
Shammas polynomial power for D(x) is A+B*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.3443633951047	0	-Inf	0
x1	26.3115827069005	0	Inf	0
x2	-111.266482808121	0	-Inf	0
x3	181.95030909552	0	Inf	0
x4	-134.574944254676	0	-Inf	0
x5	38.9832843868274	0	Inf	0
x6	-1.059229155682	0	-Inf	0
x7	77.9769988979243	0	Inf	0
x8	-74.5957821232653	0	-Inf	0
x9	-167.680254735388	0	-Inf	0
x10	0	0	NaN	NaN
x11	415.626914986668	0	Inf	0
x12	-250.328033600159	0	-Inf	0

Number of observations: 981, Error degrees of freedom: 969

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	41.0603113011163	980	0.0418982768378737		

Model	41.0603113011163	11	3.73275557282875	Inf	0
Residual	0	969	0		

```

for Q(x) A = 0.219833, B = 0.568197
for D(x) A = 1.079434, B = 0.621775
order Q(x) = 6.000000, order D(x) = 6.000000
Xmin = 2.000000 and Xmax = 100.000000
Ymin = 0.422784 and Ymax = 4.600162
List of powers for Q(x): [0.788031, 1.356228, 1.924426, 2.492623, 3.060820, 3.629018]
List of powers for D(x): [1.701209, 1.958757, 2.156380, 2.322984, 2.469766, 2.602466]
Fitting digamma(x) in range (1.000000, 2.000000)
MSS of errors squared = 1.302704e-08
Corrected MSS of errors squared = 2.605408e-08
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 3

```

Fitting digamma(x) in range (2.000000, 100.000000)
Shammas polynomial power for Q(x) is A+B*sqrt(i)+C*i
Shammas polynomial power for D(x) is A+B*i+C*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.645766915363556	0	Inf	0
x1	-12.2656556086506	0	-Inf	0
x2	32.9707614563207	0	Inf	0
x3	-27.0083731621824	0	-Inf	0
x4	0	0	NaN	NaN
x5	6.12176248559594	0	Inf	0
x6	0	0	NaN	NaN
x7	-0.464261439962956	0	-Inf	0
x8	5.75152269451178	0	Inf	0
x9	-7.6455950843903	0	-Inf	0
x10	1.90096904159872	0	Inf	0
x11	0	0	NaN	NaN
x12	2.49387525794039	0	Inf	0
x13	-1.50077255612725	0	-Inf	0

Number of observations: 981, Error degrees of freedom: 970

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	41.0603113009361	980	0.0418982768376899		
Model	41.0603113009361	10	4.10603113009361	Inf	0
Residual	0	970	0		

for Q(x) A = 0.026261, B = 0.400000, C = 0.000000
 for D(x) A = 0.024494, B = 0.100000, C = 0.095997
 order Q(x) = 7.000000, order D(x) = 6.000000
 Xmin = 2.000000 and Xmax = 100.000000
 Ymin = 0.422784 and Ymax = 4.600162
 List of powers for Q(x): [0.426261, 0.591946, 0.719081, 0.826261, 0.920688, 1.006057, 1.084561]
 List of powers for D(x): [0.220491, 0.360254, 0.490765, 0.616488, 0.739149, 0.859637]
 Fitting digamma(x) in range (1.000000, 2.000000)
 MSS of errors squared = 3.938722e-08
 Corrected MSS of errors squared = 9.647858e-08
 R-squared = 1.00000000
 R-squared Adjusted = 1.00000000
 Particle swarm AICc = -1.000000e+99
 AIC = -Inf
 AICc = -Inf

Combo Powers 4

Fitting digamma(x) in range (2.000000, 100.000000)
 Shammas polynomial power for Q(x) is A+B/i
 Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	19.0658596057256	0	Inf	0
x1	-44.6160341315527	0	-Inf	0
x2	261.58518462087	0	Inf	0
x3	-236.035370145683	0	-Inf	0
x4	86.6549544418859	0	Inf	0
x5	-255.584435204031	0	-Inf	0
x6	286.830909204881	0	Inf	0
x7	-144.578387533851	0	-Inf	0
x8	27.6773189696357	0	Inf	0

Number of observations: 981, Error degrees of freedom: 972

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	41.0603113009261	980	0.0418982768376797		
Model	41.0603113009261	8	5.13253891261577	Inf	0
Residual	0	972	0		

for Q(x) A = 0.161272, B = 0.407223
 for D(x) A = 0.657624, B = 0.162552
 order Q(x) = 3.000000, order D(x) = 5.000000
 Xmin = 2.000000 and Xmax = 100.000000
 Ymin = 0.422784 and Ymax = 4.600162
 List of powers for Q(x): [0.568496, 0.364884, 0.297013]
 List of powers for D(x): [0.820176, 0.982728, 1.145280, 1.307832, 1.470384]
 Fitting digamma(x) in range (1.000000, 2.000000)
 MSS of errors squared = 7.694990e-07
 Corrected MSS of errors squared = 1.538998e-06
 R-squared = 1.00000000
 R-squared Adjusted = 1.00000000
 Particle swarm AICc = -1.000000e+99
 AIC = -Inf
 AICc = -Inf

Combo Powers 5

Fitting digamma(x) in range (2.000000, 100.000000)
 Shammas polynomial power for Q(x) is A+B*sqrt(i)
 Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.192830411681863	0	Inf	0
x1	75.9056910968123	0	Inf	0
x2	-220.591121053968	0	-Inf	0
x3	0	0	NaN	NaN
x4	332.09518622942	0	Inf	0
x5	0	0	NaN	NaN
x6	-348.335402154567	0	-Inf	0
x7	160.732676630084	0	Inf	0
x8	3.51870844274105	0	Inf	0
x9	-4.21511947385082	0	-Inf	0
x10	1.88463160636314	0	Inf	0
x11	-0.188081755515028	0	-Inf	0

Number of observations: 981, Error degrees of freedom: 971

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	41.0603113008786	980	0.0418982768376312		
Model	41.0603113008786	9	4.56225681120873	Inf	0
Residual	0	971	0		

for Q(x) A = 0.358158, B = 0.402858

for D(x) A = 0.012701, B = 0.524121

order Q(x) = 7.000000, order D(x) = 4.000000

Xmin = 2.000000 and Xmax = 100.000000

Ymin = 0.422784 and Ymax = 4.600162

List of powers for Q(x): [0.761015, 0.927884, 1.055927, 1.163873, 1.258975, 1.344953, 1.424019]

List of powers for D(x): [0.536822, 1.060943, 1.585063, 2.109184]

Fitting digamma(x) in range (1.000000, 2.000000)

MSS of errors squared = 2.245411e-07

Corrected MSS of errors squared = 4.490822e-07

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 6

Fitting digamma(x) in range (2.000000, 100.000000)

Shammas polynomial power for Q(x) is A+B*i+C*sqrt(i)

Shammas polynomial power for D(x) is A+B*sqrt(i)+C*i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	1.50402380264851	0	Inf	0
x1	-6.63280881556371	0	-Inf	0
x2	7.80601070867322	0	Inf	0
x3	0	0	NaN	NaN
x4	-4.71488652265481	0	-Inf	0
x5	2.0376660818056	0	Inf	0
x6	11.2116592069575	0	Inf	0
x7	-30.866360897388	0	-Inf	0
x8	40.8828529010979	0	Inf	0
x9	-32.759943498498	0	-Inf	0
x10	17.3605543682734	0	Inf	0
x11	-5.49820861948314	0	-Inf	0

x12 0.669441284202157 0 Inf 0

Number of observations: 981, Error degrees of freedom: 969

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	41.0603113009407	980	0.0418982768376946		
Model	41.0603113009407	11	3.73275557281279	Inf	0
Residual	0	969	0		

for Q(x) A = 0.069028, B = 0.340839, C = 0.001541

for D(x) A = 0.091553, B = 0.209300, C = 0.209574

order Q(x) = 5.000000, order D(x) = 7.000000

Xmin = 2.000000 and Xmax = 100.000000

Ymin = 0.422784 and Ymax = 4.600162

List of powers for Q(x): [0.411408, 0.752886, 1.094215, 1.435467, 1.776670]

List of powers for D(x): [0.510427, 0.806695, 1.082793, 1.348448, 1.607431, 1.861675, 2.112326]

Fitting digamma(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.932077e-08

Corrected MSS of errors squared = 4.732604e-08

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 7

Fitting digamma(x) in range (2.000000, 100.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.913446824821286	0	Inf	0
x1	-11.3725402231091	0	-Inf	0
x2	18.9051070062282	0	Inf	0
x3	0	0	NaN	NaN
x4	-21.8356346798484	0	-Inf	0
x5	19.0900607539167	0	Inf	0
x6	-6.586129241427	0	-Inf	0
x7	0.885690347403248	0	Inf	0

x8	73.8672526394303	0	Inf	0
x9	0	0	NaN	NaN
x10	-137.54741294798	0	-Inf	0
x11	80.6182510464632	0	Inf	0
x12	-15.9380915259805	0	-Inf	0

Number of observations: 981, Error degrees of freedom: 970

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	41.060311300929	980	0.0418982768376826		
Model	41.060311300929	10	4.1060311300929	Inf	0
Residual	0	970	0		

for Q(x) A = 0.344046, B = 0.255885

for D(x) A = 0.587244, B = 1.727883

order Q(x) = 7.000000, order D(x) = 5.000000

Xmin = 2.000000 and Xmax = 100.000000

Ymin = 0.422784 and Ymax = 4.600162

List of powers for Q(x): [0.599931, 0.855815, 1.111700, 1.367584, 1.623469, 1.879354, 2.135238]

List of powers for D(x): [0.587244, 0.601433, 0.676787, 0.814269, 0.999673]

Fitting digamma(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.424903e-07

Corrected MSS of errors squared = 2.849805e-07

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 8

Fitting digamma(x) in range (2.000000, 100.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.937214643932953	0	Inf	0
x1	0.195107052957023	0	Inf	0
x2	-7.73945821074239	0	-Inf	0
x3	-4.85700208961968	0	-Inf	0

x4	68.7123538577807	0	Inf	0
x5	-57.2482554742076	0	-Inf	0
x6	191.137675860387	0	Inf	0
x7	0	0	NaN	NaN
x8	-301.635604048499	0	-Inf	0
x9	111.497968393977	0	Inf	0

Number of observations: 981, Error degrees of freedom: 972

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	41.0603113009258	980	0.0418982768376794		
Model	41.0603113009258	8	5.13253891261572	Inf	0
Residual	0	972	0		

for Q(x) A = 0.421612, B = 0.902923

for D(x) A = 0.477899, B = 0.484797

order Q(x) = 5.000000, order D(x) = 4.000000

Xmin = 2.000000 and Xmax = 100.000000

Ymin = 0.422784 and Ymax = 4.600162

List of powers for Q(x): [1.324535, 0.873074, 0.722587, 0.647343, 0.602197]

List of powers for D(x): [0.477899, 0.481880, 0.503022, 0.541596]

Fitting digamma(x) in range (1.000000, 2.000000)

MSS of errors squared = 5.373363e-07

Corrected MSS of errors squared = 1.074673e-06

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 9

Fitting digamma(x) in range (2.000000, 100.000000)

Shammas polynomial power for Q(x) is A+B*log10(i)^4

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.000474447857390117	0	-Inf	0
x1	2.37313830561927	0	Inf	0
x2	0	0	NaN	NaN
x3	0	0	NaN	NaN

x4	-7.60184758083786	0	-Inf	0
x5	7.19612220328696	0	Inf	0
x6	-1.96693829611627	0	-Inf	0
x7	7.68374553353525	0	Inf	0
x8	-22.0420795361642	0	-Inf	0
x9	32.8003008496225	0	Inf	0
x10	-27.3823352255128	0	-Inf	0
x11	12.2230243252341	0	Inf	0
x12	-2.28265613079543	0	-Inf	0

Number of observations: 981, Error degrees of freedom: 970

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	41.0603113009354	980	0.0418982768376892		
Model	41.0603113009354	10	4.10603113009354	Inf	0
Residual	0	970	0		

for Q(x) A = 1.985476, B = 0.396566

for D(x) A = 0.017199, B = 0.156923

order Q(x) = 6.000000, order D(x) = 6.000000

Xmin = 2.000000 and Xmax = 100.000000

Ymin = 0.422784 and Ymax = 4.600162

List of powers for Q(x): [1.985476, 1.988732, 2.006026, 2.037580, 2.080132, 2.130878]

List of powers for D(x): [0.174122, 0.331045, 0.487969, 0.644892, 0.801815, 0.958738]

Fitting digamma(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.108883e-07

Corrected MSS of errors squared = 2.217767e-07

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 10

Fitting digamma(x) in range (2.000000, 100.000000)

Shammas polynomial power for Q(x) is A+B*log10(i)^4

Shammas polynomial power for D(x) is A+B/i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.000941451913315336	0	-Inf	0

x1	16.359154696379	0	Inf	0
x2		0	0	NaN
x3		0	0	NaN
x4		0	0	NaN
x5	-84.1525556481487	0	-Inf	0
x6	101.15579811466	0	Inf	0
x7	-33.3614571119803	0	-Inf	0
x8	0.618891313164923	0	Inf	0
x9	-4.47266553676881	0	-Inf	0
x10	4.85377562132453	0	Inf	0

Number of observations: 981, Error degrees of freedom: 973

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	41.0603113009367	980	0.0418982768376905		
Model	41.0603113009367	7	5.86575875727667	Inf	0
Residual	0	973	0		

for Q(x) A = 1.400000, B = 0.100000

for D(x) A = 0.000000, B = 0.100000

order Q(x) = 7.000000, order D(x) = 3.000000

Xmin = 2.000000 and Xmax = 100.000000

Ymin = 0.422784 and Ymax = 4.600162

List of powers for Q(x): [1.400000, 1.400821, 1.405182, 1.413139, 1.423869, 1.436665, 1.451007]

List of powers for D(x): [0.100000, 0.050000, 0.033333]

Fitting digamma(x) in range (1.000000, 2.000000)

MSS of errors squared = 3.965116e-06

Corrected MSS of errors squared = 7.930233e-06

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Comments

The following table shows the summary results for the ten models that fit the digamma function. The results are sorted in ascending order using the MSSE values.

<i>Power Q(x)</i>	<i>Power D(x)</i>	<i>MSSE</i>	<i>Q/D Orders</i>
A+B*i	A+B*sqrt(i)	1.30270400E-08	6, 6
A+B*i+C*sqrt(i)	A+B*sqrt(i)+C*i	1.93207700E-08	5, 7
A+B*sqrt(i)+C*i	A+B*i+C*sqrt(i)	3.93872200E-08	7, 6
A+B*log10(i)^4	A+B*i	1.10888300E-07	6, 6
A+B*i	A+B*log10(i)^4	1.42490300E-07	7, 5
A+B*sqrt(i)	A+B*i	2.24541100E-07	7, 4
A+B*i	A+B/i	4.64525700E-07	4, 5
A+B/i	A+B*log10(i)^4	5.37336300E-07	5, 4
A+B/i	A+B*i	7.69499000E-07	3, 5
A+B*log10(i)^4	A+B/i	3.96511600E-06	7, 3

RESULTS FOR THE ERROR FUNCTION

Combo Powers 1

Fitting erf(x) in range (0.000000, 2.100000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B/i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-9.15124789619221e-06	0	-Inf	0
x1	4.29853062898433e-05	0	Inf	0
x2	-4.86361465761674e-05	0	-Inf	0
x3	1.63006936205325e-05	0	Inf	0
x4	-0.225843513114606	0	-Inf	0
x5	5.41416233673103	0	Inf	0
x6	-17.775293124505	0	-Inf	0
x7	13.5869728022751	0	Inf	0

Number of observations: 211, Error degrees of freedom: 203

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	18.337111282999	210	0.0873195775380907		
Model	18.337111282999	7	2.61958732614272	Inf	0
Residual	0	203	0		

for Q(x) A = 0.180216, B = 0.465781

for D(x) A = 0.009365, B = 0.198677

```

order Q(x) = 3.000000, order D(x) = 4.000000
Xmin = 0.000000 and Xmax = 2.100000
Ymin = 0.000000 and Ymax = 0.997021
List of powers for Q(x): [0.645997, 1.111778, 1.577558]
List of powers for D(x): [0.208042, 0.108704, 0.075591, 0.059034]
Fitting erf(x) in range (1.000000, 2.000000)
MSS of errors squared = 1.193036e-07
Corrected MSS of errors squared = 2.386072e-07
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 2

```

Fitting erf(x) in range (0.000000, 2.100000)
Shammas polynomial power for Q(x) is A+B*i
Shammas polynomial power for D(x) is A+B*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	1.33984551270203	0	Inf	0
x1	-187.804739254568	0	-Inf	0
x2	591.332601552696	0	Inf	0
x3	-705.97698592902	0	-Inf	0
x4	376.010343672448	0	Inf	0
x5	-74.894605180115	0	-Inf	0
x6	496.622464697075	0	Inf	0
x7	-1357.12471398502	0	-Inf	0
x8	0	0	NaN	NaN
x9	2001.32235284612	0	Inf	0
x10	0	0	NaN	NaN
x11	-2202.22441079749	0	-Inf	0
x12	1062.3978468656	0	Inf	0

Number of observations: 211, Error degrees of freedom: 200

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	18.3371112829029	210	0.0873195775376328		
Model	18.3371112829029	10	1.83371112829029	Inf	0
Residual	0	200	0		

```

for Q(x) A = 0.486538, B = 0.100000
for D(x) A = 0.720159, B = 0.201341
order Q(x) = 5.000000, order D(x) = 7.000000
Xmin = 0.000000 and Xmax = 2.100000
Ymin = 0.000000 and Ymax = 0.997021
List of powers for Q(x): [0.586538, 0.686538, 0.786538, 0.886538, 0.986538]
List of powers for D(x): [0.921500, 1.004898, 1.068892, 1.122841, 1.170372, 1.213342,
1.252858]
Fitting erf(x) in range (1.000000, 2.000000)
MSS of errors squared = 1.405351e-09
Corrected MSS of errors squared = 2.810702e-09
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 3

```

Fitting erf(x) in range (0.000000, 2.100000)
Shammas polynomial power for Q(x) is A+B*sqrt(i)+C*i
Shammas polynomial power for D(x) is A+B*i+C*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	3.00341952836115	0	Inf	0
x1	-42.9751178802413	0	-Inf	0
x2	86.1859287624297	0	Inf	0
x3	0	0	NaN	NaN
x4	-139.84863995635	0	-Inf	0
x5	130.688402623102	0	Inf	0
x6	-37.0532972652532	0	-Inf	0
x7	27.0545472077083	0	Inf	0
x8	-93.8458964852898	0	-Inf	0
x9	140.099796305558	0	Inf	0
x10	-107.789022114694	0	-Inf	0
x11	42.1087596939813	0	Inf	0
x12	-6.62888041928624	0	-Inf	0

Number of observations: 211, Error degrees of freedom: 199

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

SumSq	DF	MeanSq	F	pValue

Total	18.337111282992	210	0.0873195775380573		
Model	18.337111282992	11	1.66701011663564	Inf	0
Residual	0	199	0		

for Q(x) A = 0.078635, B = 0.202969, C = 0.085079
 for D(x) A = 0.296891, B = 0.171955, C = 0.254273
 order Q(x) = 6.000000, order D(x) = 6.000000
 Xmin = 0.000000 and Xmax = 2.100000
 Ymin = 0.000000 and Ymax = 0.997021
 List of powers for Q(x): [0.366683, 0.535835, 0.685424, 0.824889, 0.957882, 1.086279]
 List of powers for D(x): [0.723120, 1.000398, 1.253171, 1.493259, 1.725240, 1.951463]
 Fitting erf(x) in range (1.000000, 2.000000)
 MSS of errors squared = 9.355423e-09
 Corrected MSS of errors squared = 2.291601e-08
 R-squared = 1.00000000
 R-squared Adjusted = 1.00000000
 Particle swarm AICc = -1.000000e+99
 AIC = -Inf
 AICc = -Inf

Combo Powers 4

Fitting erf(x) in range (0.000000, 2.100000)
 Shammas polynomial power for Q(x) is A+B/i
 Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.000193379286462342	0	-Inf	0
x1	0.0572377238059836	0	Inf	0
x2	-0.240397320427805	0	-Inf	0
x3	0.183364340678724	0	Inf	0
x4	13.7770644483866	0	Inf	0
x5	-45.2003050993773	0	-Inf	0
x6	72.6981894448732	0	Inf	0
x7	-67.6954224393292	0	-Inf	0
x8	37.2734852138807	0	Inf	0
x9	-11.3211659027357	0	-Inf	0
x10	1.46814296953083	0	Inf	0

Number of observations: 211, Error degrees of freedom: 200

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

SumSq	DF	MeanSq	F	pValue
-------	----	--------	---	--------

Total	18.3371112829999	210	0.0873195775380948		
Model	18.3371112829999	10	1.83371112829999	Inf	0
Residual	0	200	0		

```

for Q(x) A = 0.500000, B = 0.061670
for D(x) A = 0.105833, B = 0.178333
order Q(x) = 3.000000, order D(x) = 7.000000
Xmin = 0.000000 and Xmax = 2.100000
Ymin = 0.000000 and Ymax = 0.997021
List of powers for Q(x): [0.561670, 0.530835, 0.520557]
List of powers for D(x): [0.284166, 0.462499, 0.640832, 0.819165, 0.997498, 1.175831,
1.354164]
Fitting erf(x) in range (1.000000, 2.000000)
MSS of errors squared = 1.815693e-09
Corrected MSS of errors squared = 3.631386e-09
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 5

```

Fitting erf(x) in range (0.000000, 2.100000)
Shammas polynomial power for Q(x) is A+B*sqrt(i)
Shammas polynomial power for D(x) is A+B*i

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.371934668036999	0	Inf	0
x1	-60.8202049230557	0	-Inf	0
x2	166.642815667848	0	Inf	0
x3	0	0	NaN	NaN
x4	-251.652925709736	0	-Inf	0
x5	0	0	NaN	NaN
x6	284.972740754831	0	Inf	0
x7	-139.504211208775	0	-Inf	0
x8	27.9443956119104	0	Inf	0
x9	-81.1848495315572	0	-Inf	0
x10	99.4817243959947	0	Inf	0
x11	-62.6538991772098	0	-Inf	0
x12	19.9073708947609	0	Inf	0
x13	-2.50489144280107	0	-Inf	0

Number of observations: 211, Error degrees of freedom: 199

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	18.3371112830179	210	0.0873195775381804		
Model	18.3371112830179	11	1.66701011663799	Inf	0
Residual	0	199	0		

for Q(x) A = 0.671922, B = 0.476574

for D(x) A = 0.781049, B = 0.353372

order Q(x) = 7.000000, order D(x) = 6.000000

Xmin = 0.000000 and Xmax = 2.100000

Ymin = 0.000000 and Ymax = 0.997021

List of powers for Q(x): [1.148496, 1.345899, 1.497372, 1.625070, 1.737573, 1.839285, 1.932818]

List of powers for D(x): [1.134421, 1.487793, 1.841165, 2.194537, 2.547909, 2.901281]

Fitting erf(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.431595e-08

Corrected MSS of errors squared = 2.863190e-08

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 6

Fitting erf(x) in range (0.000000, 2.100000)

Shammas polynomial power for Q(x) is A+B*i+C*sqrt(i)

Shammas polynomial power for D(x) is A+B*sqrt(i)+C*i

Linear regression model:

$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.143569122346333	0	-Inf	0
x1	1.30632941540665	0	Inf	0
x2	-2.68777934863375	0	-Inf	0
x3	2.10608979150819	0	Inf	0
x4	-0.584230506440149	0	-Inf	0
x5	84.3689133480789	0	Inf	0
x6	-382.342484843166	0	-Inf	0
x7	704.512569075588	0	Inf	0
x8	-649.237880579515	0	-Inf	0
x9	298.322054714733	0	Inf	0

x10 -54.6200119457366 0 -Inf 0

Number of observations: 211, Error degrees of freedom: 200

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	18.3371112829698	210	0.0873195775379513		
Model	18.3371112829698	10	1.83371112829698	Inf	0
Residual	0	200	0		

for Q(x) A = 0.163389, B = 0.264959, C = 0.137586

for D(x) A = 0.329156, B = 0.270741, C = 0.090717

order Q(x) = 4.000000, order D(x) = 6.000000

Xmin = 0.000000 and Xmax = 2.100000

Ymin = 0.000000 and Ymax = 0.997021

List of powers for Q(x): [0.565934, 0.887882, 1.196571, 1.498396]

List of powers for D(x): [0.690614, 0.893475, 1.070243, 1.233505, 1.388135, 1.536634]

Fitting erf(x) in range (1.000000, 2.000000)

MSS of errors squared = 2.532297e-09

Corrected MSS of errors squared = 6.202836e-09

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 7

Fitting erf(x) in range (0.000000, 2.100000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00149092310829936	0	Inf	0
x1	-0.0109052709470454	0	-Inf	0
x2	0.015475331665484	0	Inf	0
x3	-0.00618580045344699	0	-Inf	0
x4	4811.44596948761	0	Inf	0
x5	-5684.15769882701	0	-Inf	0
x6	873.711854213212	0	Inf	0

Number of observations: 211, Error degrees of freedom: 204

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	18.3371112829674	210	0.0873195775379398		
Model	18.3371112829674	6	3.05618521382789	Inf	0
Residual	0	204	0		

for Q(x) A = 0.492670, B = 0.313736

for D(x) A = 0.139977, B = 0.100000

order Q(x) = 3.000000, order D(x) = 3.000000

Xmin = 0.000000 and Xmax = 2.100000

Ymin = 0.000000 and Ymax = 0.997021

List of powers for Q(x): [0.806406, 1.120142, 1.433878]

List of powers for D(x): [0.139977, 0.140798, 0.145159]

Fitting erf(x) in range (1.000000, 2.000000)

MSS of errors squared = 6.068309e-06

Corrected MSS of errors squared = 1.213662e-05

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 8

Fitting erf(x) in range (0.000000, 2.100000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.655940403606839	0	Inf	0
x1	6.01383800464941	0	Inf	0
x2	-85.1551503368275	0	-Inf	0
x3	161.419423846374	0	Inf	0
x4	0	0	NaN	NaN
x5	-82.9340433785418	0	-Inf	0
x6	18.1640522663919	0	Inf	0
x7	0	0	NaN	NaN
x8	0	0	NaN	NaN
x9	0	0	NaN	NaN
x10	-68.0990621579982	0	-Inf	0
x11	71.8687330600118	0	Inf	0

x12 -20.9337317076599 0 -Inf 0

Number of observations: 211, Error degrees of freedom: 202

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	18.3371112829975	210	0.0873195775380836		
Model	18.3371112829975	8	2.29213891037469	Inf	0
Residual	0	202	0		

for Q(x) A = 0.100000, B = 0.100000

for D(x) A = 0.100000, B = 0.172003

order Q(x) = 5.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 2.100000

Ymin = 0.000000 and Ymax = 0.997021

List of powers for Q(x): [0.200000, 0.150000, 0.133333, 0.125000, 0.120000]

List of powers for D(x): [0.100000, 0.101412, 0.108914, 0.122599, 0.141055, 0.163066, 0.187733]

Fitting erf(x) in range (1.000000, 2.000000)

MSS of errors squared = 2.402012e-08

Corrected MSS of errors squared = 4.804024e-08

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 9

Fitting erf(x) in range (0.000000, 2.100000)

Shammas polynomial power for Q(x) is $A+B*\log_{10}(i)^4$

Shammas polynomial power for D(x) is $A+B*i$

Linear regression model:

$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.00195809419507364	0	-Inf	0
x1	2.1723092202684	0	Inf	0
x2	0	0	NaN	NaN
x3	0	0	NaN	NaN
x4	-4.7008360518937	0	-Inf	0
x5	2.53122561451591	0	Inf	0
x6	127.905425918091	0	Inf	0
x7	-475.986183548967	0	-Inf	0

x8	731.535224362962	0	Inf	0
x9	-575.482622215184	0	-Inf	0
x10	230.499948727077	0	Inf	0
x11	-37.472533932962	0	-Inf	0

Number of observations: 211, Error degrees of freedom: 201

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	18.3371112829829	210	0.0873195775380137		
Model	18.3371112829829	9	2.03745680922032	Inf	0
Residual	0	201	0		

for Q(x) A = 1.171910, B = 0.250606

for D(x) A = 0.536444, B = 0.143291

order Q(x) = 5.000000, order D(x) = 6.000000

Xmin = 0.000000 and Xmax = 2.100000

Ymin = 0.000000 and Ymax = 0.997021

List of powers for Q(x): [1.171910, 1.173968, 1.184897, 1.204837, 1.231727]

List of powers for D(x): [0.679735, 0.823026, 0.966317, 1.109608, 1.252899, 1.396190]

Fitting erf(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.242645e-08

Corrected MSS of errors squared = 2.485291e-08

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 10

Fitting erf(x) in range (0.000000, 2.100000)

Shammas polynomial power for Q(x) is $A+B*\log_{10}(i)^4$

Shammas polynomial power for D(x) is $A+B/i$

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-5.0563099207998e-05	0	-Inf	0
x1	0.821459408573613	0	Inf	0
x2	-0.972615468313783	0	-Inf	0
x3	0.151220665606085	0	Inf	0
x4	-0.00304553607815328	0	-Inf	0
x5	0.78559000525225	0	Inf	0

x6	-14.1665651753108	0	-Inf	0
x7	65.1672723244799	0	Inf	0
x8	-104.864200557467	0	-Inf	0
x9	54.0809348963532	0	Inf	0

Number of observations: 211, Error degrees of freedom: 201

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	18.3371112829977	210	0.0873195775380845		
Model	18.3371112829977	9	2.03745680922197	Inf	0
Residual	0	201	0		

for Q(x) A = 0.998613, B = 0.478066

for D(x) A = 0.000000, B = 1.445114

order Q(x) = 3.000000, order D(x) = 6.000000

Xmin = 0.000000 and Xmax = 2.100000

Ymin = 0.000000 and Ymax = 0.997021

List of powers for Q(x): [0.998613, 1.002539, 1.023387]

List of powers for D(x): [1.445114, 0.722557, 0.481705, 0.361278, 0.289023, 0.240852]

Fitting erf(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.090090e-08

Corrected MSS of errors squared = 2.180181e-08

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Comments

The following table shows the summary results for the ten models that fit the error function. The results are sorted in ascending order using the MSSE values.

<i>Power Q(x)</i>	<i>Power D(x)</i>	<i>MSSE</i>	<i>Q/D Orders</i>
A+B*i	A+B*sqrt(i)	1.40535100E-09	5, 7
A+B/i	A+B*i	1.81569300E-09	3, 7
A+B*i+C*sqrt(i)	A+B*sqrt(i)+C*i	2.53229700E-09	4, 6
A+B*sqrt(i)+C*i	A+B*i+C*sqrt(i)	9.35542300E-09	6, 6
A+B*log10(i)^4	A+B/i	1.09009000E-08	3, 6
A+B*log10(i)^4	A+B*i	1.24264500E-08	5, 6
A+B*sqrt(i)	A+B*i	1.43159500E-08	7, 6
A+B/i	A+B*log10(i)^4	2.40201200E-08	5, 7
A+B*i	A+B/i	1.19303600E-07	3, 4
A+B*i	A+B*log10(i)^4	6.06830900E-06	3, 3

RESULTS FOR THE EXPONENTIAL FUNCTION

Combo Powers 1

Fitting $\exp(x)$ in range (0.000000, 2.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B/i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.910851875479806	0	Inf	0
x1	-1.78177787398973	0	-Inf	0
x2	1.41748466030519	0	Inf	0
x3	-0.518643498781951	0	-Inf	0
x4	0.0827337624915208	0	Inf	0
x5	-0.0131250560577016	0	-Inf	0
x6	-0.937715032107551	0	-Inf	0
x7	15.8405780439118	0	Inf	0
x8	-32.4843614551063	0	-Inf	0
x9	0	0	NaN	NaN
x10	18.4839745738598	0	Inf	0

Number of observations: 201, Error degrees of freedom: 191

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	15.9274939123256	200	0.0796374695616282		
Model	15.9274939123256	9	1.76972154581396	Inf	0
Residual	0	191	0		

```

for Q(x) A = 0.056802, B = 0.470430
for D(x) A = 0.557817, B = 1.383809
order Q(x) = 4.000000, order D(x) = 6.000000
Xmin = 0.000000 and Xmax = 2.000000
Ymin = 1.000000 and Ymax = 7.389056
List of powers for Q(x): [0.527231, 0.997661, 1.468091, 1.938520]
List of powers for D(x): [1.941626, 1.249721, 1.019086, 0.903769, 0.834578, 0.788451]
Fitting exp(x) in range (1.000000, 2.000000)
MSS of errors squared = 1.768885e-12
Corrected MSS of errors squared = 3.537771e-12
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 2

```

Fitting exp(x) in range (0.000000, 2.000000)
Shammas polynomial power for Q(x) is A+B*i
Shammas polynomial power for D(x) is A+B*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.865286632496739	0	Inf	0
x1	0.0654873518175477	0	Inf	0
x2	0.0305201405039008	0	Inf	0
x3	0.00870833263659217	0	Inf	0
x4	0.00112408316308156	0	Inf	0
x5	0.000144707100310372	0	Inf	0
x6	5.52642625965518e-06	0	Inf	0
x7	7.30662771864375e-07	0	Inf	0
x8	0.214662391053865	0	Inf	0
x9	-0.289135702339599	0	-Inf	0
x10	0	0	NaN	NaN
x11	0.103195806479744	0	Inf	0

Number of observations: 201, Error degrees of freedom: 190

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	15.9274939123249	200	0.0796374695616246		

Model	15.9274939123249	10	1.59274939123249	Inf	0
Residual	0	190	0		

```

for Q(x) A = 0.225081, B = 1.596100
for D(x) A = 0.733204, B = 0.583064
order Q(x) = 7.000000, order D(x) = 4.000000
Xmin = 0.000000 and Xmax = 2.000000
Ymin = 1.000000 and Ymax = 7.389056
List of powers for Q(x): [1.821181, 3.417281, 5.013381, 6.609481, 8.205582, 9.801682,
11.397782]
List of powers for D(x): [1.316268, 1.557781, 1.743101, 1.899332]
Fitting exp(x) in range (1.000000, 2.000000)
MSS of errors squared = 2.071166e-14
Corrected MSS of errors squared = 4.142331e-14
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 3

```

Fitting exp(x) in range (0.000000, 2.000000)
Shammas polynomial power for Q(x) is A+B*sqrt(i)+C*i
Shammas polynomial power for D(x) is A+B*i+C*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.654266320224882	0	Inf	0
x1	-6.69530493340979	0	-Inf	0
x2	15.6402708497179	0	Inf	0
x3	-15.7672255021862	0	-Inf	0
x4	7.7570602805381	0	Inf	0
x5	-1.53790410551572	0	-Inf	0
x6	3.70399051277445	0	Inf	0
x7	-3.93672505561606	0	-Inf	0
x8	1.18157163304193	0	Inf	0

Number of observations: 201, Error degrees of freedom: 192

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	15.9274939123252	200	0.0796374695616258		

Model	15.9274939123252	8	1.99093673904065	Inf	0
Residual	0	192	0		

```

for Q(x) A = 0.424004, B = 0.195145, C = 0.210960
for D(x) A = 0.218834, B = 0.191084, C = 0.083614
order Q(x) = 5.000000, order D(x) = 3.000000
Xmin = 0.000000 and Xmax = 2.000000
Ymin = 1.000000 and Ymax = 7.389056
List of powers for Q(x): [0.830108, 1.121900, 1.394884, 1.658133, 1.915161]
List of powers for D(x): [0.493532, 0.719250, 0.936909]
Fitting exp(x) in range (1.000000, 2.000000)
MSS of errors squared = 7.989785e-10
Corrected MSS of errors squared = 1.957090e-09
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 4

```

Fitting exp(x) in range (0.000000, 2.000000)
Shammas polynomial power for Q(x) is A+B/i
Shammas polynomial power for D(x) is A+B*i

```

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-4.81140360088449	0	-Inf	0
x1	7.32326632791825	0	Inf	0
x2	0	0	NaN	NaN
x3	-56.3327829235121	0	-Inf	0
x4	0	0	NaN	NaN
x5	0	0	NaN	NaN
x6	53.9676687459491	0	Inf	0
x7	2.13233203032924	0	Inf	0
x8	-1.89146206480475	0	-Inf	0
x9	0.754641863290392	0	Inf	0
x10	-0.15892914375703	0	-Inf	0
x11	0.0166687651208223	0	Inf	0

Number of observations: 201, Error degrees of freedom: 192

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

SumSq	DF	MeanSq	F	pValue
-------	----	--------	---	--------

Total	15.9274939123264	200	0.0796374695616319		
Model	15.9274939123264	8	1.9909367390408	Inf	0
Residual	0	192	0		

```

for Q(x) A = 0.108181, B = 0.280959
for D(x) A = 0.204102, B = 0.434539
order Q(x) = 6.000000, order D(x) = 5.000000
Xmin = 0.000000 and Xmax = 2.000000
Ymin = 1.000000 and Ymax = 7.389056
List of powers for Q(x): [0.389140, 0.248660, 0.201834, 0.178420, 0.164373, 0.155007]
List of powers for D(x): [0.638641, 1.073181, 1.507720, 1.942260, 2.376799]
Fitting exp(x) in range (1.000000, 2.000000)
MSS of errors squared = 1.079694e-10
Corrected MSS of errors squared = 2.159387e-10
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 5

```

Fitting exp(x) in range (0.000000, 2.000000)
Shammas polynomial power for Q(x) is A+B*sqrt(i)
Shammas polynomial power for D(x) is A+B*i

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.86718828774755	0	Inf	0
x1	-0.623492719556782	0	-Inf	0
x2	3.11619668442718	0	Inf	0
x3	-3.76226358233158	0	-Inf	0
x4	0	0	NaN	NaN
x5	0	0	NaN	NaN
x6	1.1172366247619	0	Inf	0
x7	0.398256782308322	0	Inf	0
x8	-0.0453735445054166	0	-Inf	0
x9	-0.105396320901622	0	-Inf	0
x10	0.0425125843014355	0	Inf	0
x11	-0.00486479621955243	0	-Inf	0

Number of observations: 201, Error degrees of freedom: 191

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	15.9274939123251	200	0.0796374695616253		
Model	15.9274939123251	9	1.7697215458139	Inf	0
Residual	0	191	0		

```

for Q(x) A = 1.125226, B = 0.437394
for D(x) A = 1.032376, B = 0.723764
order Q(x) = 6.000000, order D(x) = 5.000000
Xmin = 0.000000 and Xmax = 2.000000
Ymin = 1.000000 and Ymax = 7.389056
List of powers for Q(x): [1.562619, 1.743794, 1.882814, 2.000013, 2.103268, 2.196617]
List of powers for D(x): [1.756141, 2.479905, 3.203670, 3.927434, 4.651199]
Fitting exp(x) in range (1.000000, 2.000000)
MSS of errors squared = 1.104377e-12
Corrected MSS of errors squared = 2.208755e-12
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 6

```

Fitting exp(x) in range (0.000000, 2.000000)
Shammas polynomial power for Q(x) is A+B*i+C*sqrt(i)
Shammas polynomial power for D(x) is A+B*sqrt(i)+C*i

```

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.890154157234972	0	Inf	0
x1	-2.51497546806431	0	-Inf	0
x2	2.04777591211565	0	Inf	0
x3	0	0	NaN	NaN
x4	-0.351881704729341	0	-Inf	0
x5	0.0589545865964199	0	Inf	0
x6	3.12816918823936	0	Inf	0
x7	-4.33285472728931	0	-Inf	0
x8	3.36172076230315	0	Inf	0
x9	-1.8017853239523	0	-Inf	0
x10	0.604009988732989	0	Inf	0
x11	-0.089287371186496	0	-Inf	0

Number of observations: 201, Error degrees of freedom: 190

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	15.9274939123251	200	0.0796374695616257		
Model	15.9274939123251	10	1.59274939123251	Inf	0
Residual	0	190	0		

for Q(x) A = 0.263868, B = 0.183144, C = 0.173535

for D(x) A = 0.243211, B = 0.103259, C = 0.337702

order Q(x) = 5.000000, order D(x) = 6.000000

Xmin = 0.000000 and Xmax = 2.000000

Ymin = 1.000000 and Ymax = 7.389056

List of powers for Q(x): [0.620546, 0.875571, 1.113870, 1.343513, 1.567623]

List of powers for D(x): [0.684172, 1.064644, 1.435166, 1.800535, 2.162613, 2.522353]

Fitting exp(x) in range (1.000000, 2.000000)

MSS of errors squared = 2.856848e-13

Corrected MSS of errors squared = 6.997821e-13

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 7

Fitting exp(x) in range (0.000000, 2.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.952025394109981	0	Inf	0
x1	-3.60506203139959	0	-Inf	0
x2	18.6492010117053	0	Inf	0
x3	-28.1113830989401	0	-Inf	0
x4	16.9062089746048	0	Inf	0
x5	-3.61307511871246	0	-Inf	0
x6	-100.096323901739	0	-Inf	0
x7	0	0	NaN	NaN
x8	197.53269847279	0	Inf	0
x9	-128.838161652246	0	-Inf	0
x10	34.702210924961	0	Inf	0
x11	-3.47833897508719	0	-Inf	0

Number of observations: 201, Error degrees of freedom: 190

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	15.9274939123131	200	0.0796374695615654		
Model	15.9274939123131	10	1.59274939123131	Inf	0
Residual	0	190	0		

for Q(x) A = 0.500000, B = 0.458372

for D(x) A = 2.000000, B = 3.500000

order Q(x) = 5.000000, order D(x) = 6.000000

Xmin = 0.000000 and Xmax = 2.000000

Ymin = 1.000000 and Ymax = 7.389056

List of powers for Q(x): [0.958372, 1.416744, 1.875116, 2.333488, 2.791860]

List of powers for D(x): [2.000000, 2.028741, 2.181377, 2.459862, 2.835415, 3.283288]

Fitting exp(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.969243e-12

Corrected MSS of errors squared = 3.938487e-12

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 8

Fitting exp(x) in range (0.000000, 2.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.940869823394305	0	Inf	0
x1	0.0427303528448961	0	Inf	0
x2	-1.95647300830949	0	-Inf	0
x3	8.7636838769191	0	Inf	0
x4	-7.66978721282998	0	-Inf	0
x5	-119.809928473783	0	-Inf	0
x6	0	0	NaN	NaN
x7	0	0	NaN	NaN
x8	0	0	NaN	NaN
x9	666.911207755414	0	Inf	0
x10	-824.416619453507	0	-Inf	0

x11 278.194316339778 0 Inf 0

Number of observations: 201, Error degrees of freedom: 192

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	15.9274939123219	200	0.0796374695616097		
Model	15.9274939123219	8	1.99093673904024	Inf	0
Residual	0	192	0		

for Q(x) A = 0.290967, B = 1.556402

for D(x) A = 0.539043, B = 0.254669

order Q(x) = 4.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 2.000000

Ymin = 1.000000 and Ymax = 7.389056

List of powers for Q(x): [1.847369, 1.069168, 0.809768, 0.680068]

List of powers for D(x): [0.539043, 0.541135, 0.552241, 0.572504, 0.599830, 0.632419, 0.668942]

Fitting exp(x) in range (1.000000, 2.000000)

MSS of errors squared = 3.350121e-11

Corrected MSS of errors squared = 6.700242e-11

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 9

Fitting exp(x) in range (0.000000, 2.000000)

Shammas polynomial power for Q(x) is $A+B*\log_{10}(i)^4$

Shammas polynomial power for D(x) is $A+B*i$

Linear regression model:

$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	3.72883230404461	0	Inf	0
x1	-150.647561577088	0	-Inf	0
x2	0	0	NaN	NaN
x3	0	0	NaN	NaN
x4	0	0	NaN	NaN
x5	715.095803594828	0	Inf	0
x6	-843.746439222547	0	-Inf	0
x7	275.953078274934	0	Inf	0

x8	1.00709275497481	0	Inf	0
x9	-0.482977609430112	0	-Inf	0
x10	0.100184717297178	0	Inf	0
x11	-0.00801323644660781	0	-Inf	0

Number of observations: 201, Error degrees of freedom: 192

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	15.9274939123203	200	0.0796374695616017		
Model	15.9274939123203	8	1.99093673904004	Inf	0
Residual	0	192	0		

for Q(x) A = 0.383549, B = 0.487169

for D(x) A = 0.223843, B = 0.816006

order Q(x) = 7.000000, order D(x) = 4.000000

Xmin = 0.000000 and Xmax = 2.000000

Ymin = 1.000000 and Ymax = 7.389056

List of powers for Q(x): [0.383549, 0.387549, 0.408795, 0.447557, 0.499831, 0.562171, 0.632038]

List of powers for D(x): [1.039849, 1.855855, 2.671861, 3.487867]

Fitting exp(x) in range (1.000000, 2.000000)

MSS of errors squared = 4.511592e-11

Corrected MSS of errors squared = 9.023184e-11

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 10

Fitting exp(x) in range (0.000000, 2.000000)

Shammas polynomial power for Q(x) is $A+B*\log_{10}(i)^4$

Shammas polynomial power for D(x) is $A+B/i$

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.84190891139136	0	Inf	0
x1	-217.061777131357	0	-Inf	0
x2	0	0	NaN	NaN
x3	401.675762179677	0	Inf	0
x4	-225.730448000599	0	-Inf	0

x5	40.2986257349981	0	Inf	0
x6	0.655020272977335	0	Inf	0
x7	-5.88996019009554	0	-Inf	0
x8	6.21086822327139	0	Inf	0

Number of observations: 201, Error degrees of freedom: 193

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	15.9274939123288	200	0.0796374695616441		
Model	15.9274939123288	7	2.27535627318983	Inf	0
Residual	0	193	0		

for Q(x) A = 0.441084, B = 0.421488

for D(x) A = 0.151345, B = 0.484959

order Q(x) = 5.000000, order D(x) = 3.000000

Xmin = 0.000000 and Xmax = 2.000000

Ymin = 1.000000 and Ymax = 7.389056

List of powers for Q(x): [0.441084, 0.444545, 0.462926, 0.496462, 0.541688]

List of powers for D(x): [0.636304, 0.393824, 0.312998]

Fitting exp(x) in range (1.000000, 2.000000)

MSS of errors squared = 6.872078e-10

Corrected MSS of errors squared = 1.374416e-09

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Comments

The following table shows the summary results for the ten models that fit the exponential function. The results are sorted in ascending order using the MSSE values.

<i>Power Q(x)</i>	<i>Power D(x)</i>	<i>MSSE</i>	<i>Q/D Orders</i>
A+B*i	A+B*sqrt(i)	2.07116600E-14	7, 4
A+B*i+C*sqrt(i)	A+B*sqrt(i)+C*i	2.85684800E-13	5, 6
A+B*sqrt(i)	A+B*i	1.10437700E-12	6, 5
A+B*i	A+B/i	1.76888500E-12	4, 6
A+B*i	A+B*log10(i)^4	1.96924300E-12	5, 6
A+B/i	A+B*log10(i)^4	3.35012100E-11	4, 7
A+B*log10(i)^4	A+B*i	4.51159200E-11	7, 4
A+B/i	A+B*i	1.07969400E-10	6, 5
A+B*log10(i)^4	A+B/i	6.87207800E-10	5, 3
A+B*sqrt(i)+C*i	A+B*i+C*sqrt(i)	7.98978500E-10	5, 3

RESULTS FOR THE FRESNEL COSINE

Combo Powers 1

Fitting FresnelCosine(x) in range (0.500000, 5.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B/i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12 + x13 + x14$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.132650367984883	0	Inf	0
x1	-1.79707597340238	0	-Inf	0
x2	5.88667186925149	0	Inf	0
x3	-9.43961589644337	0	-Inf	0
x4	8.72711339488587	0	Inf	0
x5	-4.74701535248248	0	-Inf	0
x6	1.4168972978452	0	Inf	0
x7	-0.179617346821403	0	-Inf	0
x8	0.529676600698166	0	Inf	0
x9	-220.969415723929	0	-Inf	0
x10	5674.46764380669	0	Inf	0
x11	-37567.5687900807	0	-Inf	0
x12	95382.1478617164	0	Inf	0
x13	-101991.665878225	0	-Inf	0
x14	38724.0588947478	0	Inf	0

Number of observations: 451, Error degrees of freedom: 436

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

SumSq	DF	MeanSq	F	pValue

Total	32.7345285700007	450	0.0727433968222237		
Model	32.7345285700007	14	2.33818061214291	Inf	0
Residual	0	436	0		

```

for Q(x) A = 0.269929, B = 0.477504
for D(x) A = 1.062117, B = 2.000000
order Q(x) = 7.000000, order D(x) = 7.000000
Xmin = 0.500000 and Xmax = 5.000000
Ymin = 0.402386 and Ymax = 0.977438
List of powers for Q(x): [0.747433, 1.224937, 1.702441, 2.179946, 2.657450, 3.134954,
3.612458]
List of powers for D(x): [3.062117, 2.062117, 1.728784, 1.562117, 1.462117, 1.395450,
1.347831]
Fitting FresnelCosine(x) in range (1.000000, 2.000000)
MSS of errors squared = 4.592798e-02
Corrected MSS of errors squared = 9.185596e-02
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 2

```

Fitting FresnelCosine(x) in range (0.500000, 5.000000)
Shammas polynomial power for Q(x) is A+B*i
Shammas polynomial power for D(x) is A+B*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.000152087405115594	0	Inf	0
x1	-0.00053788863692443	0	-Inf	0
x2	0.000550663952732916	0	Inf	0
x3	-0.000162667003500189	0	-Inf	0
x4	76.1774649122197	0	Inf	0
x5	-540.219465961946	0	-Inf	0
x6	1590.83536928295	0	Inf	0
x7	-2450.15835832701	0	-Inf	0
x8	2085.86374925128	0	Inf	0
x9	-933.570014278002	0	-Inf	0
x10	172.07125319636	0	Inf	0

Number of observations: 451, Error degrees of freedom: 440

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	32.7345285596173	450	0.0727433967991496		
Model	32.7345285596173	10	3.27345285596173	Inf	0
Residual	0	440	0		

for Q(x) A = 0.343573, B = 0.481292

for D(x) A = 0.074434, B = 0.770600

order Q(x) = 3.000000, order D(x) = 7.000000

Xmin = 0.500000 and Xmax = 5.000000

Ymin = 0.402386 and Ymax = 0.977438

List of powers for Q(x): [0.824865, 1.306157, 1.787449]

List of powers for D(x): [0.845034, 1.164226, 1.409152, 1.615633, 1.797547, 1.962010, 2.113249]

Fitting FresnelCosine(x) in range (1.000000, 2.000000)

MSS of errors squared = 2.183451e-01

Corrected MSS of errors squared = 4.366903e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 3

Fitting FresnelCosine(x) in range (0.500000, 5.000000)

Shammas polynomial power for Q(x) is A+B*sqrt(i)+C*i

Shammas polynomial power for D(x) is A+B*i+C*sqrt(i)

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.000189502214299147	0	-Inf	0
x1	0.00202514254586913	0	Inf	0
x2	-0.00579740481768459	0	-Inf	0
x3	0.00777702018354253	0	Inf	0
x4	-0.0055639701366552	0	-Inf	0
x5	0.00206061353631773	0	Inf	0
x6	-0.000311845852031398	0	-Inf	0
x7	20.1918123747419	0	Inf	0
x8	-90.561441021039	0	-Inf	0
x9	190.796747185145	0	Inf	0
x10	-223.430231963536	0	-Inf	0
x11	150.190471721933	0	Inf	0
x12	-54.4882869202941	0	-Inf	0

x13 8.30092857741747 0 Inf 0

Number of observations: 451, Error degrees of freedom: 437

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	32.7345285596361	450	0.0727433967991914		
Model	32.7345285596361	13	2.51804065843355	Inf	0
Residual	0	437	0		

for Q(x) A = 0.106705, B = 0.325734, C = 0.394744

for D(x) A = 0.000000, B = 0.103018, C = 0.237252

order Q(x) = 6.000000, order D(x) = 7.000000

Xmin = 0.500000 and Xmax = 5.000000

Ymin = 0.402386 and Ymax = 0.977438

List of powers for Q(x): [0.827183, 1.356850, 1.855124, 2.337148, 2.808787, 3.273049]

List of powers for D(x): [0.340271, 0.541562, 0.719988, 0.886577, 1.045603, 1.199256, 1.348838]

Fitting FresnelCosine(x) in range (1.000000, 2.000000)

MSS of errors squared = 2.235834e-02

Corrected MSS of errors squared = 5.476653e-02

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 4

Fitting FresnelCosine(x) in range (0.500000, 5.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.000153962927327445	0	Inf	0
x1	-0.000648950471412947	0	-Inf	0
x2	0.00371354893577512	0	Inf	0
x3	-0.00321849115994408	0	-Inf	0
x4	44.2821135565187	0	Inf	0
x5	-157.788604980203	0	-Inf	0
x6	241.03394054441	0	Inf	0
x7	-192.212896835443	0	-Inf	0

x8	78.9304233063022	0	Inf	0
x9	-13.2449756516788	0	-Inf	0

Number of observations: 451, Error degrees of freedom: 441

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	32.7345285596235	450	0.0727433967991634		
Model	32.7345285596235	9	3.63716983995817	Inf	0
Residual	0	441	0		

for Q(x) A = 0.186873, B = 0.209713

for D(x) A = 0.174814, B = 0.093855

order Q(x) = 3.000000, order D(x) = 6.000000

Xmin = 0.500000 and Xmax = 5.000000

Ymin = 0.402386 and Ymax = 0.977438

List of powers for Q(x): [0.396586, 0.291730, 0.256778]

List of powers for D(x): [0.268669, 0.362523, 0.456378, 0.550233, 0.644087, 0.737942]

Fitting FresnelCosine(x) in range (1.000000, 2.000000)

MSS of errors squared = 2.186184e-01

Corrected MSS of errors squared = 4.372367e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 5

Fitting FresnelCosine(x) in range (0.500000, 5.000000)

Shammas polynomial power for Q(x) is A+B*sqrt(i)

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00673534231689435	0	Inf	0
x1	-2.28138480980989	0	-Inf	0
x2	7.87118829122557	0	Inf	0
x3	-8.84178033563497	0	-Inf	0
x4	3.24527543536644	0	Inf	0
x5	18.7903308894117	0	Inf	0
x6	-61.8214909375064	0	-Inf	0
x7	95.3547936077025	0	Inf	0

x8	-83.6056672029906	0	-Inf	0
x9	42.9039909045375	0	Inf	0
x10	-12.0625226343905	0	-Inf	0
x11	1.44053648470749	0	Inf	0

Number of observations: 451, Error degrees of freedom: 439

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	32.734528559637	450	0.0727433967991934		
Model	32.734528559637	11	2.97586623269428	Inf	0
Residual	0	439	0		

for Q(x) A = 1.067252, B = 0.261877

for D(x) A = 0.452686, B = 0.460402

order Q(x) = 4.000000, order D(x) = 7.000000

Xmin = 0.500000 and Xmax = 5.000000

Ymin = 0.402386 and Ymax = 0.977438

List of powers for Q(x): [1.329129, 1.437602, 1.520836, 1.591006]

List of powers for D(x): [0.913088, 1.373490, 1.833893, 2.294295, 2.754697, 3.215100, 3.675502]

Fitting FresnelCosine(x) in range (1.000000, 2.000000)

MSS of errors squared = 6.576477e-01

Corrected MSS of errors squared = 1.315295e+00

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 6

Fitting FresnelCosine(x) in range (0.500000, 5.000000)

Shammas polynomial power for Q(x) is A+B*i+C*sqrt(i)

Shammas polynomial power for D(x) is A+B*sqrt(i)+C*i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00035490367807404	0	Inf	0
x1	-0.00252278243046764	0	-Inf	0
x2	0.00426620265245057	0	Inf	0
x3	-0.00270970201062809	0	-Inf	0
x4	0.000612025207308615	0	Inf	0

x5	363.022175292731	0	Inf	0
x6	-2506.62670293031	0	-Inf	0
x7	6940.7530952887	0	Inf	0
x8	-9982.49097815183	0	-Inf	0
x9	7930.7417391599	0	Inf	0
x10	-3316.66129485541	0	-Inf	0
x11	572.261965643567	0	Inf	0

Number of observations: 451, Error degrees of freedom: 439

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	32.7345285596035	450	0.072743396799119		
Model	32.7345285596035	11	2.97586623269123	Inf	0
Residual	0	439	0		

for Q(x) A = 0.340000, B = 0.349875, C = 0.000000

for D(x) A = 0.396098, B = 0.307461, C = 0.042650

order Q(x) = 4.000000, order D(x) = 7.000000

Xmin = 0.500000 and Xmax = 5.000000

Ymin = 0.402386 and Ymax = 0.977438

List of powers for Q(x): [0.689875, 1.039751, 1.389626, 1.739501]

List of powers for D(x): [0.746209, 0.916214, 1.056587, 1.181621, 1.296853, 1.405122, 1.508115]

Fitting FresnelCosine(x) in range (1.000000, 2.000000)

MSS of errors squared = Inf

Corrected MSS of errors squared = Inf

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 7

Fitting FresnelCosine(x) in range (0.500000, 5.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12 + x13$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.13091029649408	0	Inf	0
x1	-49.3031355307734	0	-Inf	0

x2	204.158763107195	0	Inf	0
x3	-345.172546310338	0	-Inf	0
x4	296.396941116773	0	Inf	0
x5	-128.83562590172	0	-Inf	0
x6	22.6246932435582	0	Inf	0
x7	299.271965125709	0	Inf	0
x8	0	0	NaN	NaN
x9	-711.18762090028	0	-Inf	0
x10	654.094179982058	0	Inf	0
x11	-315.257315461687	0	-Inf	0
x12	83.8009734263498	0	Inf	0
x13	-9.72218218948741	0	-Inf	0

Number of observations: 451, Error degrees of freedom: 438

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	32.7345285595942	450	0.0727433967990981		
Model	32.7345285595942	12	2.72787737996618	Inf	0
Residual	0	438	0		

for Q(x) A = 0.500000, B = 0.100000

for D(x) A = 0.385861, B = 0.973797

order Q(x) = 6.000000, order D(x) = 7.000000

Xmin = 0.500000 and Xmax = 5.000000

Ymin = 0.402386 and Ymax = 0.977438

List of powers for Q(x): [0.600000, 0.700000, 0.800000, 0.900000, 1.000000, 1.100000]

List of powers for D(x): [0.385861, 0.393858, 0.436326, 0.513808, 0.618297, 0.742908, 0.882565]

Fitting FresnelCosine(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.344653e-01

Corrected MSS of errors squared = 2.689306e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 8

Fitting FresnelCosine(x) in range (0.500000, 5.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00592351218056234	0	Inf	0
x1	1.04318965101162	0	Inf	0
x2	-11.935247135315	0	-Inf	0
x3	18.7856257925305	0	Inf	0
x4	0	0	NaN	NaN
x5	0	0	NaN	NaN
x6	-7.89949178220922	0	-Inf	0
x7	20731.7608357755	0	Inf	0
x8	-26056.502562077	0	-Inf	0
x9	6117.77947990923	0	Inf	0
x10	-792.037753637567	0	-Inf	0

Number of observations: 451, Error degrees of freedom: 442

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	32.734528559112	450	0.0727433967980267		
Model	32.734528559112	8	4.091816069889	Inf	0
Residual	0	442	0		

for Q(x) A = 0.352256, B = 0.100000

for D(x) A = 0.100000, B = 0.100000

order Q(x) = 6.000000, order D(x) = 4.000000

Xmin = 0.500000 and Xmax = 5.000000

Ymin = 0.402386 and Ymax = 0.977438

List of powers for Q(x): [0.452256, 0.402256, 0.385590, 0.377256, 0.372256, 0.368923]

List of powers for D(x): [0.100000, 0.100821, 0.105182, 0.113139]

Fitting FresnelCosine(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.029446e-01

Corrected MSS of errors squared = 2.058892e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 9

Fitting FresnelCosine(x) in range (0.500000, 5.000000)

Shammas polynomial power for Q(x) is A+B*log10(i)^4

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12 + x13

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	1.0539032981074e-06	0	Inf	0
x1	-0.0293673620536556	0	-Inf	0
x2	0	0	NaN	NaN
x3	0	0	NaN	NaN
x4	0.0990831843727402	0	Inf	0
x5	-0.0980918869042928	0	-Inf	0
x6	0.0283750187458774	0	Inf	0
x7	6.50524945830149	0	Inf	0
x8	-18.1312227480538	0	-Inf	0
x9	28.0666525648895	0	Inf	0
x10	-26.0600508762723	0	-Inf	0
x11	14.5136964078315	0	Inf	0
x12	-4.48923407415249	0	-Inf	0
x13	0.594909260550267	0	Inf	0

Number of observations: 451, Error degrees of freedom: 439

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	32.7345285596263	450	0.0727433967991696		
Model	32.7345285596263	11	2.9758662326933	Inf	0
Residual	0	439	0		

for Q(x) A = 1.724109, B = 0.199587

for D(x) A = 0.000000, B = 0.173797

order Q(x) = 6.000000, order D(x) = 7.000000

Xmin = 0.500000 and Xmax = 5.000000

Ymin = 0.402386 and Ymax = 0.977438

List of powers for Q(x): [1.724109, 1.725748, 1.734452, 1.750333, 1.771749, 1.797288]

List of powers for D(x): [0.173797, 0.347595, 0.521392, 0.695189, 0.868987, 1.042784, 1.216581]

Fitting FresnelCosine(x) in range (1.000000, 2.000000)

MSS of errors squared = 7.978905e-02

Corrected MSS of errors squared = 1.595781e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 10

Fitting FresnelCosine(x) in range (0.500000, 5.000000)

Shammas polynomial power for Q(x) is $A+B*\log_{10}(i)^4$

Shammas polynomial power for $D(x)$ is $A+B/i$

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.074094189886048	0	-Inf	0
x1	0	0	NaN	NaN
x2	0.0878506278440617	0	Inf	0
x3	-0.0137575790291096	0	-Inf	0
x4	0.261122166560951	0	Inf	0
x5	-111.039180074302	0	-Inf	0
x6	2893.83383927723	0	Inf	0
x7	-19366.5933292982	0	-Inf	0
x8	49570.850757887	0	Inf	0
x9	-53338.84982508	0	-Inf	0
x10	20352.5366160148	0	Inf	0

Number of observations: 451, Error degrees of freedom: 441

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	32.7345285595222	450	0.0727433967989382		
Model	32.7345285595222	9	3.63716983994691	Inf	0
Residual	0	441	0		

for $Q(x)$ $A = 0.000000$, $B = 0.500000$

for $D(x)$ $A = 0.856880$, $B = 2.000000$

order $Q(x) = 3.000000$, order $D(x) = 7.000000$

Xmin = 0.500000 and Xmax = 5.000000

Ymin = 0.402386 and Ymax = 0.977438

List of powers for $Q(x)$: [0.000000, 0.004106, 0.025911]

List of powers for $D(x)$: [2.856880, 1.856880, 1.523546, 1.356880, 1.256880, 1.190213, 1.142594]

Fitting FresnelCosine(x) in range (1.000000, 2.000000)

MSS of errors squared = 3.593008e-01

Corrected MSS of errors squared = 7.186015e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Comments

The following table shows the summary results for the ten models that fit the Fresnel cosine. The results are sorted in ascending order using the MSSE values.

<i>Power Q(x)</i>	<i>Power D(x)</i>	<i>MSSE</i>	<i>Q/D Orders</i>
A+B*sqrt(i)+C*i	A+B*i+C*sqrt(i)	2.23583400E-02	6, 7
A+B*i	A+B/i	4.59279800E-02	7, 7
A+B*log10(i)^4	A+B*i	7.97890500E-02	6, 7
A+B/i	A+B*log10(i)^4	1.02944600E-01	6, 4
A+B*i	A+B*log10(i)^4	1.34465300E-01	6, 7
A+B*i	A+B*sqrt(i)	2.18345100E-01	3, 7
A+B/i	A+B*i	2.18618400E-01	3, 6
A+B*log10(i)^4	A+B/i	3.59300800E-01	3, 7
A+B*sqrt(i)	A+B*i	6.57647700E-01	4, 7
A+B*i+C*sqrt(i)	A+B*sqrt(i)+C*i	Inf	4, 7

RESULTS FOR THE FRESNEL SINE

Combo Powers 1

Fitting FresnelSine(x) in range (1.000000, 5.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B/i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.000971046069507743	0	-Inf	0
x1	0.0284460462280111	0	Inf	0
x2	-0.065581694036304	0	-Inf	0
x3	0.0525376922746047	0	Inf	0
x4	-0.0144305237845185	0	-Inf	0
x5	36.4443201817369	0	Inf	0
x6	-1528.68292259429	0	-Inf	0
x7	8588.28983651851	0	Inf	0
x8	-14345.1299010575	0	-Inf	0
x9	7250.07866647234	0	Inf	0

Number of observations: 401, Error degrees of freedom: 391

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	21.5526862273626	400	0.0538817155684064		
Model	21.5526862273626	9	2.3947429141514	Inf	0
Residual	0	391	0		

```

for Q(x) A = 0.401210, B = 0.138527
for D(x) A = 0.279948, B = 0.199873
order Q(x) = 4.000000, order D(x) = 5.000000
Xmin = 1.000000 and Xmax = 5.000000
Ymin = 0.310268 and Ymax = 0.894821
List of powers for Q(x): [0.539737, 0.678264, 0.816791, 0.955318]
List of powers for D(x): [0.479821, 0.379884, 0.346572, 0.329916, 0.319923]
Fitting FresnelSine(x) in range (1.000000, 2.000000)
MSS of errors squared = 7.450995e-01
Corrected MSS of errors squared = 1.490199e+00
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 2

```

Fitting FresnelSine(x) in range (1.000000, 5.000000)
Shammas polynomial power for Q(x) is A+B*i
Shammas polynomial power for D(x) is A+B*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.00360757429453173	0	-Inf	0
x1	0.0581959991560685	0	Inf	0
x2	-0.154711634607017	0	-Inf	0
x3	0.171688953355355	0	Inf	0
x4	-0.0899127589714286	0	-Inf	0
x5	0.018347021080594	0	Inf	0
x6	37.0640101693365	0	Inf	0
x7	-165.093773478811	0	-Inf	0
x8	236.669647574345	0	Inf	0
x9	0	0	NaN	NaN
x10	-304.072042217835	0	-Inf	0
x11	271.550328829237	0	Inf	0
x12	-75.1181708819973	0	-Inf	0

Number of observations: 401, Error degrees of freedom: 389

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	21.5526862273338	400	0.0538817155683345		

Model	21.5526862273338	11	1.9593351115758	Inf	0
Residual	0	389	0		

```

for Q(x) A = 0.112983, B = 0.100000
for D(x) A = 0.000000, B = 0.174482
order Q(x) = 5.000000, order D(x) = 7.000000
Xmin = 1.000000 and Xmax = 5.000000
Ymin = 0.310268 and Ymax = 0.894821
List of powers for Q(x): [0.212983, 0.312983, 0.412983, 0.512983, 0.612983]
List of powers for D(x): [0.174482, 0.246755, 0.302212, 0.348965, 0.390154, 0.427393,
0.461637]
Fitting FresnelSine(x) in range (1.000000, 2.000000)
MSS of errors squared = 9.403716e-02
Corrected MSS of errors squared = 1.880743e-01
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 3

```

Fitting FresnelSine(x) in range (1.000000, 5.000000)
Shammas polynomial power for Q(x) is A+B*sqrt(i)+C*i
Shammas polynomial power for D(x) is A+B*i+C*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	4.28868625940959e-05	0	Inf	0
x1	-0.000215982776128646	0	-Inf	0
x2	0.000261981219689427	0	Inf	0
x3	-8.80940214898831e-05	0	-Inf	0
x4	8.08629717217465	0	Inf	0
x5	-22.915360383439	0	-Inf	0
x6	34.0047240771097	0	Inf	0
x7	-29.7750956496791	0	-Inf	0
x8	15.5251402822586	0	Inf	0
x9	-4.47744661477059	0	-Inf	0
x10	0.551740287076993	0	Inf	0

Number of observations: 401, Error degrees of freedom: 390

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

SumSq	DF	MeanSq	F	pValue
-------	----	--------	---	--------

Total	21.5526862273338	400	0.0538817155683345		
Model	21.5526862273338	10	2.15526862273338	Inf	0
Residual	0	390	0		

for Q(x) A = 0.550000, B = 0.252103, C = 0.363907
 for D(x) A = 0.080912, B = 0.390000, C = 0.000000
 order Q(x) = 3.000000, order D(x) = 7.000000
 Xmin = 1.000000 and Xmax = 5.000000
 Ymin = 0.310268 and Ymax = 0.894821
 List of powers for Q(x): [1.166010, 1.634342, 2.078376]
 List of powers for D(x): [0.470912, 0.860912, 1.250912, 1.640912, 2.030912, 2.420912, 2.810912]
 Fitting FresnelSine(x) in range (1.000000, 2.000000)
 MSS of errors squared = 7.514929e-02
 Corrected MSS of errors squared = 1.840774e-01
 R-squared = 1.00000000
 R-squared Adjusted = 1.00000000
 Particle swarm AICc = -1.000000e+99
 AIC = -Inf
 AICc = -Inf

Combo Powers 4

Fitting FresnelSine(x) in range (1.000000, 5.000000)
 Shammas polynomial power for Q(x) is A+B/i
 Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00202091144553017	0	Inf	0
x1	0.332741545979756	0	Inf	0
x2	-4.10946685926232	0	-Inf	0
x3	7.3059236299363	0	Inf	0
x4	0	0	NaN	NaN
x5	-3.53121899397459	0	-Inf	0
x6	200.880105742123	0	Inf	0
x7	-503.034103162209	0	-Inf	0
x8	0	0	NaN	NaN
x9	1235.71220081388	0	Inf	0
x10	-1623.38992209266	0	-Inf	0
x11	861.586163071713	0	Inf	0
x12	-170.754444606072	0	-Inf	0

Number of observations: 401, Error degrees of freedom: 390

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	21.5526862273439	400	0.0538817155683599		
Model	21.5526862273439	10	2.15526862273439	Inf	0
Residual	0	390	0		

for Q(x) A = 0.434963, B = 0.124028

for D(x) A = 0.332194, B = 0.065741

order Q(x) = 5.000000, order D(x) = 7.000000

Xmin = 1.000000 and Xmax = 5.000000

Ymin = 0.310268 and Ymax = 0.894821

List of powers for Q(x): [0.558991, 0.496977, 0.476306, 0.465970, 0.459769]

List of powers for D(x): [0.397934, 0.463675, 0.529416, 0.595156, 0.660897, 0.726638, 0.792378]

Fitting FresnelSine(x) in range (1.000000, 2.000000)

MSS of errors squared = 5.876497e-02

Corrected MSS of errors squared = 1.175299e-01

R-squared = 1.000000000

R-squared Adjusted = 1.000000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 5

Fitting FresnelSine(x) in range (1.000000, 5.000000)

Shammas polynomial power for Q(x) is A+B*sqrt(i)

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	5.38072682728206e-10	0	Inf	0
x1	-9.79183978077393e-08	0	-Inf	0
x2	3.24122053511088e-07	0	Inf	0
x3	-3.53169473313958e-07	0	-Inf	0
x4	1.26435508264316e-07	0	Inf	0
x5	5.91791221268524	0	Inf	0
x6	-14.5923335181241	0	-Inf	0
x7	19.1901620499009	0	Inf	0
x8	-14.1956076821374	0	-Inf	0
x9	5.60050222067549	0	Inf	0
x10	-0.920635283007988	0	-Inf	0

Number of observations: 401, Error degrees of freedom: 390

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	21.5526862273343	400	0.0538817155683358		
Model	21.5526862273343	10	2.15526862273343	Inf	0
Residual	0	390	0		

for Q(x) A = 1.400000, B = 0.422261

for D(x) A = 0.000000, B = 0.035436

order Q(x) = 4.000000, order D(x) = 6.000000

Xmin = 1.000000 and Xmax = 5.000000

Ymin = 0.310268 and Ymax = 0.894821

List of powers for Q(x): [1.822261, 1.997167, 2.131377, 2.244521]

List of powers for D(x): [0.035436, 0.070872, 0.106308, 0.141744, 0.177180, 0.212615]

Fitting FresnelSine(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.243277e-01

Corrected MSS of errors squared = 2.486555e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 6

Fitting FresnelSine(x) in range (1.000000, 5.000000)

Shammas polynomial power for Q(x) is A+B*i+C*sqrt(i)

Shammas polynomial power for D(x) is A+B*sqrt(i)+C*i

Linear regression model:

$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13}$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.000800864598797527	0	-Inf	0
x1	0.0132432292131737	0	Inf	0
x2	-0.0400358776781098	0	-Inf	0
x3	0.0544291860883024	0	Inf	0
x4	-0.0392492048913825	0	-Inf	0
x5	0.0146505832075113	0	Inf	0
x6	-0.00223673696670707	0	-Inf	0
x7	30.2244593172428	0	Inf	0
x8	-157.955029565131	0	-Inf	0
x9	370.92742998739	0	Inf	0
x10	-473.264048805374	0	-Inf	0

x11	341.704511698301	0	Inf	0
x12	-131.842320507956	0	-Inf	0
x13	21.2049975609484	0	Inf	0

Number of observations: 401, Error degrees of freedom: 387

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	21.5526862273028	400	0.053881715568257		
Model	21.5526862273028	13	1.65789894056175	Inf	0
Residual	0	387	0		

for Q(x) A = 0.340000, B = 0.390000, C = 0.000000

for D(x) A = 0.000000, B = 0.400000, C = 0.075973

order Q(x) = 6.000000, order D(x) = 7.000000

Xmin = 1.000000 and Xmax = 5.000000

Ymin = 0.310268 and Ymax = 0.894821

List of powers for Q(x): [0.730000, 1.120000, 1.510000, 1.900000, 2.290000, 2.680000]

List of powers for D(x): [0.475973, 0.717632, 0.920741, 1.103894, 1.274294, 1.435636, 1.590114]

Fitting FresnelSine(x) in range (1.000000, 2.000000)

MSS of errors squared = 6.793127e-02

Corrected MSS of errors squared = 1.663969e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 7

Fitting FresnelSine(x) in range (1.000000, 5.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	8.43605784989207e-13	0	Inf	0
x1	-1.81056608503972e-11	0	-Inf	0
x2	3.95648434179477e-11	0	Inf	0
x3	-3.02372019174529e-11	0	-Inf	0
x4	7.93141622850143e-12	0	Inf	0
x5	1.00000001024196	0	Inf	0

x6	-1.21666398750476e-08	0	-Inf	0
x7	1.92468096679327e-09	0	Inf	0

Number of observations: 401, Error degrees of freedom: 393

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	21.5526862273342	400	0.0538817155683355		
Model	21.5526862273342	7	3.07895517533346	Inf	0
Residual	0	393	0		

for Q(x) A = 0.500000, B = 0.217850

for D(x) A = 0.000000, B = 0.100000

order Q(x) = 4.000000, order D(x) = 3.000000

Xmin = 1.000000 and Xmax = 5.000000

Ymin = 0.310268 and Ymax = 0.894821

List of powers for Q(x): [0.717850, 0.935699, 1.153549, 1.371398]

List of powers for D(x): [0.000000, 0.000821, 0.005182]

Fitting FresnelSine(x) in range (1.000000, 2.000000)

MSS of errors squared = 2.020288e-01

Corrected MSS of errors squared = 4.040576e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 8

Fitting FresnelSine(x) in range (1.000000, 5.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.000102113196365802	0	Inf	0
x1	-2.65673116777674e-06	0	-Inf	0
x2	0.000402647146931278	0	Inf	0
x3	-0.00370930675529781	0	-Inf	0
x4	0.00821213354137645	0	Inf	0
x5	-0.00500488068131598	0	-Inf	0
x6	300.265193161375	0	Inf	0
x7	-365.06519515208	0	-Inf	0

x8	73.4922128465971	0	Inf	0
x9	-7.69221089935956	0	-Inf	0

Number of observations: 401, Error degrees of freedom: 391

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	21.5526862273347	400	0.0538817155683366		
Model	21.5526862273347	9	2.39474291414829	Inf	0
Residual	0	391	0		

for Q(x) A = 0.500000, B = 3.500000

for D(x) A = 0.100000, B = 0.500000

order Q(x) = 5.000000, order D(x) = 4.000000

Xmin = 1.000000 and Xmax = 5.000000

Ymin = 0.310268 and Ymax = 0.894821

List of powers for Q(x): [4.000000, 2.250000, 1.666667, 1.375000, 1.200000]

List of powers for D(x): [0.100000, 0.104106, 0.125911, 0.165695]

Fitting FresnelSine(x) in range (1.000000, 2.000000)

MSS of errors squared = 4.589759e-02

Corrected MSS of errors squared = 9.179518e-02

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 9

Fitting FresnelSine(x) in range (1.000000, 5.000000)

Shammas polynomial power for Q(x) is A+B*log10(i)^4

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	1.2926136929259	0	Inf	0
x1	-5560.13320128874	0	-Inf	0
x2	6939.51090691932	0	Inf	0
x3	-1573.14581230068	0	-Inf	0
x4	192.475502709103	0	Inf	0
x5	5.10202665222868	0	Inf	0
x6	-10.8377588699326	0	-Inf	0
x7	12.2685062231795	0	Inf	0

x8	-7.80580582499981	0	-Inf	0
x9	2.64661831092131	0	Inf	0
x10	-0.373596170552425	0	-Inf	0

Number of observations: 401, Error degrees of freedom: 390

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	21.5526862273086	400	0.0538817155682714		
Model	21.5526862273086	10	2.15526862273086	Inf	0
Residual	0	390	0		

for Q(x) A = 0.216638, B = 0.372577

for D(x) A = 0.000000, B = 0.381474

order Q(x) = 4.000000, order D(x) = 6.000000

Xmin = 1.000000 and Xmax = 5.000000

Ymin = 0.310268 and Ymax = 0.894821

List of powers for Q(x): [0.216638, 0.219698, 0.235946, 0.265591]

List of powers for D(x): [0.381474, 0.762947, 1.144421, 1.525894, 1.907368, 2.288841]

Fitting FresnelSine(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.413378e-01

Corrected MSS of errors squared = 2.826756e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 10

Fitting FresnelSine(x) in range (1.000000, 5.000000)

Shammas polynomial power for Q(x) is A+B*log10(i)^4

Shammas polynomial power for D(x) is A+B/i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.166146964731801	0	-Inf	0
x1	0	0	NaN	NaN
x2	0.197353177414609	0	Inf	0
x3	-0.0312062950569383	0	-Inf	0
x4	0.0119910096045551	0	Inf	0
x5	-5.81959236106034	0	-Inf	0
x6	167.120037344253	0	Inf	0

x7	-1203.66481485219	0	-Inf	0
x8	3263.1846278607	0	Inf	0
x9	-3677.34098699643	0	-Inf	0
x10	1457.50873807549	0	Inf	0

Number of observations: 401, Error degrees of freedom: 391

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	21.5526862273318	400	0.0538817155683296		
Model	21.5526862273318	9	2.39474291414798	Inf	0
Residual	0	391	0		

for Q(x) A = 0.000000, B = 0.100000

for D(x) A = 0.290069, B = 2.000000

order Q(x) = 3.000000, order D(x) = 7.000000

Xmin = 1.000000 and Xmax = 5.000000

Ymin = 0.310268 and Ymax = 0.894821

List of powers for Q(x): [0.000000, 0.000821, 0.005182]

List of powers for D(x): [2.290069, 1.290069, 0.956735, 0.790069, 0.690069, 0.623402, 0.575783]

Fitting FresnelSine(x) in range (1.000000, 2.000000)

MSS of errors squared = 9.127878e-02

Corrected MSS of errors squared = 1.825576e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Comments

The following table shows the summary results for the ten models that fit the Fresnel sine function. The results are sorted in ascending order using the MSSE values.

<i>Power Q(x)</i>	<i>Power D(x)</i>	<i>MSSE</i>	<i>Q/D Orders</i>
A+B/i	A+B*log10(i)^4	4.58975900E-02	5, 4
A+B/i	A+B*i	5.87649700E-02	5, 7
A+B*i+C*sqrt(i)	A+B*sqrt(i)+C*i	6.79312700E-02	6, 7
A+B*sqrt(i)+C*i	A+B*i+C*sqrt(i)	7.51492900E-02	3, 7
A+B*log10(i)^4	A+B/i	9.12787800E-02	3, 7
A+B*i	A+B*sqrt(i)	9.40371600E-02	5, 7
A+B*sqrt(i)	A+B*i	1.24327700E-01	4, 6
A+B*log10(i)^4	A+B*i	1.41337800E-01	4, 6
A+B*i	A+B*log10(i)^4	2.02028800E-01	4, 3
A+B*i	A+B/i	7.45099500E-01	4, 5

RESULTS FOR THE BESSEL JO(X)

Combo Powers 1

Fitting JO(x) in range (2.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B/i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12 + x13 + x14$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	2.38329562003581e-08	0	Inf	0
x1	-3.96434518650235e-07	0	-Inf	0
x2	1.22732583173316e-06	0	Inf	0
x3	-1.77940379726592e-06	0	-Inf	0
x4	1.46448846935267e-06	0	Inf	0
x5	-7.04746024911857e-07	0	-Inf	0
x6	1.85742532210608e-07	0	Inf	0
x7	-2.08052072461949e-08	0	-Inf	0
x8	0.020166113346286	0	Inf	0
x9	-1.04946272343231	0	-Inf	0
x10	5.00793510629033	0	Inf	0
x11	0	0	NaN	NaN
x12	-12.9364666983095	0	-Inf	0
x13	0	0	NaN	NaN
x14	9.95782820210501	0	Inf	0

Number of observations: 281, Error degrees of freedom: 268

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

SumSq	DF	MeanSq	F	pValue

Total	16.0887001722663	280	0.0574596434723796		
Model	16.0887001722663	12	1.34072501435552	Inf	0
Residual	0	268	0		

```

for Q(x) A = 0.484020, B = 0.500000
for D(x) A = 0.000000, B = 0.100000
order Q(x) = 7.000000, order D(x) = 7.000000
Xmin = 2.000000 and Xmax = 30.000000
Ymin = -0.402556 and Ymax = 0.300079
List of powers for Q(x): [0.984020, 1.484020, 1.984020, 2.484020, 2.984020, 3.484020,
3.984020]
List of powers for D(x): [0.100000, 0.050000, 0.033333, 0.025000, 0.020000, 0.016667,
0.014286]
Fitting J0(x) in range (1.000000, 2.000000)
MSS of errors squared = Inf
Corrected MSS of errors squared = Inf
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 2

```

Fitting J0(x) in range (2.000000, 30.000000)
Shammas polynomial power for Q(x) is A+B*i
Shammas polynomial power for D(x) is A+B*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00197711111516558	0	Inf	0
x1	-0.0147649239305027	0	-Inf	0
x2	0.0339449301231127	0	Inf	0
x3	-0.0375913095199522	0	-Inf	0
x4	0.0226728284481442	0	Inf	0
x5	-0.00717464519117283	0	-Inf	0
x6	0.000935896044314924	0	Inf	0
x7	132.957012101622	0	Inf	0
x8	-655.032389545416	0	-Inf	0
x9	979.937277861257	0	Inf	0
x10	0	0	NaN	NaN
x11	-1305.8097386246	0	-Inf	0
x12	1175.86874103707	0	Inf	0
x13	-326.920902782299	0	-Inf	0

Number of observations: 281, Error degrees of freedom: 268

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	16.0887001722594	280	0.0574596434723551		
Model	16.0887001722594	12	1.34072501435495	Inf	0
Residual	0	268	0		

for Q(x) A = 0.286570, B = 0.720811

for D(x) A = 0.234825, B = 0.361346

order Q(x) = 6.000000, order D(x) = 7.000000

Xmin = 2.000000 and Xmax = 30.000000

Ymin = -0.402556 and Ymax = 0.300079

List of powers for Q(x): [1.007380, 1.728191, 2.449002, 3.169812, 3.890623, 4.611434]

List of powers for D(x): [0.596170, 0.745845, 0.860694, 0.957516, 1.042818, 1.119937, 1.190856]

Fitting J0(x) in range (1.000000, 2.000000)

MSS of errors squared = 4.030675e+01

Corrected MSS of errors squared = 8.061349e+01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 3

Fitting J0(x) in range (2.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B*sqrt(i)+C*i

Shammas polynomial power for D(x) is A+B*i+C*sqrt(i)

Linear regression model:

$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.0154359408205616	0	Inf	0
x1	-0.472087055688907	0	-Inf	0
x2	1.61471770126303	0	Inf	0
x3	-2.19182458219404	0	-Inf	0
x4	1.34667320986817	0	Inf	0
x5	-0.312915147760843	0	-Inf	0
x6	14.9405716052431	0	Inf	0
x7	-40.5514825458801	0	-Inf	0
x8	46.2087383738769	0	Inf	0
x9	-24.7297434390902	0	-Inf	0

x10 5.1319159571171 0 Inf 0

Number of observations: 281, Error degrees of freedom: 270

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	16.0887001722675	280	0.057459643472384		
Model	16.0887001722675	10	1.60887001722675	Inf	0
Residual	0	270	0		

for Q(x) A = 0.250856, B = 0.400000, C = 0.090733

for D(x) A = 0.101670, B = 0.104739, C = 0.051488

order Q(x) = 5.000000, order D(x) = 5.000000

Xmin = 2.000000 and Xmax = 30.000000

Ymin = -0.402556 and Ymax = 0.300079

List of powers for Q(x): [0.741589, 0.998007, 1.215875, 1.413787, 1.598947]

List of powers for D(x): [0.257896, 0.383962, 0.505066, 0.623601, 0.740494]

Fitting J0(x) in range (1.000000, 2.000000)

MSS of errors squared = 6.862900e-02

Corrected MSS of errors squared = 1.681060e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 4

Fitting J0(x) in range (2.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-1.91574314456353e-06	0	-Inf	0
x1	-0.000244799099037003	0	-Inf	0
x2	0.00345119761582957	0	Inf	0
x3	-0.00822730249377396	0	-Inf	0
x4	0.00502281941697389	0	Inf	0
x5	209.533476281241	0	Inf	0
x6	-984.777434770466	0	-Inf	0
x7	2004.45936299387	0	Inf	0
x8	-2235.31028195641	0	-Inf	0

x9	1430.21444793314	0	Inf	0
x10	-495.516679274871	0	-Inf	0
x11	72.3971087936403	0	Inf	0

Number of observations: 281, Error degrees of freedom: 269

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	16.0887001723192	280	0.0574596434725685		
Model	16.0887001723192	11	1.46260910657447	Inf	0
Residual	0	269	0		

for Q(x) A = 0.409700, B = 0.131975

for D(x) A = 0.236983, B = 0.075653

order Q(x) = 4.000000, order D(x) = 7.000000

Xmin = 2.000000 and Xmax = 30.000000

Ymin = -0.402556 and Ymax = 0.300079

List of powers for Q(x): [0.541675, 0.475688, 0.453692, 0.442694]

List of powers for D(x): [0.312636, 0.388289, 0.463941, 0.539594, 0.615247, 0.690900, 0.766553]

Fitting J0(x) in range (1.000000, 2.000000)

MSS of errors squared = 5.648957e-01

Corrected MSS of errors squared = 1.129791e+00

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 5

Fitting J0(x) in range (2.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B*sqrt(i)

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.000147926150552383	0	-Inf	0
x1	0.00213038500142823	0	Inf	0
x2	-0.00518678675446334	0	-Inf	0
x3	0.00388926342785333	0	Inf	0
x4	0	0	NaN	NaN
x5	-0.000684935524977846	0	-Inf	0

x6	3.9910427280353	0	Inf	0
x7	-4.97762263701515	0	-Inf	0
x8	0	0	NaN	NaN
x9	4.95530860545783	0	Inf	0
x10	-3.95534030258915	0	-Inf	0
x11	0.986611606111543	0	Inf	0

Number of observations: 281, Error degrees of freedom: 271

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	16.0887001722662	280	0.0574596434723793		
Model	16.0887001722662	9	1.78763335247402	Inf	0
Residual	0	271	0		

for Q(x) A = 0.000000, B = 0.045231

for D(x) A = 0.000000, B = 0.009611

order Q(x) = 5.000000, order D(x) = 6.000000

Xmin = 2.000000 and Xmax = 30.000000

Ymin = -0.402556 and Ymax = 0.300079

List of powers for Q(x): [0.045231, 0.063967, 0.078343, 0.090463, 0.101141]

List of powers for D(x): [0.009611, 0.019223, 0.028834, 0.038445, 0.048056, 0.057668]

Fitting J0(x) in range (1.000000, 2.000000)

MSS of errors squared = Inf

Corrected MSS of errors squared = Inf

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 6

Fitting J0(x) in range (2.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B*i+C*sqrt(i)

Shammas polynomial power for D(x) is A+B*sqrt(i)+C*i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-4.95802816240703e-08	0	-Inf	0
x1	9.31366062197269e-08	0	Inf	0
x2	1.88369895265938e-07	0	Inf	0
x3	-3.98777672049162e-07	0	-Inf	0

x4	1.65970912595486e-07	0	Inf	0
x5	121.573170813125	0	Inf	0
x6	-651.566227874868	0	-Inf	0
x7	1490.16382973872	0	Inf	0
x8	-1835.05212003363	0	-Inf	0
x9	1278.59884613965	0	Inf	0
x10	-477.161514874121	0	-Inf	0
x11	74.4440160915229	0	Inf	0

Number of observations: 281, Error degrees of freedom: 269

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	16.088700172317	280	0.0574596434725607		
Model	16.088700172317	11	1.46260910657427	Inf	0
Residual	0	269	0		

for Q(x) A = 0.340000, B = 0.179366, C = 0.200386

for D(x) A = 0.184652, B = 0.100000, C = 0.065147

order Q(x) = 4.000000, order D(x) = 7.000000

Xmin = 2.000000 and Xmax = 30.000000

Ymin = -0.402556 and Ymax = 0.300079

List of powers for Q(x): [0.719751, 0.982120, 1.225175, 1.458234]

List of powers for D(x): [0.349800, 0.456368, 0.553299, 0.645242, 0.733996, 0.820485, 0.905259]

Fitting J0(x) in range (1.000000, 2.000000)

MSS of errors squared = 2.404869e+00

Corrected MSS of errors squared = 5.890703e+00

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 7

Fitting J0(x) in range (2.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	1.50609186354728e-10	0	Inf	0

x1	-1.22725624094938e-09	0	-Inf	0
x2	2.25444567097705e-09	0	Inf	0
x3	-1.56518116927432e-09	0	-Inf	0
x4	3.87385136915475e-10	0	Inf	0
x5	0.999999689657741	0	Inf	0
x6	4.18918382674608e-07	0	Inf	0
x7	-1.49108476084582e-07	0	-Inf	0
x8	5.28058852192058e-08	0	Inf	0
x9	-1.47038476142859e-08	0	-Inf	0
x10	2.65679810202154e-09	0	Inf	0
x11	-2.26486349415936e-10	0	-Inf	0

Number of observations: 281, Error degrees of freedom: 269

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	16.0887001722662	280	0.0574596434723794		
Model	16.0887001722662	11	1.46260910656966	Inf	0
Residual	0	269	0		

for Q(x) A = 0.152422, B = 0.168771

for D(x) A = 0.000000, B = 3.500000

order Q(x) = 4.000000, order D(x) = 7.000000

Xmin = 2.000000 and Xmax = 30.000000

Ymin = -0.402556 and Ymax = 0.300079

List of powers for Q(x): [0.321193, 0.489964, 0.658735, 0.827506]

List of powers for D(x): [0.000000, 0.028741, 0.181377, 0.459862, 0.835415, 1.283288, 1.785239]

Fitting J0(x) in range (1.000000, 2.000000)

MSS of errors squared = 7.344504e-01

Corrected MSS of errors squared = 1.468901e+00

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 8

Fitting J0(x) in range (2.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12 + x13 + x14$

Estimated Coefficients:

Estimate	SE	tStat	pValue
----------	----	-------	--------

(Intercept)	0.00423968344107825	0	Inf	0
x1	0.534038748225273	0	Inf	0
x2	-5.91875941206198	0	-Inf	0
x3	8.76676890172072	0	Inf	0
x4	0	0	NaN	NaN
x5	0	0	NaN	NaN
x6	0	0	NaN	NaN
x7	-3.38628793122854	0	-Inf	0
x8	43314.0477630356	0	Inf	0
x9	0	0	NaN	NaN
x10	-113770.204867794	0	-Inf	0
x11	119249.452198656	0	Inf	0
x12	-66436.3489204052	0	-Inf	0
x13	20330.6850810062	0	Inf	0
x14	-2686.63125449472	0	-Inf	0

Number of observations: 281, Error degrees of freedom: 270

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	16.0887001708834	280	0.0574596434674406		
Model	16.0887001708834	10	1.60887001708834	Inf	0
Residual	0	270	0		

for Q(x) A = 0.310405, B = 0.100000

for D(x) A = 0.453108, B = 0.306752

order Q(x) = 7.000000, order D(x) = 7.000000

Xmin = 2.000000 and Xmax = 30.000000

Ymin = -0.402556 and Ymax = 0.300079

List of powers for Q(x): [0.410405, 0.360405, 0.343739, 0.335405, 0.330405, 0.327072, 0.324691]

List of powers for D(x): [0.453108, 0.455627, 0.469005, 0.493412, 0.526327, 0.565580, 0.609573]

Fitting J0(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.485605e-01

Corrected MSS of errors squared = 2.971211e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 9

Fitting J0(x) in range (2.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B*log10(i)^4

Shammas polynomial power for $D(x)$ is $A+B*i$

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0	0	NaN	NaN
x1	-2.05795832692495e-08	0	-Inf	0
x2	2.60546547762144e-08	0	Inf	0
x3	-6.35212426733848e-09	0	-Inf	0
x4	8.77053149207535e-10	0	Inf	0
x5	1	0	Inf	0
x6	0	0	NaN	NaN
x7	0	0	NaN	NaN
x8	0	0	NaN	NaN
x9	0	0	NaN	NaN
x10	0	0	NaN	NaN
x11	0	0	NaN	NaN

Number of observations: 281, Error degrees of freedom: 276

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	16.0887001722662	280	0.0574596434723794		
Model	16.0887001722662	4	4.02217504306656	Inf	0
Residual	0	276	0		

for $Q(x)$ $A = 0.000000$, $B = 0.392368$

for $D(x)$ $A = 0.000000$, $B = 0.000000$

order $Q(x) = 4.000000$, order $D(x) = 7.000000$

Xmin = 2.000000 and Xmax = 30.000000

Ymin = -0.402556 and Ymax = 0.300079

List of powers for $Q(x)$: [0.000000, 0.003222, 0.020333, 0.051553]

List of powers for $D(x)$: [0.000000, 0.000000, 0.000000, 0.000000, 0.000000, 0.000000, 0.000000]

Fitting $J_0(x)$ in range (1.000000, 2.000000)

MSS of errors squared = 1.328796e-01

Corrected MSS of errors squared = 2.657591e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 10

Fitting $J_0(x)$ in range (2.000000, 30.000000)

Shammas polynomial power for $Q(x)$ is $A+B*\log_{10}(i)^4$

Shammas polynomial power for $D(x)$ is $A+B/i$

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.00099418038021312	0	-Inf	0
x1	0.706269597174027	0	Inf	0
x2	0	0	NaN	NaN
x3	-1.09622756122187	0	-Inf	0
x4	0	0	NaN	NaN
x5	0.801280307636249	0	Inf	0
x6	-0.515695055039276	0	-Inf	0
x7	0.105366889500934	0	Inf	0
x8	0.00492711909839454	0	Inf	0
x9	-2.42703084074423	0	-Inf	0
x10	71.5822029594818	0	Inf	0
x11	-528.804111504415	0	-Inf	0
x12	1466.46122464408	0	Inf	0
x13	-1686.01751748774	0	-Inf	0
x14	680.20030511134	0	Inf	0

Number of observations: 281, Error degrees of freedom: 268

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	16.0887001722616	280	0.0574596434723627		
Model	16.0887001722616	12	1.34072501435513	Inf	0
Residual	0	268	0		

for $Q(x)$ $A = 0.446844$, $B = 0.500000$

for $D(x)$ $A = 0.153802$, $B = 1.746071$

order $Q(x) = 7.000000$, order $D(x) = 7.000000$

Xmin = 2.000000 and Xmax = 30.000000

Ymin = -0.402556 and Ymax = 0.300079

List of powers for $Q(x)$: [0.446844, 0.450949, 0.472755, 0.512538, 0.566189, 0.630170, 0.701878]

List of powers for $D(x)$: [1.899873, 1.026837, 0.735825, 0.590319, 0.503016, 0.444813, 0.403240]

Fitting $J_0(x)$ in range (1.000000, 2.000000)

```
MSS of errors squared = 3.668817e-01
Corrected MSS of errors squared = 7.337633e-01
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf
```

Comments

The following table shows the summary results for the ten models that fit the Bessel $J_0(x)$ function. The results are sorted in ascending order using the MSSE values.

<i>Power Q(x)</i>	<i>Power D(x)</i>	<i>MSSE</i>	<i>Q/D Orders</i>
A+B*sqrt(i)+C*i	A+B*i+C*sqrt(i)	6.86290000E-02	5, 5
A+B*log10(i)^4	A+B*i	1.32879600E-01	4, 7
A+B/i	A+B*log10(i)^4	1.48560500E-01	7, 7
A+B*log10(i)^4	A+B/i	3.66881700E-01	7, 7
A+B/i	A+B*i	5.64895700E-01	4, 7
A+B*i	A+B*log10(i)^4	7.34450400E-01	4, 7
A+B*i+C*sqrt(i)	A+B*sqrt(i)+C*i	2.40486900E+00	4, 7
A+B*i	A+B*sqrt(i)	4.03067500E+01	6, 7
A+B*i	A+B/i	Inf	7, 7
A+B*sqrt(i)	A+B*i	Inf	5, 6

RESULTS FOR THE BESSEL J1(X)

Combo Powers 1

Fitting J1(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B/i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00147946038201581	0	Inf	0
x1	-0.163170707419611	0	-Inf	0
x2	0.527865441996054	0	Inf	0
x3	-0.658382639540998	0	-Inf	0
x4	0.372465737697665	0	Inf	0
x5	-0.0802572748644415	0	-Inf	0
x6	0.507302023245158	0	Inf	0
x7	-182.004522060166	0	-Inf	0
x8	4477.59139066643	0	Inf	0
x9	-29091.7158962018	0	-Inf	0
x10	73121.9023512723	0	Inf	0
x11	-77713.9353254091	0	-Inf	0
x12	29388.6546996958	0	Inf	0

Number of observations: 301, Error degrees of freedom: 288

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	14.5140717815458	300	0.0483802392718192		

Model	14.5140717815458	12	1.20950598179548	Inf	0
Residual	0	288	0		

for Q(x) A = 0.443188, B = 0.109454
 for D(x) A = 0.507408, B = 1.111978
 order Q(x) = 5.000000, order D(x) = 7.000000
 Xmin = 0.000000 and Xmax = 30.000000
 Ymin = -0.345961 and Ymax = 0.581517
 List of powers for Q(x): [0.552642, 0.662096, 0.771550, 0.881003, 0.990457]
 List of powers for D(x): [1.619385, 1.063397, 0.878067, 0.785402, 0.729803, 0.692737, 0.666262]
 Fitting J1(x) in range (1.000000, 2.000000)
 MSS of errors squared = 1.714217e-01
 Corrected MSS of errors squared = 3.428433e-01
 R-squared = 1.00000000
 R-squared Adjusted = 1.00000000
 Particle swarm AICc = -1.000000e+99
 AIC = -Inf
 AICc = -Inf

Combo Powers 2

Fitting J1(x) in range (0.000000, 30.000000)
 Shammas polynomial power for Q(x) is A+B*i
 Shammas polynomial power for D(x) is A+B*sqrt(i)

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12 + x13$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-3.81586968932491e-05	0	-Inf	0
x1	0.00014126427748092	0	Inf	0
x2	-0.000192785822390246	0	-Inf	0
x3	0.000128899772191112	0	Inf	0
x4	-4.54027966989393e-05	0	-Inf	0
x5	8.07983613348965e-06	0	Inf	0
x6	-5.71738782099571e-07	0	-Inf	0
x7	56.8414896496162	0	Inf	0
x8	-386.11098014669	0	-Inf	0
x9	1108.47508457896	0	Inf	0
x10	-1675.07117815249	0	-Inf	0
x11	1403.78765758053	0	Inf	0
x12	-619.755666876762	0	-Inf	0
x13	112.833592368428	0	Inf	0

Number of observations: 301, Error degrees of freedom: 287

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	14.5140717819326	300	0.0483802392731086		
Model	14.5140717819326	13	1.11646706014866	Inf	0
Residual	0	287	0		

for Q(x) A = 0.000000, B = 2.000000
 for D(x) A = 0.000000, B = 1.000000
 order Q(x) = 6.000000, order D(x) = 7.000000
 Xmin = 0.000000 and Xmax = 30.000000
 Ymin = -0.345961 and Ymax = 0.581517
 List of powers for Q(x): [2.000000, 4.000000, 6.000000, 8.000000, 10.000000, 12.000000]
 List of powers for D(x): [1.000000, 1.414214, 1.732051, 2.000000, 2.236068, 2.449490, 2.645751]
 Fitting J1(x) in range (1.000000, 2.000000)
 MSS of errors squared = 8.380167e-02
 Corrected MSS of errors squared = 1.676033e-01
 R-squared = 1.00000000
 R-squared Adjusted = 1.00000000
 Particle swarm AICc = -1.000000e+99
 AIC = -Inf
 AICc = -Inf

Combo Powers 3

Fitting J1(x) in range (0.000000, 30.000000)
 Shammas polynomial power for Q(x) is A+B*sqrt(i)+C*i
 Shammas polynomial power for D(x) is A+B*i+C*sqrt(i)

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.000316210195278438	0	Inf	0
x1	-0.00332015011005725	0	-Inf	0
x2	0.00853626129710834	0	Inf	0
x3	-0.00959317773124362	0	-Inf	0
x4	0.00512541649171486	0	Inf	0
x5	-0.00106376684058991	0	-Inf	0
x6	14.1643401159051	0	Inf	0
x7	-48.4986395077304	0	-Inf	0
x8	80.7440494286806	0	Inf	0
x9	-76.9141778187856	0	-Inf	0
x10	42.9168899388658	0	Inf	0
x11	-13.1130272937839	0	-Inf	0

x12 1.70056454947723 0 Inf 0

Number of observations: 301, Error degrees of freedom: 288

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	14.5140717819771	300	0.0483802392732571		
Model	14.5140717819771	12	1.20950598183143	Inf	0
Residual	0	288	0		

for Q(x) A = 0.215113, B = 0.100000, C = 0.400000

for D(x) A = 0.157193, B = 0.335205, C = 0.133088

order Q(x) = 5.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.345961 and Ymax = 0.581517

List of powers for Q(x): [0.715113, 1.156534, 1.588318, 2.015113, 2.438720]

List of powers for D(x): [0.625486, 1.015819, 1.393324, 1.764190, 2.130814, 2.494423, 2.855748]

Fitting J1(x) in range (1.000000, 2.000000)

MSS of errors squared = 5.433896e-01

Corrected MSS of errors squared = 1.331027e+00

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 4

Fitting J1(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00191742043632798	0	Inf	0
x1	-0.0117494238348941	0	-Inf	0
x2	0.0636586014014433	0	Inf	0
x3	-0.0538261865406682	0	-Inf	0
x4	14.2489422314474	0	Inf	0
x5	-46.8920926090522	0	-Inf	0
x6	75.1807494330681	0	Inf	0
x7	-69.5251278142749	0	-Inf	0

x8	37.905682114486	0	Inf	0
x9	-11.3712780016852	0	-Inf	0
x10	1.45312435064012	0	Inf	0

Number of observations: 301, Error degrees of freedom: 290

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	14.5140717819772	300	0.0483802392732572		
Model	14.5140717819772	10	1.45140717819772	Inf	0
Residual	0	290	0		

for Q(x) A = 0.211824, B = 0.188198

for D(x) A = 0.205156, B = 0.330412

order Q(x) = 3.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.345961 and Ymax = 0.581517

List of powers for Q(x): [0.400021, 0.305922, 0.274556]

List of powers for D(x): [0.535568, 0.865980, 1.196392, 1.526804, 1.857215, 2.187627, 2.518039]

Fitting J1(x) in range (1.000000, 2.000000)

MSS of errors squared = 9.824380e-01

Corrected MSS of errors squared = 1.964876e+00

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 5

Fitting J1(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B*sqrt(i)

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.0335131717975142	0	Inf	0
x1	-11.6949784492421	0	-Inf	0
x2	58.6558460850406	0	Inf	0
x3	-105.594192938196	0	-Inf	0
x4	81.9253008938954	0	Inf	0
x5	-23.3254886450769	0	-Inf	0

x6	61.6055370281793	0	Inf	0
x7	-254.874359520344	0	-Inf	0
x8	469.933497306183	0	Inf	0
x9	-481.933026316664	0	-Inf	0
x10	286.102278450089	0	Inf	0
x11	-92.4902886415814	0	-Inf	0
x12	12.6563616074729	0	Inf	0

Number of observations: 301, Error degrees of freedom: 288

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	14.5140717819736	300	0.0483802392732453		
Model	14.5140717819736	12	1.20950598183113	Inf	0
Residual	0	288	0		

for Q(x) A = 0.298548, B = 0.179832

for D(x) A = 0.401608, B = 0.210198

order Q(x) = 5.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.345961 and Ymax = 0.581517

List of powers for Q(x): [0.478380, 0.552868, 0.610025, 0.658211, 0.700664]

List of powers for D(x): [0.611806, 0.822005, 1.032203, 1.242401, 1.452600, 1.662798, 1.872997]

Fitting J1(x) in range (1.000000, 2.000000)

MSS of errors squared = 9.774654e-02

Corrected MSS of errors squared = 1.954931e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 6

Fitting J1(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B*i+C*sqrt(i)

Shammas polynomial power for D(x) is A+B*sqrt(i)+C*i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.000212033005682047	0	-Inf	0
x1	0.00347067820450396	0	Inf	0

x2	-0.00958851021446641	0	-Inf	0
x3	0.0109318502324043	0	Inf	0
x4	-0.00577184318420036	0	-Inf	0
x5	0.00116985618160942	0	Inf	0
x6	4000.95109831064	0	Inf	0
x7	-23339.1826034636	0	-Inf	0
x8	38848.53096061	0	Inf	0
x9	0	0	NaN	NaN
x10	-59709.2266896235	0	-Inf	0
x11	56711.4784875197	0	Inf	0
x12	-16511.5512533529	0	-Inf	0

Number of observations: 301, Error degrees of freedom: 289

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	14.5140717815052	300	0.0483802392716839		
Model	14.5140717815052	11	1.31946107104592	Inf	0
Residual	0	289	0		

for Q(x) A = 0.316559, B = 0.248865, C = 0.360000

for D(x) A = 0.317134, B = 0.100000, C = 0.000000

order Q(x) = 5.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.345961 and Ymax = 0.581517

List of powers for Q(x): [0.925423, 1.323405, 1.686691, 2.032017, 2.365866]

List of powers for D(x): [0.417134, 0.458555, 0.490339, 0.517134, 0.540740, 0.562082, 0.581709]

Fitting J1(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.905565e-01

Corrected MSS of errors squared = 4.667663e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 7

Fitting J1(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12 + x13

Estimated Coefficients:

Estimate	SE	tStat	pValue
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(Intercept)	-0.202310594581138	0	-Inf	0
x1	3.0202371105096	0	Inf	0
x2	-10.4732386281519	0	-Inf	0
x3	17.7819482992874	0	Inf	0
x4	-17.4336079274997	0	-Inf	0
x5	10.0771527122616	0	Inf	0
x6	-3.20385445656625	0	-Inf	0
x7	0.433673493540893	0	Inf	0
x8	64632.3223485419	0	Inf	0
x9	0	0	NaN	NaN
x10	-155066.980066066	0	-Inf	0
x11	137596.156471214	0	Inf	0
x12	-56629.3198578402	0	-Inf	0
x13	9468.82110414438	0	Inf	0

Number of observations: 301, Error degrees of freedom: 288

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	14.5140717847223	300	0.0483802392824076		
Model	14.5140717847223	12	1.20950598206019	Inf	0
Residual	0	288	0		

for Q(x) A = 0.126607, B = 0.219911

for D(x) A = 0.234518, B = 0.100000

order Q(x) = 7.000000, order D(x) = 6.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.345961 and Ymax = 0.581517

List of powers for Q(x): [0.346518, 0.566429, 0.786340, 1.006251, 1.226162, 1.446072, 1.665983]

List of powers for D(x): [0.234518, 0.235339, 0.239700, 0.247657, 0.258387, 0.271184]

Fitting J1(x) in range (1.000000, 2.000000)

MSS of errors squared = 5.307852e-02

Corrected MSS of errors squared = 1.061570e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 8

Fitting J1(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.535468692943155	0	Inf	0
x1	0.0118009925595528	0	Inf	0
x2	-2.16067048754664	0	-Inf	0
x3	32.8079144332778	0	Inf	0
x4	-134.120395899291	0	-Inf	0
x5	196.591508508592	0	Inf	0
x6	-93.6656262378497	0	-Inf	0
x7	2026.07251351612	0	Inf	0
x8	0	0	NaN	NaN
x9	-5103.82179927768	0	-Inf	0
x10	5053.48641027549	0	Inf	0
x11	-2636.14212521743	0	-Inf	0
x12	755.195775122633	0	Inf	0
x13	-93.7907744209614	0	-Inf	0

Number of observations: 301, Error degrees of freedom: 288

R-squared: 1, Adjusted R-squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	14.5140717819918	300	0.0483802392733058		
Model	14.5140717819918	12	1.20950598183265	Inf	0
Residual	0	288	0		

for Q(x) A = 0.100000, B = 1.197854

for D(x) A = 0.341604, B = 0.500000

order Q(x) = 6.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.345961 and Ymax = 0.581517

List of powers for Q(x): [1.297854, 0.698927, 0.499285, 0.399464, 0.339571, 0.299642]

List of powers for D(x): [0.341604, 0.345710, 0.367515, 0.407298, 0.460949, 0.524931, 0.596638]

Fitting J1(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.419162e+00

Corrected MSS of errors squared = 2.838323e+00

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 9

Fitting J1(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is $A+B*\log_{10}(i)^4$

Shammas polynomial power for D(x) is $A+B*i$

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	4.80368318009677e-14	0	Inf	0
x1	-1.23431413280145e-08	0	-Inf	0
x2	0	0	NaN	NaN
x3	2.02153970665085e-08	0	Inf	0
x4	0	0	NaN	NaN
x5	-1.75417279899914e-08	0	-Inf	0
x6	1.2498292951443e-08	0	Inf	0
x7	-2.82886871371688e-09	0	-Inf	0
x8	1	0	Inf	0
x9	0	0	NaN	NaN
x10	0	0	NaN	NaN
x11	0	0	NaN	NaN
x12	0	0	NaN	NaN
x13	0	0	NaN	NaN
x14	0	0	NaN	NaN

Number of observations: 301, Error degrees of freedom: 294

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	14.5140717819765	300	0.0483802392732549		
Model	14.5140717819765	6	2.41901196366275	Inf	0
Residual	0	294	0		

for Q(x) A = 1.428556, B = 0.348420

for D(x) A = 0.000000, B = 0.000000

order Q(x) = 7.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.345961 and Ymax = 0.581517

List of powers for Q(x): [1.428556, 1.431417, 1.446611, 1.474334, 1.511720, 1.556305, 1.606274]

List of powers for D(x): [0.000000, 0.000000, 0.000000, 0.000000, 0.000000, 0.000000, 0.000000]

Fitting J1(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.114395e-01
 Corrected MSS of errors squared = 2.228790e-01
 R-squared = 1.00000000
 R-squared Adjusted = 1.00000000
 Particle swarm AICc = -1.000000e+99
 AIC = -Inf
 AICc = -Inf

Combo Powers 10

Fitting J1(x) in range (0.000000, 30.000000)
 Shammas polynomial power for Q(x) is $A+B*\log_{10}(i)^4$
 Shammas polynomial power for D(x) is $A+B/i$

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	37.5380453438599	0	Inf	0
x1	-657.165729676572	0	-Inf	0
x2	0	0	NaN	NaN
x3	796.780559815027	0	Inf	0
x4	0	0	NaN	NaN
x5	-253.422431014981	0	-Inf	0
x6	76.2695560792113	0	Inf	0
x7	930.542312504281	0	Inf	0
x8	-27202.2922688242	0	-Inf	0
x9	90229.8332185709	0	Inf	0
x10	0	0	NaN	NaN
x11	-144777.458217587	0	-Inf	0
x12	0	0	NaN	NaN
x13	80820.374954937	0	Inf	0

Number of observations: 301, Error degrees of freedom: 291

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	14.5140717834643	300	0.0483802392782142		
Model	14.5140717834643	9	1.61267464260714	Inf	0
Residual	0	291	0		

for Q(x) A = 0.044646, B = 0.191182
 for D(x) A = 0.371662, B = 0.100000
 order Q(x) = 6.000000, order D(x) = 7.000000
 Xmin = 0.000000 and Xmax = 30.000000

```
Ymin = -0.345961 and Ymax = 0.581517
List of powers for Q(x): [0.044646, 0.046215, 0.054553, 0.069765, 0.090279, 0.114743]
List of powers for D(x): [0.471662, 0.421662, 0.404995, 0.396662, 0.391662, 0.388328,
0.385948]
Fitting J1(x) in range (1.000000, 2.000000)
MSS of errors squared = Inf
Corrected MSS of errors squared = Inf
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf
```

Comments

The following table shows the summary results for the ten models that fit the Bessel $J_1(x)$ function. The results are sorted in ascending order using the MSSE values.

<i>Power Q(x)</i>	<i>Power D(x)</i>	<i>MSSE</i>	<i>Q/D Orders</i>
A+B*i	A+B*log10(i)^4	5.30785200E-02	7, 6
A+B*i	A+B*sqrt(i)	8.38016700E-02	6, 7
A+B*sqrt(i)	A+B*i	9.77465400E-02	5, 7
A+B*log10(i)^4	A+B*i	1.11439500E-01	7, 7
A+B*i	A+B/i	1.71421700E-01	5, 7
A+B*i+C*sqrt(i)	A+B*sqrt(i)+C*i	1.90556500E-01	5, 7
A+B*sqrt(i)+C*i	A+B*i+C*sqrt(i)	5.43389600E-01	5, 7
A+B/i	A+B*i	9.82438000E-01	3, 7
A+B/i	A+B*log10(i)^4	1.41916200E+00	6, 7
A+B*log10(i)^4	A+B/i	Inf	6, 7

RESULTS FOR THE BESSEL J2(X)

Combo Powers 1

Fitting J2(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B/i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12 + x13 + x14$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.000856462614125072	0	Inf	0
x1	-0.00499393458803197	0	-Inf	0
x2	0.0124624590014294	0	Inf	0
x3	-0.0172538504770549	0	-Inf	0
x4	0.0143121349218328	0	Inf	0
x5	-0.00711295735745314	0	-Inf	0
x6	0.00196106189407583	0	Inf	0
x7	-0.000231374535403437	0	-Inf	0
x8	0.000670011742554377	0	Inf	0
x9	-0.371614036260413	0	-Inf	0
x10	11.9450108959548	0	Inf	0
x11	-95.0487492145923	0	-Inf	0
x12	281.93204139539	0	Inf	0
x13	-344.927524342109	0	-Inf	0
x14	147.470165288795	0	Inf	0

Number of observations: 301, Error degrees of freedom: 286

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

SumSq	DF	MeanSq	F	pValue

Total	16.2950552976667	300	0.0543168509922222		
Model	16.2950552976667	14	1.1639325212619	Inf	0
Residual	0	286	0		

```

for Q(x) A = 0.000000, B = 0.500000
for D(x) A = 0.000000, B = 2.000000
order Q(x) = 7.000000, order D(x) = 7.000000
Xmin = 0.000000 and Xmax = 30.000000
Ymin = -0.313525 and Ymax = 0.486207
List of powers for Q(x): [0.500000, 1.000000, 1.500000, 2.000000, 2.500000, 3.000000,
3.500000]
List of powers for D(x): [2.000000, 1.000000, 0.666667, 0.500000, 0.400000, 0.333333,
0.285714]
Fitting J2(x) in range (1.000000, 2.000000)
MSS of errors squared = 4.787318e-02
Corrected MSS of errors squared = 9.574635e-02
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 2

```

Fitting J2(x) in range (0.000000, 30.000000)
Shammas polynomial power for Q(x) is A+B*i
Shammas polynomial power for D(x) is A+B*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.000666540343556078	0	Inf	0
x1	-0.00320936405386968	0	-Inf	0
x2	0.00628128730273392	0	Inf	0
x3	-0.00665825476125622	0	-Inf	0
x4	0.00420832591628872	0	Inf	0
x5	-0.00159180131399697	0	-Inf	0
x6	0.000333869927953493	0	Inf	0
x7	-2.99449048043009e-05	0	-Inf	0
x8	61.2543740074695	0	Inf	0
x9	-430.04104285422	0	-Inf	0
x10	1267.24859515918	0	Inf	0
x11	-1958.660224167	0	-Inf	0
x12	1675.04608393698	0	Inf	0
x13	-753.426605257848	0	-Inf	0
x14	139.578818683098	0	Inf	0

Number of observations: 301, Error degrees of freedom: 286

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	16.2950552976202	300	0.0543168509920673		
Model	16.2950552976202	14	1.16393252125859	Inf	0
Residual	0	286	0		

for Q(x) A = 0.096332, B = 1.162829

for D(x) A = 0.000000, B = 0.766906

order Q(x) = 7.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.313525 and Ymax = 0.486207

List of powers for Q(x): [1.259161, 2.421990, 3.584819, 4.747647, 5.910476, 7.073305, 8.236134]

List of powers for D(x): [0.766906, 1.084569, 1.328321, 1.533813, 1.714855, 1.878529, 2.029043]

Fitting J2(x) in range (1.000000, 2.000000)

MSS of errors squared = 8.021262e-02

Corrected MSS of errors squared = 1.604252e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 3

Fitting J2(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B*sqrt(i)+C*i

Shammas polynomial power for D(x) is A+B*i+C*sqrt(i)

Linear regression model:

$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00414343000600488	0	Inf	0
x1	-0.0279456556521503	0	-Inf	0
x2	0.0598837317477082	0	Inf	0
x3	-0.058478118139271	0	-Inf	0
x4	0.0274268144463701	0	Inf	0
x5	-0.00502937530926964	0	-Inf	0
x6	43.442197029154	0	Inf	0
x7	-130.306719865778	0	-Inf	0

x8	154.330205239125	0	Inf	0
x9	-84.0525438986178	0	-Inf	0
x10	17.5868608886441	0	Inf	0

Number of observations: 301, Error degrees of freedom: 290

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	16.2950552976551	300	0.0543168509921838		
Model	16.2950552976551	10	1.62950552976551	Inf	0
Residual	0	290	0		

for Q(x) A = 0.000000, B = 0.400000, C = 0.382098

for D(x) A = 0.245661, B = 0.100000, C = 0.000000

order Q(x) = 5.000000, order D(x) = 5.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.313525 and Ymax = 0.486207

List of powers for Q(x): [0.782098, 1.329882, 1.839115, 2.328393, 2.804919]

List of powers for D(x): [0.345661, 0.445661, 0.545661, 0.645661, 0.745661]

Fitting J2(x) in range (1.000000, 2.000000)

MSS of errors squared = 3.282345e-01

Corrected MSS of errors squared = 8.040071e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 4

Fitting J2(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.00660062788343477	0	-Inf	0
x1	-7.39429127210928	0	-Inf	0
x2	93.9575569962385	0	Inf	0
x3	-215.762405089385	0	-Inf	0
x4	129.205740675113	0	Inf	0
x5	12.5354702584934	0	Inf	0
x6	-36.1569804927562	0	-Inf	0

x7	48.5506401216093	0	Inf	0
x8	-35.2074788507859	0	-Inf	0
x9	13.3753260333017	0	Inf	0
x10	-2.09697758943947	0	-Inf	0

Number of observations: 301, Error degrees of freedom: 290

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	16.2950552976569	300	0.0543168509921898		
Model	16.2950552976569	10	1.62950552976569	Inf	0
Residual	0	290	0		

for Q(x) A = 0.251127, B = 0.047695

for D(x) A = 0.142809, B = 0.215802

order Q(x) = 4.000000, order D(x) = 6.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.313525 and Ymax = 0.486207

List of powers for Q(x): [0.298822, 0.274974, 0.267025, 0.263050]

List of powers for D(x): [0.358611, 0.574413, 0.790215, 1.006018, 1.221820, 1.437622]

Fitting J2(x) in range (1.000000, 2.000000)

MSS of errors squared = 3.050154e-01

Corrected MSS of errors squared = 6.100308e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 5

Fitting J2(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B*sqrt(i)

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	7.62354393050065e-05	0	Inf	0
x1	-0.0112233625608459	0	-Inf	0
x2	0.0367361659914732	0	Inf	0
x3	-0.0397358900324646	0	-Inf	0
x4	0.0141474318401013	0	Inf	0
x5	6.07228057023005	0	Inf	0

x6	-15.7839386486404	0	-Inf	0
x7	22.765688923858	0	Inf	0
x8	-19.677004590449	0	-Inf	0
x9	10.1915059662018	0	Inf	0
x10	-2.92876658639988	0	-Inf	0
x11	0.360233917989326	0	Inf	0

Number of observations: 301, Error degrees of freedom: 289

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	16.2950552976597	300	0.054316850992199		
Model	16.2950552976597	11	1.48136866342361	Inf	0
Residual	0	289	0		

for Q(x) A = 1.252920, B = 0.447141

for D(x) A = 0.000000, B = 0.379631

order Q(x) = 4.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.313525 and Ymax = 0.486207

List of powers for Q(x): [1.700061, 1.885272, 2.027391, 2.147202]

List of powers for D(x): [0.379631, 0.759263, 1.138894, 1.518526, 1.898157, 2.277789, 2.657420]

Fitting J2(x) in range (1.000000, 2.000000)

MSS of errors squared = 5.148291e-01

Corrected MSS of errors squared = 1.029658e+00

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 6

Fitting J2(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B*i+C*sqrt(i)

Shammas polynomial power for D(x) is A+B*sqrt(i)+C*i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12 + x13 + x14$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	53.9460792671638	0	Inf	0
x1	-1685.0981357734	0	-Inf	0
x2	8455.19601113907	0	Inf	0

x3	-19094.5045972044	0	-Inf	0
x4	23525.0818677864	0	Inf	0
x5	-16458.4230779998	0	-Inf	0
x6	6170.16094869697	0	Inf	0
x7	-966.359094490861	0	-Inf	0
x8	9.64454514914336	0	Inf	0
x9	-30.0148465373473	0	-Inf	0
x10	47.4448212362104	0	Inf	0
x11	-43.3861574402814	0	-Inf	0
x12	23.3292361816534	0	Inf	0
x13	-6.87883702130033	0	-Inf	0
x14	0.861237441623155	0	Inf	0

Number of observations: 301, Error degrees of freedom: 286

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	16.2950552975325	300	0.0543168509917749		
Model	16.2950552975325	14	1.16393252125232	Inf	0
Residual	0	286	0		

for Q(x) A = 0.043402, B = 0.100000, C = 0.360000

for D(x) A = 0.033798, B = 0.222158, C = 0.400000

order Q(x) = 7.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.313525 and Ymax = 0.486207

List of powers for Q(x): [0.503402, 0.752519, 0.966941, 1.163402, 1.348387, 1.525219, 1.695873]

List of powers for D(x): [0.655956, 1.147977, 1.618587, 2.078114, 2.530558, 2.977971, 3.421573]

Fitting J2(x) in range (1.000000, 2.000000)

MSS of errors squared = 8.029042e-02

Corrected MSS of errors squared = 1.966706e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 7

Fitting J2(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-5.38557933469542e-12	0	-Inf	0
x1	3.99994440241065e-11	0	Inf	0
x2	-8.60812524090268e-11	0	-Inf	0
x3	8.29186474012011e-11	0	Inf	0
x4	-3.84189240455304e-11	0	-Inf	0
x5	6.9689071620607e-12	0	Inf	0
x6	0.999996062975923	0	Inf	0
x7	4.98871688926847e-06	0	Inf	0
x8	-1.22179668563746e-06	0	-Inf	0
x9	1.70103872477592e-07	0	Inf	0

Number of observations: 301, Error degrees of freedom: 291

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	16.2950552976593	300	0.0543168509921977		
Model	16.2950552976593	9	1.81056169973992	Inf	0
Residual	0	291	0		

for Q(x) A = 0.174898, B = 0.429611

for D(x) A = 0.000000, B = 0.100000

order Q(x) = 5.000000, order D(x) = 4.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.313525 and Ymax = 0.486207

List of powers for Q(x): [0.604510, 1.034121, 1.463732, 1.893344, 2.322955]

List of powers for D(x): [0.000000, 0.000821, 0.005182, 0.013139]

Fitting J2(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.574918e-01

Corrected MSS of errors squared = 3.149836e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 8

Fitting J2(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-4.89297124112054e-07	0	-Inf	0
x1	-9.28596187839445e-07	0	-Inf	0
x2	1.7478960958454e-05	0	Inf	0
x3	-4.81359394579513e-05	0	-Inf	0
x4	3.20748921927914e-05	0	Inf	0
x5	356.910974104249	0	Inf	0
x6	0	0	NaN	NaN
x7	-861.44660825124	0	-Inf	0
x8	810.580316083215	0	Inf	0
x9	-402.397986951356	0	-Inf	0
x10	110.686998365473	0	Inf	0
x11	-13.333693350358	0	-Inf	0

Number of observations: 301, Error degrees of freedom: 290

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	16.2950552976486	300	0.054316850992162		
Model	16.2950552976486	10	1.62950552976486	Inf	0
Residual	0	290	0		

for Q(x) A = 0.185623, B = 0.421746

for D(x) A = 0.100000, B = 0.246135

order Q(x) = 4.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.313525 and Ymax = 0.486207

List of powers for Q(x): [0.607369, 0.396496, 0.326205, 0.291059]

List of powers for D(x): [0.100000, 0.102021, 0.112755, 0.132339, 0.158750, 0.190246, 0.225546]

Fitting J2(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.757294e-01

Corrected MSS of errors squared = 3.514589e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 9

Fitting J2(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is $A+B*\log_{10}(i)^4$

Shammas polynomial power for D(x) is $A+B*i$

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	1.45886842799446e-05	0	Inf	0
x1	-0.307577208499461	0	-Inf	0
x2	0.364287841943887	0	Inf	0
x3	-0.0567249171453878	0	-Inf	0
x4	5.49794313291558	0	Inf	0
x5	-12.5870002844758	0	-Inf	0
x6	15.3591657137182	0	Inf	0
x7	-10.5354274932064	0	-Inf	0
x8	3.85162787153166	0	Inf	0
x9	-0.586309175513437	0	-Inf	0

Number of observations: 301, Error degrees of freedom: 291

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	16.2950552976593	300	0.0543168509921978		
Model	16.2950552976593	9	1.81056169973993	Inf	0
Residual	0	291	0		

for Q(x) A = 1.559270, B = 0.384713

for D(x) A = 0.000000, B = 0.241640

order Q(x) = 3.000000, order D(x) = 6.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.313525 and Ymax = 0.486207

List of powers for Q(x): [1.559270, 1.562430, 1.579207]

List of powers for D(x): [0.241640, 0.483280, 0.724920, 0.966560, 1.208200, 1.449840]

Fitting J2(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.440178e-01

Corrected MSS of errors squared = 2.880355e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 10

Fitting J2(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B*log10(i)^4

Shammas polynomial power for D(x) is A+B/i

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	5.85280959710724e-05	0	Inf	0
x1	0	0	NaN	NaN
x2	-6.94385167438367e-05	0	-Inf	0
x3	1.09108346004475e-05	0	Inf	0
x4	0.00071967702078793	0	Inf	0
x5	-0.385214890517842	0	-Inf	0
x6	12.2353700015541	0	Inf	0
x7	-96.7784602953558	0	-Inf	0
x8	286.032216436359	0	Inf	0
x9	-349.104892953003	0	-Inf	0
x10	149.000262023571	0	Inf	0

Number of observations: 301, Error degrees of freedom: 291

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	16.2950552976534	300	0.0543168509921779		
Model	16.2950552976534	9	1.81056169973926	Inf	0
Residual	0	291	0		

for Q(x) A = 0.000000, B = 0.500000

for D(x) A = 0.000000, B = 1.792747

order Q(x) = 3.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.313525 and Ymax = 0.486207

List of powers for Q(x): [0.000000, 0.004106, 0.025911]

List of powers for D(x): [1.792747, 0.896374, 0.597582, 0.448187, 0.358549, 0.298791, 0.256107]

Fitting J2(x) in range (1.000000, 2.000000)

MSS of errors squared = 2.011136e-01

Corrected MSS of errors squared = 4.022273e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Comments

The following table shows the summary results for the ten models that fit the Bessel $J_2(x)$ function. The results are sorted in ascending order using the MSSE values.

<i>Power Q(x)</i>	<i>Power D(x)</i>	<i>MSSE</i>	<i>Q/D Orders</i>
A+B*i	A+B/i	4.78731800E-02	7, 7
A+B*i	A+B*sqrt(i)	8.02126200E-02	7, 7
A+B*i+C*sqrt(i)	A+B*sqrt(i)+C*i	8.02904200E-02	7, 7
A+B*log10(i)^4	A+B*i	1.44017800E-01	3, 6
A+B*i	A+B*log10(i)^4	1.57491800E-01	5, 4
A+B/i	A+B*log10(i)^4	1.75729400E-01	4, 7
A+B*log10(i)^4	A+B/i	2.01113600E-01	3, 7
A+B/i	A+B*i	3.05015400E-01	4, 6
A+B*sqrt(i)+C*i	A+B*i+C*sqrt(i)	3.28234500E-01	5, 5
A+B*sqrt(i)	A+B*i	5.14829100E-01	4, 7

RESULTS FOR THE BESSEL J3(X)

Combo Powers 1

Fitting J3(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B/i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12 + x13 + x14$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.105815436754389	0	Inf	0
x1	-1.43509729041015	0	-Inf	0
x2	4.42653864748761	0	Inf	0
x3	-6.57383170031216	0	-Inf	0
x4	5.590941491171	0	Inf	0
x5	-2.78577206933893	0	-Inf	0
x6	0.758832678085858	0	Inf	0
x7	-0.0874268331078495	0	-Inf	0
x8	-0.980235603305949	0	-Inf	0
x9	134.404187303991	0	Inf	0
x10	-1621.75748888807	0	-Inf	0
x11	5209.79990653918	0	Inf	0
x12	-5199.09640962681	0	-Inf	0
x13	0	0	NaN	NaN
x14	1478.63004001562	0	Inf	0

Number of observations: 301, Error degrees of freedom: 287

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

SumSq	DF	MeanSq	F	pValue

Total	17.4514907484554	300	0.0581716358281847		
Model	17.4514907484554	13	1.3424223652658	Inf	0
Residual	0	287	0		

```

for Q(x) A = 0.251119, B = 0.366919
for D(x) A = 0.453491, B = 0.906219
order Q(x) = 7.000000, order D(x) = 7.000000
Xmin = 0.000000 and Xmax = 30.000000
Ymin = -0.291132 and Ymax = 0.434394
List of powers for Q(x): [0.618039, 0.984958, 1.351878, 1.718797, 2.085717, 2.452636,
2.819556]
List of powers for D(x): [1.359710, 0.906600, 0.755564, 0.680046, 0.634735, 0.604527,
0.582951]
Fitting J3(x) in range (1.000000, 2.000000)
MSS of errors squared = 7.396576e-02
Corrected MSS of errors squared = 1.479315e-01
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 2

```

Fitting J3(x) in range (0.000000, 30.000000)
Shammas polynomial power for Q(x) is A+B*i
Shammas polynomial power for D(x) is A+B*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	2.92648117088656	0	Inf	0
x1	-14.0133618779831	0	-Inf	0
x2	26.8366433622279	0	Inf	0
x3	-25.6928910399438	0	-Inf	0
x4	12.2969195141997	0	Inf	0
x5	-2.35379099374227	0	-Inf	0
x6	65.1309926977357	0	Inf	0
x7	-469.329106152689	0	-Inf	0
x8	1411.32360969126	0	Inf	0
x9	-2219.30914841409	0	-Inf	0
x10	1927.29012295837	0	Inf	0
x11	-879.087551552668	0	-Inf	0
x12	164.981080672202	0	Inf	0

Number of observations: 301, Error degrees of freedom: 288

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	17.4514907484598	300	0.0581716358281994		
Model	17.4514907484598	12	1.45429089570498	Inf	0
Residual	0	288	0		

for Q(x) A = 0.000000, B = 0.100000

for D(x) A = 0.000000, B = 0.540878

order Q(x) = 5.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.291132 and Ymax = 0.434394

List of powers for Q(x): [0.100000, 0.200000, 0.300000, 0.400000, 0.500000]

List of powers for D(x): [0.540878, 0.764917, 0.936828, 1.081756, 1.209440, 1.324875, 1.431029]

Fitting J3(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.379624e-01

Corrected MSS of errors squared = 2.759248e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 3

Fitting J3(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B*sqrt(i)+C*i

Shammas polynomial power for D(x) is A+B*i+C*sqrt(i)

Linear regression model:

$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.000114038240658417	0	-Inf	0
x1	0.000344016406395114	0	Inf	0
x2	-0.000329609475575353	0	-Inf	0
x3	9.91542739075696e-05	0	Inf	0
x4	7.85598899457821	0	Inf	0
x5	-23.1388835109519	0	-Inf	0
x6	36.0383784411274	0	Inf	0
x7	-32.9813364677287	0	-Inf	0
x8	17.8962920290747	0	Inf	0
x9	-5.35261392756582	0	-Inf	0
x10	0.682174744551758	0	Inf	0

Number of observations: 301, Error degrees of freedom: 290

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	17.4514907484611	300	0.0581716358282036		
Model	17.4514907484611	10	1.74514907484611	Inf	0
Residual	0	290	0		

for Q(x) A = 0.052759, B = 0.102836, C = 0.400000

for D(x) A = 0.000000, B = 0.361979, C = 0.109307

order Q(x) = 3.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.291132 and Ymax = 0.434394

List of powers for Q(x): [0.555594, 0.998190, 1.430876]

List of powers for D(x): [0.471286, 0.878541, 1.275261, 1.666528, 2.054311, 2.439618, 2.823049]

Fitting J3(x) in range (1.000000, 2.000000)

MSS of errors squared = 3.045918e-01

Corrected MSS of errors squared = 7.460946e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 4

Fitting J3(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12 + x13$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	2.34327797598656	0	Inf	0
x1	7.70548946981198	0	Inf	0
x2	-113.472714918718	0	-Inf	0
x3	200.389405968393	0	Inf	0
x4	0	0	NaN	NaN
x5	0	0	NaN	NaN
x6	-96.9654561983383	0	-Inf	0
x7	0	0	NaN	NaN
x8	10.1381069828503	0	Inf	0

x9	-27.6028883395995	0	-Inf	0
x10	35.7853967294528	0	Inf	0
x11	-25.2539803427127	0	-Inf	0
x12	9.37165501320934	0	Inf	0
x13	-1.43829178466603	0	-Inf	0

Number of observations: 301, Error degrees of freedom: 290

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	17.4514907484591	300	0.0581716358281972		
Model	17.4514907484591	10	1.74514907484591	Inf	0
Residual	0	290	0		

for Q(x) A = 0.100000, B = 0.146011

for D(x) A = 0.125079, B = 0.250671

order Q(x) = 7.000000, order D(x) = 6.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.291132 and Ymax = 0.434394

List of powers for Q(x): [0.246011, 0.173006, 0.148670, 0.136503, 0.129202, 0.124335, 0.120859]

List of powers for D(x): [0.375750, 0.626422, 0.877093, 1.127765, 1.378436, 1.629108]

Fitting J3(x) in range (1.000000, 2.000000)

MSS of errors squared = 4.045858e-01

Corrected MSS of errors squared = 8.091715e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 5

Fitting J3(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B*sqrt(i)

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00323167926845959	0	Inf	0
x1	-1.18350464412488	0	-Inf	0
x2	5.6687455233531	0	Inf	0
x3	-9.8108692407365	0	-Inf	0

x4	7.34759077581321	0	Inf	0
x5	-2.02519199245851	0	-Inf	0
x6	5.9659693272454	0	Inf	0
x7	-15.249075532682	0	-Inf	0
x8	21.644456851831	0	Inf	0
x9	-18.4232714372955	0	-Inf	0
x10	9.40274027657472	0	Inf	0
x11	-2.66400974407061	0	-Inf	0
x12	0.323188695655042	0	Inf	0

Number of observations: 301, Error degrees of freedom: 288

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	17.4514907484616	300	0.0581716358282053		
Model	17.4514907484616	12	1.45429089570513	Inf	0
Residual	0	288	0		

for Q(x) A = 1.110211, B = 0.500000

for D(x) A = 0.000000, B = 0.449787

order Q(x) = 5.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.291132 and Ymax = 0.434394

List of powers for Q(x): [1.610211, 1.817318, 1.976236, 2.110211, 2.228245]

List of powers for D(x): [0.449787, 0.899574, 1.349361, 1.799148, 2.248936, 2.698723, 3.148510]

Fitting J3(x) in range (1.000000, 2.000000)

MSS of errors squared = 2.405990e-01

Corrected MSS of errors squared = 4.811980e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 6

Fitting J3(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is $A+B*i+C*\sqrt{i}$

Shammas polynomial power for D(x) is $A+B*\sqrt{i}+C*i$

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12 + x13 + x14$$

Estimated Coefficients:

Estimate	SE	tStat	pValue

(Intercept)	10.2145826182601	0	Inf	0
x1	-64.0933055106026	0	-Inf	0
x2	172.249172382076	0	Inf	0
x3	-257.014434897533	0	-Inf	0
x4	229.951828008837	0	Inf	0
x5	-123.365764404461	0	-Inf	0
x6	36.745715614373	0	Inf	0
x7	-4.68779339659633	0	-Inf	0
x8	69.2043973165806	0	Inf	0
x9	-363.164185257934	0	-Inf	0
x10	822.300037038698	0	Inf	0
x11	-1003.45770061317	0	-Inf	0
x12	692.089550533499	0	Inf	0
x13	-255.274790450782	0	-Inf	0
x14	39.3026911364101	0	Inf	0

Number of observations: 301, Error degrees of freedom: 286

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	17.4514907484629	300	0.0581716358282097		
Model	17.4514907484629	14	1.24653505346164	Inf	0
Residual	0	286	0		

for Q(x) A = 0.000000, B = 0.289666, C = 0.000000

for D(x) A = 0.271429, B = 0.324121, C = 0.122188

order Q(x) = 7.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.291132 and Ymax = 0.434394

List of powers for Q(x): [0.289666, 0.579333, 0.868999, 1.158666, 1.448332, 1.737999, 2.027665]

List of powers for D(x): [0.717738, 0.974182, 1.199388, 1.408424, 1.607127, 1.798489, 1.984290]

Fitting J3(x) in range (1.000000, 2.000000)

MSS of errors squared = 6.495733e-02

Corrected MSS of errors squared = 1.591123e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 7

Fitting J3(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00213541765454805	0	Inf	0
x1	-0.087675346523962	0	-Inf	0
x2	0.230279936899344	0	Inf	0
x3	-0.227830396115196	0	-Inf	0
x4	0.0979396171706259	0	Inf	0
x5	-0.0148491027975458	0	-Inf	0
x6	2.70465299733853	0	Inf	0
x7	-1.81983065103422	0	-Inf	0
x8	0.121849329335728	0	Inf	0
x9	-0.00667176587068697	0	-Inf	0

Number of observations: 301, Error degrees of freedom: 291

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	17.4514907484613	300	0.0581716358282044		
Model	17.4514907484613	9	1.93905452760681	Inf	0
Residual	0	291	0		

for Q(x) A = 0.381880, B = 0.117820

for D(x) A = 0.033273, B = 3.500000

order Q(x) = 5.000000, order D(x) = 4.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.291132 and Ymax = 0.434394

List of powers for Q(x): [0.499700, 0.617519, 0.735339, 0.853158, 0.970978]

List of powers for D(x): [0.033273, 0.062014, 0.214650, 0.493135]

Fitting J3(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.465062e-01

Corrected MSS of errors squared = 2.930124e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 8

Fitting J3(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	3.8054570860185e-05	0	Inf	0
x1	-9.07755994721418e-07	0	-Inf	0
x2	9.4484862163521e-05	0	Inf	0
x3	-0.00082160187596544	0	-Inf	0
x4	0.00182064870114798	0	Inf	0
x5	-0.00113067646629264	0	-Inf	0
x6	140.65833531523	0	Inf	0
x7	0	0	NaN	NaN
x8	-300.265773066422	0	-Inf	0
x9	229.285439923535	0	Inf	0
x10	-80.1595948767908	0	-Inf	0
x11	11.4815927029462	0	Inf	0

Number of observations: 301, Error degrees of freedom: 290

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	17.4514907484589	300	0.0581716358281965		
Model	17.4514907484589	10	1.74514907484589	Inf	0
Residual	0	290	0		

for Q(x) A = 0.106592, B = 2.693969

for D(x) A = 0.100000, B = 0.279066

order Q(x) = 5.000000, order D(x) = 6.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.291132 and Ymax = 0.434394

List of powers for Q(x): [2.800561, 1.453576, 1.004582, 0.780084, 0.645386]

List of powers for D(x): [0.100000, 0.102292, 0.114462, 0.136666, 0.166610, 0.202321]

Fitting J3(x) in range (1.000000, 2.000000)

MSS of errors squared = 9.813203e-01

Corrected MSS of errors squared = 1.962641e+00

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 9

Fitting J3(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is $A+B*\log_{10}(i)^4$

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	6.14724317222272e-06	0	Inf	0
x1	-125.603566630193	0	-Inf	0
x2	0	0	NaN	NaN
x3	0	0	NaN	NaN
x4	434.134430304127	0	Inf	0
x5	-438.280144280731	0	-Inf	0
x6	129.749274529375	0	Inf	0
x7	17.9378993528129	0	Inf	0
x8	-39.5537662545435	0	-Inf	0
x9	31.158540193196	0	Inf	0
x10	-8.54267334446047	0	-Inf	0

Number of observations: 301, Error degrees of freedom: 292

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	17.4514907484578	300	0.0581716358281927		
Model	17.4514907484578	8	2.18143634355723	Inf	0
Residual	0	292	0		

for Q(x) A = 2.200000, B = 0.026652

for D(x) A = 0.043459, B = 0.023306

order Q(x) = 6.000000, order D(x) = 4.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.291132 and Ymax = 0.434394

List of powers for Q(x): [2.200000, 2.200219, 2.201381, 2.203502, 2.206362, 2.209772]

List of powers for D(x): [0.066766, 0.090072, 0.113378, 0.136684]

Fitting J3(x) in range (1.000000, 2.000000)

MSS of errors squared = 3.103942e-01

Corrected MSS of errors squared = 6.207884e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 10

Fitting J3(x) in range (0.000000, 30.000000)

Shammas polynomial power for $Q(x)$ is $A+B*\log_{10}(i)^4$

Shammas polynomial power for $D(x)$ is $A+B/i$

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0	0	NaN	NaN
x1	0.181011724035144	0	Inf	0
x2	0	0	NaN	NaN
x3	-0.443720118639931	0	-Inf	0
x4	0.40656832335915	0	Inf	0
x5	-0.174445290071426	0	-Inf	0
x6	0.0305853613265506	0	Inf	0
x7	0.0317745300392642	0	Inf	0
x8	-1.95272041077756	0	-Inf	0
x9	13.2864615601415	0	Inf	0
x10	-21.0797867661875	0	-Inf	0
x11	0	0	NaN	NaN
x12	10.7142710867772	0	Inf	0

Number of observations: 301, Error degrees of freedom: 291

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	17.4514907484612	300	0.0581716358282039		
Model	17.4514907484612	9	1.9390545276068	Inf	0
Residual	0	291	0		

for $Q(x)$ $A = 0.000000$, $B = 0.100000$

for $D(x)$ $A = 0.000000$, $B = 0.100000$

order $Q(x) = 6.000000$, order $D(x) = 6.000000$

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.291132 and Ymax = 0.434394

List of powers for $Q(x)$: [0.000000, 0.000821, 0.005182, 0.013139, 0.023869, 0.036665]

List of powers for $D(x)$: [0.100000, 0.050000, 0.033333, 0.025000, 0.020000, 0.016667]

Fitting J3(x) in range (1.000000, 2.000000)

MSS of errors squared = Inf

Corrected MSS of errors squared = Inf

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Comments

The following table shows the summary results for the ten models that fit the Bessel $J_3(x)$ function. The results are sorted in ascending order using the MSSE values.

<i>Power Q(x)</i>	<i>Power D(x)</i>	<i>MSSE</i>	<i>Q/D Orders</i>
A+B*i+C*sqrt(i)	A+B*sqrt(i)+C*i	6.49573300E-02	7, 7
A+B*i	A+B/i	7.39657600E-02	7, 7
A+B*i	A+B*sqrt(i)	1.37962400E-01	5, 7
A+B*i	A+B*log10(i)^4	1.46506200E-01	5, 4
A+B*sqrt(i)	A+B*i	2.40599000E-01	5, 7
A+B*sqrt(i)+C*i	A+B*i+C*sqrt(i)	3.04591800E-01	3, 7
A+B*log10(i)^4	A+B*i	3.10394200E-01	6, 4
A+B/i	A+B*i	4.04585800E-01	7, 6
A+B/i	A+B*log10(i)^4	9.81320300E-01	5, 6
A+B*log10(i)^4	A+B/i	Inf	6, 6

RESULTS FOR THE BESSEL J4(X)

Combo Powers 1

Fitting J4(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B/i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12 + x13 + x14$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.0107484015313061	0	Inf	0
x1	-0.460719401037082	0	-Inf	0
x2	1.67915743473963	0	Inf	0
x3	-2.72306926670345	0	-Inf	0
x4	2.45294930112733	0	Inf	0
x5	-1.27825237794406	0	-Inf	0
x6	0.362869290002897	0	Inf	0
x7	-0.0436832062771219	0	-Inf	0
x8	26.834770544752	0	Inf	0
x9	-904.624680551182	0	-Inf	0
x10	3183.40806197793	0	Inf	0
x11	0	0	NaN	NaN
x12	-5386.17266462574	0	-Inf	0
x13	0	0	NaN	NaN
x14	3081.55451252861	0	Inf	0

Number of observations: 301, Error degrees of freedom: 288

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

SumSq	DF	MeanSq	F	pValue

Total	18.4699692867817	300	0.0615665642892724		
Model	18.4699692867817	12	1.53916410723181	Inf	0
Residual	0	288	0		

```

for Q(x) A = 0.500000, B = 0.263855
for D(x) A = 0.260488, B = 0.192381
order Q(x) = 7.000000, order D(x) = 7.000000
Xmin = 0.000000 and Xmax = 30.000000
Ymin = -0.274347 and Ymax = 0.399625
List of powers for Q(x): [0.763855, 1.027709, 1.291564, 1.555418, 1.819273, 2.083128,
2.346982]
List of powers for D(x): [0.452869, 0.356678, 0.324615, 0.308583, 0.298964, 0.292551,
0.287971]
Fitting J4(x) in range (1.000000, 2.000000)
MSS of errors squared = 6.187011e-02
Corrected MSS of errors squared = 1.237402e-01
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 2

```

Fitting J4(x) in range (0.000000, 30.000000)
Shammas polynomial power for Q(x) is A+B*i
Shammas polynomial power for D(x) is A+B*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	7.24797971730278e-06	0	Inf	0
x1	-0.000127882545688898	0	-Inf	0
x2	0.000274232211934023	0	Inf	0
x3	-0.000208665003163725	0	-Inf	0
x4	5.50677513225333e-05	0	Inf	0
x5	86.2142921015774	0	Inf	0
x6	-422.125435061151	0	-Inf	0
x7	635.404094513879	0	Inf	0
x8	0	0	NaN	NaN
x9	-864.22725022807	0	-Inf	0
x10	787.052887555542	0	Inf	0
x11	-221.318588882075	0	-Inf	0

Number of observations: 301, Error degrees of freedom: 290

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	18.4699692869699	300	0.0615665642898998		
Model	18.4699692869699	10	1.84699692869699	Inf	0
Residual	0	290	0		

for Q(x) A = 0.292428, B = 0.142256

for D(x) A = 0.035590, B = 0.100000

order Q(x) = 4.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.274347 and Ymax = 0.399625

List of powers for Q(x): [0.434683, 0.576939, 0.719194, 0.861450]

List of powers for D(x): [0.135590, 0.177011, 0.208795, 0.235590, 0.259197, 0.280539, 0.300165]

Fitting J4(x) in range (1.000000, 2.000000)

MSS of errors squared = Inf

Corrected MSS of errors squared = Inf

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 3

Fitting J4(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B*sqrt(i)+C*i

Shammas polynomial power for D(x) is A+B*i+C*sqrt(i)

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.246842683522786	0	Inf	0
x1	-3.01794642609054	0	-Inf	0
x2	10.9308839416092	0	Inf	0
x3	-19.7008605467659	0	-Inf	0
x4	20.3051751432505	0	Inf	0
x5	-12.2215515561222	0	-Inf	0
x6	4.0163056886596	0	Inf	0
x7	-0.558848914322088	0	-Inf	0
x8	140.030484297931	0	Inf	0
x9	-643.546276154096	0	-Inf	0
x10	1286.183455121	0	Inf	0
x11	-1409.97735253721	0	-Inf	0

x12	887.158220123676	0	Inf	0
x13	-302.277493010423	0	-Inf	0
x14	43.4289621493026	0	Inf	0

Number of observations: 301, Error degrees of freedom: 286

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	18.4699692869963	300	0.0615665642899877		
Model	18.4699692869963	14	1.31928352049974	Inf	0
Residual	0	286	0		

for Q(x) A = 0.000000, B = 0.196238, C = 0.199679

for D(x) A = 0.338931, B = 0.122320, C = 0.014355

order Q(x) = 7.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.274347 and Ymax = 0.399625

List of powers for Q(x): [0.395917, 0.676880, 0.938931, 1.191192, 1.437196, 1.678757, 1.916949]

List of powers for D(x): [0.475606, 0.603872, 0.730754, 0.856921, 0.982629, 1.108013, 1.233150]

Fitting J4(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.232816e-01

Corrected MSS of errors squared = 3.019770e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 4

Fitting J4(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-1.36442253173659e-08	0	-Inf	0
x1	0	0	NaN	NaN
x2	1.19862305483523e-08	0	Inf	0
x3	0	0	NaN	NaN
x4	0	0	NaN	NaN

x5	16.1228746685818	0	Inf	0
x6	-56.4751634085124	0	-Inf	0
x7	96.1093282092415	0	Inf	0
x8	-94.2903114333788	0	-Inf	0
x9	54.5431127700756	0	Inf	0
x10	-17.3663559858513	0	-Inf	0
x11	2.35651518052117	0	Inf	0

Number of observations: 301, Error degrees of freedom: 292

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	18.4699692869879	300	0.0615665642899596		
Model	18.4699692869879	8	2.30874616087349	Inf	0
Residual	0	292	0		

for Q(x) A = 0.250819, B = 0.000000

for D(x) A = 0.117335, B = 0.181163

order Q(x) = 4.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.274347 and Ymax = 0.399625

List of powers for Q(x): [0.250819, 0.250819, 0.250819, 0.250819]

List of powers for D(x): [0.298498, 0.479661, 0.660823, 0.841986, 1.023149, 1.204311, 1.385474]

Fitting J4(x) in range (1.000000, 2.000000)

MSS of errors squared = 2.363494e+00

Corrected MSS of errors squared = 4.726988e+00

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 5

Fitting J4(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B*sqrt(i)

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-3.50164789600697e-08	0	-Inf	0
x1	2.6196340247823e-06	0	Inf	0

x2	-5.43911471399277e-06	0	-Inf	0
x3	2.8544119894792e-06	0	Inf	0
x4	5.86861503422076	0	Inf	0
x5	-14.3496128937791	0	-Inf	0
x6	18.7121534174091	0	Inf	0
x7	-13.7249345545033	0	-Inf	0
x8	5.36878526154051	0	Inf	0
x9	-0.875006264834712	0	-Inf	0

Number of observations: 301, Error degrees of freedom: 291

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	18.4699692869884	300	0.0615665642899612		
Model	18.4699692869884	9	2.05221880966537	Inf	0
Residual	0	291	0		

for Q(x) A = 0.506188, B = 0.132677

for D(x) A = 0.000000, B = 0.064955

order Q(x) = 3.000000, order D(x) = 6.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.274347 and Ymax = 0.399625

List of powers for Q(x): [0.638865, 0.693822, 0.735992]

List of powers for D(x): [0.064955, 0.129911, 0.194866, 0.259821, 0.324776, 0.389732]

Fitting J4(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.793725e-01

Corrected MSS of errors squared = 3.587450e-01

R-squared = 1.000000000

R-squared Adjusted = 1.000000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 6

Fitting J4(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B*i+C*sqrt(i)

Shammas polynomial power for D(x) is A+B*sqrt(i)+C*i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-2.14186845092003e-05	0	-Inf	0
x1	0.000129951107547765	0	Inf	0

x2	-0.000264478328358066	0	-Inf	0
x3	0.000249268240179395	0	Inf	0
x4	-0.000113687103449374	0	-Inf	0
x5	2.03617351297267e-05	0	Inf	0
x6	38.4411103310032	0	Inf	0
x7	-173.787344462135	0	-Inf	0
x8	251.958328369522	0	Inf	0
x9	0	0	NaN	NaN
x10	-329.496370517606	0	-Inf	0
x11	296.449474232344	0	Inf	0
x12	-82.5651979509785	0	-Inf	0

Number of observations: 301, Error degrees of freedom: 289

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	18.4699692869913	300	0.0615665642899709		
Model	18.4699692869913	11	1.67908811699921	Inf	0
Residual	0	289	0		

for Q(x) A = 0.000000, B = 0.384784, C = 0.281796

for D(x) A = 0.000000, B = 0.162463, C = 0.000000

order Q(x) = 5.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.274347 and Ymax = 0.399625

List of powers for Q(x): [0.666581, 1.168089, 1.642438, 2.102729, 2.554037]

List of powers for D(x): [0.162463, 0.229758, 0.281395, 0.324927, 0.363279, 0.397953, 0.429838]

Fitting J4(x) in range (1.000000, 2.000000)

MSS of errors squared = 2.072107e-01

Corrected MSS of errors squared = 5.075605e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 7

Fitting J4(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12 + x13 + x14

Estimated Coefficients:

Estimate	SE	tStat	pValue
----------	----	-------	--------

(Intercept)	0.0022504381454102	0	Inf	0
x1	-0.044154309465992	0	-Inf	0
x2	0.147316412553417	0	Inf	0
x3	-0.229856569318679	0	-Inf	0
x4	0.203560243562313	0	Inf	0
x5	-0.105389843587164	0	-Inf	0
x6	0.0298656188076038	0	Inf	0
x7	-0.00359200450393556	0	-Inf	0
x8	2202.14745763491	0	Inf	0
x9	-2889.94574118313	0	-Inf	0
x10	899.532227081833	0	Inf	0
x11	-262.416190336494	0	-Inf	0
x12	60.1889324751145	0	Inf	0
x13	-9.18945394632122	0	-Inf	0
x14	0.682768283619571	0	Inf	0

Number of observations: 301, Error degrees of freedom: 286

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	18.4699692869426	300	0.0615665642898088		
Model	18.4699692869426	14	1.3192835204959	Inf	0
Residual	0	286	0		

for Q(x) A = 0.500000, B = 0.500000

for D(x) A = 0.569842, B = 2.286293

order Q(x) = 7.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.274347 and Ymax = 0.399625

List of powers for Q(x): [1.000000, 1.500000, 2.000000, 2.500000, 3.000000, 3.500000, 4.000000]

List of powers for D(x): [0.569842, 0.588617, 0.688323, 0.870236, 1.115557, 1.408120, 1.736008]

Fitting J4(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.303015e-01

Corrected MSS of errors squared = 2.606030e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 8

Fitting J4(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for $D(x)$ is $A+B*\log_{10}(i)^4$

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-2.41890281578121e-05	0	-Inf	0
x1	1.96675512080535e-05	0	Inf	0
x2	-0.000161196134512048	0	-Inf	0
x3	0.000165667152709146	0	Inf	0
x4	4526.28869027128	0	Inf	0
x5	-5654.76216735036	0	-Inf	0
x6	1288.71980977249	0	Inf	0
x7	-159.24633266837	0	-Inf	0

Number of observations: 301, Error degrees of freedom: 293

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	18.4699692868167	300	0.061566564289389		
Model	18.4699692868167	7	2.63856704097382	Inf	0
Residual	0	293	0		

for $Q(x)$ $A = 0.331746$, $B = 1.246103$

for $D(x)$ $A = 0.100000$, $B = 0.173944$

order $Q(x) = 3.000000$, order $D(x) = 4.000000$

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.274347 and Ymax = 0.399625

List of powers for $Q(x)$: [1.577849, 0.954797, 0.747113]

List of powers for $D(x)$: [0.100000, 0.101428, 0.109014, 0.122854]

Fitting $J_4(x)$ in range (1.000000, 2.000000)

MSS of errors squared = 1.940634e+00

Corrected MSS of errors squared = 3.881267e+00

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 9

Fitting $J_4(x)$ in range (0.000000, 30.000000)

Shammas polynomial power for $Q(x)$ is $A+B*\log_{10}(i)^4$

Shammas polynomial power for $D(x)$ is $A+B*i$

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	1.10619187644777e-08	0	Inf	0
x1	-1.23914851782875e-06	0	-Inf	0
x2	1.47446095192996e-06	0	Inf	0
x3	-2.72482105191822e-07	0	-Inf	0
x4	2.61077502325969e-08	0	Inf	0
x5	1	0	Inf	0
x6	0	0	NaN	NaN
x7	0	0	NaN	NaN
x8	0	0	NaN	NaN
x9	0	0	NaN	NaN
x10	0	0	NaN	NaN
x11	0	0	NaN	NaN

Number of observations: 301, Error degrees of freedom: 295

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	18.4699692869885	300	0.0615665642899615		
Model	18.4699692869885	5	3.69399385739769	Inf	0
Residual	0	295	0		

for Q(x) A = 0.064079, B = 0.500000

for D(x) A = 0.000000, B = 0.000000

order Q(x) = 4.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.274347 and Ymax = 0.399625

List of powers for Q(x): [0.064079, 0.068185, 0.089990, 0.129773]

List of powers for D(x): [0.000000, 0.000000, 0.000000, 0.000000, 0.000000, 0.000000, 0.000000]

Fitting J4(x) in range (1.000000, 2.000000)

MSS of errors squared = 3.652978e-02

Corrected MSS of errors squared = 7.305955e-02

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 10

Fitting J4(x) in range (0.000000, 30.000000)

Shammas polynomial power for $Q(x)$ is $A+B*\log_{10}(i)^4$

Shammas polynomial power for $D(x)$ is $A+B/i$

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.00061823949511068	0	-Inf	0
x1	36.0871082255783	0	Inf	0
x2	0	0	NaN	NaN
x3	-86.1171212861651	0	-Inf	0
x4	75.7793021627819	0	Inf	0
x5	-30.8330547403758	0	-Inf	0
x6	5.08438385417513	0	Inf	0
x7	-0.0391550946173524	0	-Inf	0
x8	6.18204288576264	0	Inf	0
x9	-88.5235594582022	0	-Inf	0
x10	349.672069707486	0	Inf	0
x11	-500.502544399261	0	-Inf	0
x12	234.211146375334	0	Inf	0

Number of observations: 301, Error degrees of freedom: 289

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	18.4699692869926	300	0.0615665642899754		
Model	18.4699692869926	11	1.67908811699933	Inf	0
Residual	0	289	0		

for $Q(x)$ $A = 1.079519$, $B = 0.434560$

for $D(x)$ $A = 0.091824$, $B = 0.889020$

order $Q(x) = 6.000000$, order $D(x) = 6.000000$

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.274347 and Ymax = 0.399625

List of powers for $Q(x)$: [1.079519, 1.083088, 1.102039, 1.136616, 1.183244, 1.238852]

List of powers for $D(x)$: [0.980844, 0.536334, 0.388164, 0.314079, 0.269628, 0.239994]

Fitting $J_4(x)$ in range (1.000000, 2.000000)

MSS of errors squared = 2.262301e-01

Corrected MSS of errors squared = 4.524603e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Comments

The following table shows the summary results for the ten models that fit the Bessel $J_4(x)$ function. The results are sorted in ascending order using the MSSE values.

<i>Power Q(x)</i>	<i>Power D(x)</i>	<i>MSSE</i>	<i>Q/D Orders</i>
$A+B*\log_{10}(i)^4$	$A+B*i$	3.65297800E-02	4, 7
$A+B*i$	$A+B/i$	6.18701100E-02	7, 7
$A+B*\sqrt{i}+C*i$	$A+B*i+C*\sqrt{i}$	1.23281600E-01	7, 7
$A+B*i$	$A+B*\log_{10}(i)^4$	1.30301500E-01	7, 7
$A+B*\sqrt{i}$	$A+B*i$	1.79372500E-01	3, 6
$A+B*i+C*\sqrt{i}$	$A+B*\sqrt{i}+C*i$	2.07210700E-01	5, 7
$A+B*\log_{10}(i)^4$	$A+B/i$	2.26230100E-01	6, 6
$A+B/i$	$A+B*\log_{10}(i)^4$	1.94063400E+00	3, 4
$A+B/i$	$A+B*i$	2.36349400E+00	4, 7
$A+B*i$	$A+B*\sqrt{i}$	Inf	4, 7

RESULTS FOR THE BESSEL J5(X)

Combo Powers 1

Fitting J5(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is $A+B*i$

Shammas polynomial power for D(x) is $A+B/i$

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.0575077496078273	0	Inf	0
x1	-0.953779488240243	0	-Inf	0
x2	2.98987820321447	0	Inf	0
x3	-4.40677394957995	0	-Inf	0
x4	3.69302502609999	0	Inf	0
x5	-1.81020967983377	0	-Inf	0
x6	0.485619326930105	0	Inf	0
x7	-0.0552652062402845	0	-Inf	0
x8	-3.02968046764825	0	-Inf	0
x9	392.49248549512	0	Inf	0
x10	-4581.58193854709	0	-Inf	0
x11	14403.290755307	0	Inf	0
x12	-14156.8131212367	0	-Inf	0
x13	0	0	NaN	NaN
x14	3946.64149804204	0	Inf	0

Number of observations: 301, Error degrees of freedom: 287

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

SumSq	DF	MeanSq	F	pValue

Total	18.6374025327746	300	0.0621246751092488		
Model	18.6374025327746	13	1.43364634867497	Inf	0
Residual	0	287	0		

```

for Q(x) A = 0.466389, B = 0.500000
for D(x) A = 0.626752, B = 0.880943
order Q(x) = 7.000000, order D(x) = 7.000000
Xmin = 0.000000 and Xmax = 30.000000
Ymin = -0.261053 and Ymax = 0.374075
List of powers for Q(x): [0.966389, 1.466389, 1.966389, 2.466389, 2.966389, 3.466389,
3.966389]
List of powers for D(x): [1.507695, 1.067223, 0.920399, 0.846987, 0.802940, 0.773575,
0.752601]
Fitting J5(x) in range (1.000000, 2.000000)
MSS of errors squared = 1.700335e-01
Corrected MSS of errors squared = 3.400671e-01
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 2

```

Fitting J5(x) in range (0.000000, 30.000000)
Shammas polynomial power for Q(x) is A+B*i
Shammas polynomial power for D(x) is A+B*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	8.34121573071508e-05	0	Inf	0
x1	-0.000356775079318128	0	-Inf	0
x2	0.00059092865885111	0	Inf	0
x3	-0.000507738296677077	0	-Inf	0
x4	0.000253838638192437	0	Inf	0
x5	-7.45291021047296e-05	0	-Inf	0
x6	1.19514985013424e-05	0	Inf	0
x7	-8.09999668848738e-07	0	-Inf	0
x8	485.937778890013	0	Inf	0
x9	-2055.59117427341	0	-Inf	0
x10	2486.50326062949	0	Inf	0
x11	0	0	NaN	NaN
x12	-1665.74130977208	0	-Inf	0
x13	749.891444323173	0	Inf	0

Number of observations: 301, Error degrees of freedom: 288

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	18.6374025325168	300	0.0621246751083894		
Model	18.6374025325168	12	1.55311687770973	Inf	0
Residual	0	288	0		

for Q(x) A = 0.253708, B = 1.474487

for D(x) A = 0.221660, B = 0.100000

order Q(x) = 7.000000, order D(x) = 6.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.261053 and Ymax = 0.374075

List of powers for Q(x): [1.728195, 3.202682, 4.677169, 6.151656, 7.626143, 9.100630, 10.575117]

List of powers for D(x): [0.321660, 0.363082, 0.394865, 0.421660, 0.445267, 0.466609]

Fitting J5(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.152415e-01

Corrected MSS of errors squared = 2.304830e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 3

Fitting J5(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B*sqrt(i)+C*i

Shammas polynomial power for D(x) is A+B*i+C*sqrt(i)

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-5.81367663868084e-05	0	-Inf	0
x1	0.00026440854550085	0	Inf	0
x2	-0.000293378024381285	0	-Inf	0
x3	8.59650022338149e-05	0	Inf	0
x4	14.4142237592816	0	Inf	0
x5	-51.8387310678505	0	-Inf	0
x6	90.5656704913435	0	Inf	0
x7	-90.0417928158079	0	-Inf	0
x8	52.2090054060966	0	Inf	0
x9	-16.5219896580632	0	-Inf	0

x10 2.21361464759538 0 Inf 0

Number of observations: 301, Error degrees of freedom: 290

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	18.6374025324216	300	0.062124675108072		
Model	18.6374025324216	10	1.86374025324216	Inf	0
Residual	0	290	0		

for Q(x) A = 0.292453, B = 0.100000, C = 0.172648

for D(x) A = 0.090691, B = 0.287928, C = 0.221171

order Q(x) = 3.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.261053 and Ymax = 0.374075

List of powers for Q(x): [0.565101, 0.779170, 0.983601]

List of powers for D(x): [0.599790, 0.979331, 1.337556, 1.684747, 2.024887, 2.360018, 2.691354]

Fitting J5(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.328108e-01

Corrected MSS of errors squared = 3.253188e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 4

Fitting J5(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	63.7849009892758	0	Inf	0
x1	-84.7040892991869	0	-Inf	0
x2	4444.75861737437	0	Inf	0
x3	-27972.9648287546	0	-Inf	0
x4	50099.5062732811	0	Inf	0
x5	-26550.3808681087	0	-Inf	0
x6	8.20199397743193	0	Inf	0
x7	-20.4929843699919	0	-Inf	0

x8	24.8459349392076	0	Inf	0
x9	-16.476463308359	0	-Inf	0
x10	5.75136782374677	0	Inf	0
x11	-0.829852973282148	0	-Inf	0

Number of observations: 301, Error degrees of freedom: 289

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	18.6374025324114	300	0.062124675108038		
Model	18.6374025324114	11	1.69430932112831	Inf	0
Residual	0	289	0		

for Q(x) A = 0.146382, B = 0.331635

for D(x) A = 0.100000, B = 0.266993

order Q(x) = 5.000000, order D(x) = 6.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.261053 and Ymax = 0.374075

List of powers for Q(x): [0.478016, 0.312199, 0.256926, 0.229290, 0.212708]

List of powers for D(x): [0.366993, 0.633986, 0.900980, 1.167973, 1.434966, 1.701959]

Fitting J5(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.327408e-01

Corrected MSS of errors squared = 2.654815e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 5

Fitting J5(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B*sqrt(i)

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	2.80953519712958e-11	0	Inf	0
x1	-9.76018629984509e-10	0	-Inf	0
x2	2.9160803296942e-09	0	Inf	0
x3	-3.00798551319069e-09	0	-Inf	0
x4	1.03982943831469e-09	0	Inf	0
x5	1	0	Inf	0

x6	0	0	NaN	NaN
x7	0	0	NaN	NaN
x8	0	0	NaN	NaN

Number of observations: 301, Error degrees of freedom: 295

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	18.6374025324205	300	0.0621246751080683		
Model	18.6374025324205	5	3.7274805064841	Inf	0
Residual	0	295	0		

for Q(x) A = 0.206516, B = 0.327887

for D(x) A = 0.000000, B = 0.000000

order Q(x) = 4.000000, order D(x) = 4.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.261053 and Ymax = 0.374075

List of powers for Q(x): [0.534403, 0.670218, 0.774433, 0.862290]

List of powers for D(x): [0.000000, 0.000000, 0.000000, 0.000000]

Fitting J5(x) in range (1.000000, 2.000000)

MSS of errors squared = 6.025644e-02

Corrected MSS of errors squared = 1.205129e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 6

Fitting J5(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B*i+C*sqrt(i)

Shammas polynomial power for D(x) is A+B*sqrt(i)+C*i

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.0318924220215429	0	Inf	0
x1	-0.706891421965557	0	-Inf	0
x2	2.64467694034174	0	Inf	0
x3	-4.58167504000283	0	-Inf	0
x4	4.44139447489112	0	Inf	0
x5	-2.49044497072259	0	-Inf	0
x6	0.758571688903384	0	Inf	0

x7	-0.09752406042933	0	-Inf	0
x8	36.3832765547587	0	Inf	0
x9	-181.19612529717	0	-Inf	0
x10	402.17735922155	0	Inf	0
x11	-486.103244830862	0	-Inf	0
x12	333.562730098169	0	Inf	0
x13	-122.680406244977	0	-Inf	0
x14	18.8564104721749	0	Inf	0

Number of observations: 301, Error degrees of freedom: 286

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	18.6374025324412	300	0.0621246751081373		
Model	18.6374025324412	14	1.33124303803151	Inf	0
Residual	0	286	0		

for Q(x) A = 0.340000, B = 0.372592, C = 0.305653

for D(x) A = 0.109078, B = 0.391778, C = 0.121304

order Q(x) = 7.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.261053 and Ymax = 0.374075

List of powers for Q(x): [1.018245, 1.517443, 1.987183, 2.441675, 2.886422, 3.324248, 3.756828]

List of powers for D(x): [0.622160, 0.905744, 1.151569, 1.377850, 1.591640, 1.796558, 1.994753]

Fitting J5(x) in range (1.000000, 2.000000)

MSS of errors squared = 6.322183e-02

Corrected MSS of errors squared = 1.548612e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 7

Fitting J5(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9$$

Estimated Coefficients:

Estimate	SE	tStat	pValue
----------	----	-------	--------

(Intercept)	-7.37013384018788e-08	0	-Inf	0
x1	4.06971738723725e-07	0	Inf	0
x2	-9.35861035831498e-07	0	-Inf	0
x3	1.14716732528502e-06	0	Inf	0
x4	-7.90559214818279e-07	0	-Inf	0
x5	2.90409420858211e-07	0	Inf	0
x6	-4.44268990098989e-08	0	-Inf	0
x7	0.999999999999657	0	Inf	0
x8	3.87283153783336e-13	0	Inf	0
x9	-4.4954080806316e-14	0	-Inf	0

Number of observations: 301, Error degrees of freedom: 291

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	18.6374025324205	300	0.0621246751080683		
Model	18.6374025324205	9	2.07082250360228	Inf	0
Residual	0	291	0		

for Q(x) A = 0.000000, B = 0.226568

for D(x) A = 0.000000, B = 3.500000

order Q(x) = 6.000000, order D(x) = 3.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.261053 and Ymax = 0.374075

List of powers for Q(x): [0.226568, 0.453137, 0.679705, 0.906273, 1.132842, 1.359410]

List of powers for D(x): [0.000000, 0.028741, 0.181377]

Fitting J5(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.775093e+00

Corrected MSS of errors squared = 3.550185e+00

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 8

Fitting J5(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12

Estimated Coefficients:

Estimate	SE	tStat	pValue
----------	----	-------	--------

(Intercept)	0.0723862956825245	0	Inf	0
x1	-0.00137491639569866	0	-Inf	0
x2	0.714360406062452	0	Inf	0
x3	-20.7027452287089	0	-Inf	0
x4	148.722756598497	0	Inf	0
x5	-400.874356390204	0	-Inf	0
x6	448.830175206952	0	Inf	0
x7	-176.761201841144	0	-Inf	0
x8	872.797392649815	0	Inf	0
x9	0	0	NaN	NaN
x10	-1746.75142281965	0	-Inf	0
x11	1112.0349306423	0	Inf	0
x12	-237.080900565033	0	-Inf	0

Number of observations: 301, Error degrees of freedom: 289

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	18.6374025324152	300	0.0621246751080506		
Model	18.6374025324152	11	1.69430932112865	Inf	0
Residual	0	289	0		

for Q(x) A = 0.500000, B = 2.560048

for D(x) A = 0.100000, B = 0.100000

order Q(x) = 7.000000, order D(x) = 5.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.261053 and Ymax = 0.374075

List of powers for Q(x): [3.060048, 1.780024, 1.353349, 1.140012, 1.012010, 0.926675, 0.865721]

List of powers for D(x): [0.100000, 0.100821, 0.105182, 0.113139, 0.123869]

Fitting J5(x) in range (1.000000, 2.000000)

MSS of errors squared = 7.411589e-01

Corrected MSS of errors squared = 1.482318e+00

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 9

Fitting J5(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is A+B*log10(i)^4

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0	0	NaN	NaN
x1	-2.48358158169787e-16	0	-Inf	0
x2	0	0	NaN	NaN
x3	0	0	NaN	NaN
x4	0	0	NaN	NaN
x5	1	0	Inf	0
x6	0	0	NaN	NaN
x7	0	0	NaN	NaN
x8	0	0	NaN	NaN
x9	0	0	NaN	NaN

Number of observations: 301, Error degrees of freedom: 299

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	18.6374025324205	300	0.0621246751080683		
Model	18.6374025324205	1	18.6374025324205	Inf	0
Residual	0	299	0		

for Q(x) A = 0.000000, B = 0.000000

for D(x) A = 0.000000, B = 0.000000

order Q(x) = 4.000000, order D(x) = 5.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.261053 and Ymax = 0.374075

List of powers for Q(x): [0.000000, 0.000000, 0.000000, 0.000000]

List of powers for D(x): [0.000000, 0.000000, 0.000000, 0.000000, 0.000000]

Fitting J5(x) in range (1.000000, 2.000000)

MSS of errors squared = 2.457734e-02

Corrected MSS of errors squared = 4.915467e-02

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 10

Fitting J5(x) in range (0.000000, 30.000000)

Shammas polynomial power for Q(x) is $A+B*\log_{10}(i)^4$

Shammas polynomial power for D(x) is $A+B/i$

Linear regression model:

$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12 + x13 + x14$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-96.7472331050579	0	-Inf	0
x1	0	0	NaN	NaN
x2	0	0	NaN	NaN
x3	159.012055123727	0	Inf	0
x4	0	0	NaN	NaN
x5	-139.646820409083	0	-Inf	0
x6	100.255077265616	0	Inf	0
x7	-22.8730828468935	0	-Inf	0
x8	0.319795823535066	0	Inf	0
x9	-134.665918697973	0	-Inf	0
x10	3489.33749355973	0	Inf	0
x11	-23260.5629021614	0	-Inf	0
x12	59369.1305241566	0	Inf	0
x13	-63744.651346714	0	-Inf	0
x14	24282.0923568011	0	Inf	0

Number of observations: 301, Error degrees of freedom: 289

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	18.6374025328585	300	0.0621246751095284		
Model	18.6374025328585	11	1.69430932116896	Inf	0
Residual	0	289	0		

for Q(x) A = 0.000000, B = 0.367514

for D(x) A = 0.906990, B = 2.000000

order Q(x) = 7.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 30.000000

Ymin = -0.261053 and Ymax = 0.374075

List of powers for Q(x): [0.000000, 0.003018, 0.019045, 0.048287, 0.087722, 0.134751, 0.187457]

List of powers for D(x): [2.906990, 1.906990, 1.573657, 1.406990, 1.306990, 1.240323, 1.192704]

Fitting J5(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.903997e-01

Corrected MSS of errors squared = 3.807994e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Comments

The following table shows the summary results for the ten models that fit the Bessel $J_5(x)$ function. The results are sorted in ascending order using the MSSE values.

<i>Power Q(x)</i>	<i>Power D(x)</i>	<i>MSSE</i>	<i>Q/D Orders</i>
$A+B*\log_{10}(i)^4$	$A+B*i$	2.45773400E-02	4, 5
$A+B*\sqrt{i}$	$A+B*i$	6.02564400E-02	4, 4
$A+B*i+C*\sqrt{i}$	$A+B*\sqrt{i}+C*i$	6.32218300E-02	7, 7
$A+B*i$	$A+B*\sqrt{i}$	1.15241500E-01	7, 6
$A+B/i$	$A+B*i$	1.32740800E-01	5, 6
$A+B*\sqrt{i}+C*i$	$A+B*i+C*\sqrt{i}$	1.32810800E-01	3, 7
$A+B*i$	$A+B/i$	1.70033500E-01	7, 7
$A+B*\log_{10}(i)^4$	$A+B/i$	1.90399700E-01	7, 7
$A+B/i$	$A+B*\log_{10}(i)^4$	7.41158900E-01	7, 5
$A+B*i$	$A+B*\log_{10}(i)^4$	1.77509300E+00	6, 3

RESULTS FOR THE NATURAL LOGRITHM

Combo Powers 1

Fitting $\ln(x)$ in range (1.000000, 10.000000)

Shammas polynomial power for Q(x) is $A+B*i$

Shammas polynomial power for D(x) is $A+B/i$

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.233029875428266	0	Inf	0
x1	-4.11646313751633	0	-Inf	0
x2	9.20507612903065	0	Inf	0
x3	-7.88054670156007	0	-Inf	0
x4	3.01237547425312	0	Inf	0
x5	-0.453624975248713	0	-Inf	0
x6	0.719139670925778	0	Inf	0
x7	-3.57100762712479	0	-Inf	0
x8	0	0	NaN	NaN
x9	3.85202129203499	0	Inf	0

Number of observations: 901, Error degrees of freedom: 892

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	58.9211587781912	900	0.0654679541979902		
Model	58.9211587781912	8	7.3651448472739	Inf	0
Residual	0	892	0		

```

for Q(x) A = 0.432576, B = 0.226446
for D(x) A = 0.058577, B = 0.546285
order Q(x) = 5.000000, order D(x) = 4.000000
Xmin = 1.000000 and Xmax = 10.000000
Ymin = 0.000000 and Ymax = 2.302585
List of powers for Q(x): [0.659022, 0.885468, 1.111913, 1.338359, 1.564805]
List of powers for D(x): [0.604862, 0.331719, 0.240672, 0.195148]
Fitting ln(x) in range (1.000000, 2.000000)
MSS of errors squared = 3.672432e-09
Corrected MSS of errors squared = 7.344863e-09
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 2

```

Fitting ln(x) in range (1.000000, 10.000000)
Shammas polynomial power for Q(x) is A+B*i
Shammas polynomial power for D(x) is A+B*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	5.35828796785817e-05	0	Inf	0
x1	-7.97163873439972e-05	0	-Inf	0
x2	1.75167933633237e-05	0	Inf	0
x3	-8.70081182651421e-07	0	-Inf	0
x4	6.82599714265085	0	Inf	0
x5	0	0	NaN	NaN
x6	-17.3157744668562	0	-Inf	0
x7	0	0	NaN	NaN
x8	17.6546858962591	0	Inf	0
x9	0	0	NaN	NaN
x10	-6.16489908599534	0	-Inf	0

Number of observations: 901, Error degrees of freedom: 893

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	58.9211587781876	900	0.0654679541979863		
Model	58.9211587781876	7	8.41730839688395	Inf	0
Residual	0	893	0		

```

for Q(x) A = 0.134430, B = 1.625684
for D(x) A = 0.000000, B = 0.100000
order Q(x) = 3.000000, order D(x) = 7.000000
Xmin = 1.000000 and Xmax = 10.000000
Ymin = 0.000000 and Ymax = 2.302585
List of powers for Q(x): [1.760114, 3.385797, 5.011481]
List of powers for D(x): [0.100000, 0.141421, 0.173205, 0.200000, 0.223607, 0.244949,
0.264575]
Fitting ln(x) in range (1.000000, 2.000000)
MSS of errors squared = 2.852731e-07
Corrected MSS of errors squared = 5.705463e-07
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 3

```

Fitting ln(x) in range (1.000000, 10.000000)
Shammas polynomial power for Q(x) is A+B*sqrt(i)+C*i
Shammas polynomial power for D(x) is A+B*i+C*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-1.33955662630117	0	-Inf	0
x1	33.6397423737053	0	Inf	0
x2	-103.938211454916	0	-Inf	0
x3	113.033101125871	0	Inf	0
x4	-41.407069319292	0	-Inf	0
x5	3.26454266989256	0	Inf	0
x6	2.8965905617848	0	Inf	0
x7	-13.3530090833566	0	-Inf	0
x8	10.4031432894223	0	Inf	0
x9	-2.46265281886711	0	-Inf	0
x10	0.263379281265176	0	Inf	0

Number of observations: 901, Error degrees of freedom: 890

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	58.9211587782015	900	0.0654679541980017		

Model	58.9211587782015	10	5.89211587782015	Inf	0
Residual	0	890	0		

for Q(x) A = 0.070698, B = 0.315816, C = 0.240131
 for D(x) A = 0.176588, B = 0.363147, C = 0.172102
 order Q(x) = 4.000000, order D(x) = 6.000000
 Xmin = 1.000000 and Xmax = 10.000000
 Ymin = 0.000000 and Ymax = 2.302585
 List of powers for Q(x): [0.626644, 0.997590, 1.338099, 1.662852]
 List of powers for D(x): [0.711837, 1.146271, 1.564118, 1.973379, 2.377154, 2.777031]
 Fitting ln(x) in range (1.000000, 2.000000)
 MSS of errors squared = 1.479743e-10
 Corrected MSS of errors squared = 3.624614e-10
 R-squared = 1.00000000
 R-squared Adjusted = 1.00000000
 Particle swarm AICc = -1.000000e+99
 AIC = -Inf
 AICc = -Inf

Combo Powers 4

Fitting ln(x) in range (1.000000, 10.000000)
 Shammas polynomial power for Q(x) is A+B/i
 Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-67.3054363268716	0	-Inf	0
x1	109.416612266441	0	Inf	0
x2	-650.212783416566	0	-Inf	0
x3	608.130172425682	0	Inf	0
x4	2.37643554264786	0	Inf	0
x5	-0.681751005720955	0	-Inf	0
x6	-1.48383371643545	0	-Inf	0
x7	1.02068692730771	0	Inf	0
x8	-0.293215767641828	0	-Inf	0
x9	0.0331131046937479	0	Inf	0

Number of observations: 901, Error degrees of freedom: 891

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	58.9211587782792	900	0.065467954198088		

Model	58.9211587782792	9	6.5467954198088	Inf	0
Residual	0	891	0		

for Q(x) A = 0.100000, B = 0.487607
 for D(x) A = 0.100000, B = 0.625614
 order Q(x) = 3.000000, order D(x) = 6.000000
 Xmin = 1.000000 and Xmax = 10.000000
 Ymin = 0.000000 and Ymax = 2.302585
 List of powers for Q(x): [0.587607, 0.343804, 0.262536]
 List of powers for D(x): [0.725614, 1.351227, 1.976841, 2.602455, 3.228068, 3.853682]
 Fitting ln(x) in range (1.000000, 2.000000)
 MSS of errors squared = 2.900962e-09
 Corrected MSS of errors squared = 5.801925e-09
 R-squared = 1.00000000
 R-squared Adjusted = 1.00000000
 Particle swarm AICc = -1.000000e+99
 AIC = -Inf
 AICc = -Inf

Combo Powers 5

Fitting ln(x) in range (1.000000, 10.000000)
 Shammas polynomial power for Q(x) is A+B*sqrt(i)
 Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	1.30711962687764	0	Inf	0
x1	134.458322998648	0	Inf	0
x2	-723.027700490448	0	-Inf	0
x3	1064.81760219178	0	Inf	0
x4	-477.594183480527	0	-Inf	0
x5	7.81484987668121	0	Inf	0
x6	-13.0757141373823	0	-Inf	0
x7	6.72184267989107	0	Inf	0
x8	-0.442692407611809	0	-Inf	0
x9	0.0205531314975203	0	Inf	0

Number of observations: 901, Error degrees of freedom: 891

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	58.9211587780999	900	0.0654679541978887		

Model	58.9211587780999	9	6.54679541978887	Inf	0
Residual	0	891	0		

```

for Q(x) A = 0.891108, B = 0.500000
for D(x) A = 0.391727, B = 0.648454
order Q(x) = 4.000000, order D(x) = 5.000000
Xmin = 1.000000 and Xmax = 10.000000
Ymin = 0.000000 and Ymax = 2.302585
List of powers for Q(x): [1.391108, 1.598215, 1.757134, 1.891108]
List of powers for D(x): [1.040181, 1.688635, 2.337089, 2.985543, 3.633997]
Fitting ln(x) in range (1.000000, 2.000000)
MSS of errors squared = 6.284332e-10
Corrected MSS of errors squared = 1.256866e-09
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 6

```

Fitting ln(x) in range (1.000000, 10.000000)
Shammas polynomial power for Q(x) is A+B*i+C*sqrt(i)
Shammas polynomial power for D(x) is A+B*sqrt(i)+C*i

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-284.572458028446	0	-Inf	0
x1	543.811612316563	0	Inf	0
x2	0	0	NaN	NaN
x3	-338.834706464343	0	-Inf	0
x4	0	0	NaN	NaN
x5	0	0	NaN	NaN
x6	79.6313522700964	0	Inf	0
x7	7.90114003323989	0	Inf	0
x8	-12.9315288840091	0	-Inf	0
x9	8.24827244162926	0	Inf	0
x10	-2.57195433348389	0	-Inf	0
x11	0.314839483849247	0	Inf	0
x12	0.00343120177787167	0	Inf	0

Number of observations: 901, Error degrees of freedom: 891

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

SumSq	DF	MeanSq	F	pValue
-------	----	--------	---	--------

Total	58.9211587781512	900	0.0654679541979458		
Model	58.9211587781512	9	6.54679541979458	Inf	0
Residual	0	891	0		

```

for Q(x) A = 0.000000, B = 0.100000, C = 0.000000
for D(x) A = 0.550000, B = 0.255503, C = 0.400000
order Q(x) = 6.000000, order D(x) = 6.000000
Xmin = 1.000000 and Xmax = 10.000000
Ymin = 0.000000 and Ymax = 2.302585
List of powers for Q(x): [0.100000, 0.200000, 0.300000, 0.400000, 0.500000, 0.600000]
List of powers for D(x): [1.205503, 1.711335, 2.192544, 2.661005, 3.121321, 3.575851]
Fitting ln(x) in range (1.000000, 2.000000)
MSS of errors squared = 2.536576e-09
Corrected MSS of errors squared = 6.213318e-09
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 7

```

Fitting ln(x) in range (1.000000, 10.000000)
Shammas polynomial power for Q(x) is A+B*i
Shammas polynomial power for D(x) is A+B*log10(i)^4

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-32.2928467808326	0	-Inf	0
x1	264.044962322828	0	Inf	0
x2	-398.877829928541	0	-Inf	0
x3	167.165292477341	0	Inf	0
x4	107.796826760125	0	Inf	0
x5	0	0	NaN	NaN
x6	-191.131000932939	0	-Inf	0
x7	104.673275116107	0	Inf	0
x8	-21.8984154484421	0	-Inf	0
x9	1.51973652912345	0	Inf	0

Number of observations: 901, Error degrees of freedom: 892

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

SumSq	DF	MeanSq	F	pValue
-------	----	--------	---	--------

Total	58.9211587781863	900	0.0654679541979847		
Model	58.9211587781863	8	7.36514484727328	Inf	0
Residual	0	892	0		

```

for Q(x) A = 0.177699, B = 0.161380
for D(x) A = 1.395223, B = 2.107991
order Q(x) = 3.000000, order D(x) = 6.000000
Xmin = 1.000000 and Xmax = 10.000000
Ymin = 0.000000 and Ymax = 2.302585
List of powers for Q(x): [0.339079, 0.500459, 0.661838]
List of powers for D(x): [1.395223, 1.412533, 1.504463, 1.672190, 1.898379, 2.168126]
Fitting ln(x) in range (1.000000, 2.000000)
MSS of errors squared = 7.386934e-09
Corrected MSS of errors squared = 1.477387e-08
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 8

```

Fitting ln(x) in range (1.000000, 10.000000)
Shammas polynomial power for Q(x) is A+B/i
Shammas polynomial power for D(x) is A+B*log10(i)^4

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.80322481436259	0	Inf	0
x1	1.58468658672342	0	Inf	0
x2	-20.537703719921	0	-Inf	0
x3	33.6994212752478	0	Inf	0
x4	0	0	NaN	NaN
x5	0	0	NaN	NaN
x6	0	0	NaN	NaN
x7	-15.5496243332064	0	-Inf	0
x8	46.0864590801127	0	Inf	0
x9	0	0	NaN	NaN
x10	-84.3860179023106	0	-Inf	0
x11	49.067449033796	0	Inf	0
x12	-9.767894834803	0	-Inf	0

Number of observations: 901, Error degrees of freedom: 892

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	58.9211587781873	900	0.0654679541979859		
Model	58.9211587781873	8	7.36514484727342	Inf	0
Residual	0	892	0		

for Q(x) A = 0.100000, B = 0.245407

for D(x) A = 0.100000, B = 0.388358

order Q(x) = 7.000000, order D(x) = 5.000000

Xmin = 1.000000 and Xmax = 10.000000

Ymin = 0.000000 and Ymax = 2.302585

List of powers for Q(x): [0.345407, 0.222704, 0.181802, 0.161352, 0.149081, 0.140901, 0.135058]

List of powers for D(x): [0.100000, 0.103189, 0.120126, 0.151026, 0.192697]

Fitting ln(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.066390e-09

Corrected MSS of errors squared = 2.132780e-09

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 9

Fitting ln(x) in range (1.000000, 10.000000)

Shammas polynomial power for Q(x) is A+B*log10(i)^4

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.517510704758161	0	Inf	0
x1	-14515.8513369913	0	-Inf	0
x2	0	0	NaN	NaN
x3	23075.3557766932	0	Inf	0
x4	0	0	NaN	NaN
x5	-17995.1657195485	0	-Inf	0
x6	11926.59906738	0	Inf	0
x7	-2491.43710111309	0	-Inf	0
x8	-17.2375470253757	0	-Inf	0
x9	47.037540636236	0	Inf	0
x10	-28.8181906774545	0	-Inf	0

Number of observations: 901, Error degrees of freedom: 892

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	58.9211587808001	900	0.065467954200889		
Model	58.9211587808001	8	7.36514484760001	Inf	0
Residual	0	892	0		

for Q(x) A = 2.200000, B = 0.500000

for D(x) A = 0.443818, B = 0.062853

order Q(x) = 7.000000, order D(x) = 3.000000

Xmin = 1.000000 and Xmax = 10.000000

Ymin = 0.000000 and Ymax = 2.302585

List of powers for Q(x): [2.200000, 2.204106, 2.225911, 2.265695, 2.319345, 2.383327, 2.455034]

List of powers for D(x): [0.506670, 0.569523, 0.632375]

Fitting ln(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.019591e-08

Corrected MSS of errors squared = 2.039182e-08

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 10

Fitting ln(x) in range (1.000000, 10.000000)

Shammas polynomial power for Q(x) is A+B*log10(i)^4

Shammas polynomial power for D(x) is A+B/i

Linear regression model:

$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-13.9381143996285	0	-Inf	0
x1	60365.3654714927	0	Inf	0
x2	0	0	NaN	NaN
x3	0	0	NaN	NaN
x4	-133238.921808335	0	-Inf	0
x5	72887.5872971349	0	Inf	0
x6	6.2613373002404	0	Inf	0
x7	-459.989760975416	0	-Inf	0
x8	3121.45354940265	0	Inf	0
x9	-5723.06462369492	0	-Inf	0

x10 3056.24665213528 0 Inf 0

Number of observations: 901, Error degrees of freedom: 892

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	58.9211587490444	900	0.0654679541656049		
Model	58.9211587490444	8	7.36514484363055	Inf	0
Residual	0	892	0		

for Q(x) A = 0.883954, B = 0.101636

for D(x) A = 2.191026, B = 1.081100

order Q(x) = 5.000000, order D(x) = 5.000000

Xmin = 1.000000 and Xmax = 10.000000

Ymin = 0.000000 and Ymax = 2.302585

List of powers for Q(x): [0.883954, 0.884789, 0.889221, 0.897308, 0.908214]

List of powers for D(x): [3.272126, 2.731576, 2.551393, 2.461301, 2.407246]

Fitting ln(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.852730e-09

Corrected MSS of errors squared = 3.705461e-09

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Comments

The following table shows the summary results for the ten models that fit the natural logarithm function. The results are sorted in ascending order using the MSSE values.

<i>Power Q(x)</i>	<i>Power D(x)</i>	<i>MSSE</i>	<i>Q/D Orders</i>
$A+B*\sqrt{i}+C*i$	$A+B*i+C*\sqrt{i}$	1.47974300E-10	4, 6
$A+B*\sqrt{i}$	$A+B*i$	6.28433200E-10	4, 5
$A+B/i$	$A+B*\log_{10}(i)^4$	1.06639000E-09	7, 5
$A+B*\log_{10}(i)^4$	$A+B/i$	1.85273000E-09	5, 5
$A+B*i+C*\sqrt{i}$	$A+B*\sqrt{i}+C*i$	2.53657600E-09	6, 6
$A+B/i$	$A+B*i$	2.90096200E-09	3, 6
$A+B*i$	$A+B/i$	3.67243200E-09	5, 4
$A+B*i$	$A+B*\log_{10}(i)^4$	7.38693400E-09	3, 6
$A+B*\log_{10}(i)^4$	$A+B*i$	1.01959100E-08	7, 3
$A+B*i$	$A+B*\sqrt{i}$	2.85273100E-07	3, 7

RESULTS FOR THE COMMON LOGARITHM

Combo Powers 1

Fitting $\log(x)$ in range (1.000000, 10.000000)

Shammas polynomial power for $Q(x)$ is $A+B*i$

Shammas polynomial power for $D(x)$ is $A+B/i$

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	184.287240527789	0	Inf	0
x1	-1367.67622240212	0	-Inf	0
x2	2298.93535111266	0	Inf	0
x3	0	0	NaN	NaN
x4	-3385.48829419803	0	-Inf	0
x5	3227.8861358049	0	Inf	0
x6	-957.956437458083	0	-Inf	0
x7	-247.121292321496	0	-Inf	0
x8	3466.06640198992	0	Inf	0
x9	-6349.26372853979	0	-Inf	0
x10	0	0	NaN	NaN
x11	3131.33084548106	0	Inf	0

Number of observations: 901, Error degrees of freedom: 891

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	58.921158778172	900	0.0654679541979689		
Model	58.921158778172	9	6.54679541979689	Inf	0

Total	58.9211587782017	900	0.0654679541980019		
Model	58.9211587782017	10	5.89211587782017	Inf	0
Residual	0	890	0		

for Q(x) A = 0.222995, B = 0.210331, C = 0.250329
 for D(x) A = 0.110260, B = 0.243140, C = 0.279330
 order Q(x) = 6.000000, order D(x) = 6.000000
 Xmin = 1.000000 and Xmax = 10.000000
 Ymin = 0.000000 and Ymax = 1.000000
 List of powers for Q(x): [0.683655, 1.021106, 1.338287, 1.644974, 1.944955, 2.240174]
 List of powers for D(x): [0.632730, 0.991572, 1.323493, 1.641479, 1.950560, 2.253315]
 Fitting log(x) in range (1.000000, 2.000000)
 MSS of errors squared = 5.331502e-11
 Corrected MSS of errors squared = 1.305946e-10
 R-squared = 1.00000000
 R-squared Adjusted = 1.00000000
 Particle swarm AICc = -1.000000e+99
 AIC = -Inf
 AICc = -Inf

Combo Powers 4

Fitting log(x) in range (1.000000, 10.000000)
 Shammas polynomial power for Q(x) is A+B/i
 Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-30.2265392382888	0	-Inf	0
x1	325.261239663103	0	Inf	0
x2	-1147.37705261145	0	-Inf	0
x3	0	0	NaN	NaN
x4	852.392984141287	0	Inf	0
x5	4.53754608569155	0	Inf	0
x6	-1.15861260427278	0	-Inf	0
x7	-6.49770626288175	0	-Inf	0
x8	5.88038777140755	0	Inf	0
x9	-2.0938974919139	0	-Inf	0
x10	0.281650605621219	0	Inf	0

Number of observations: 901, Error degrees of freedom: 891

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

SumSq	DF	MeanSq	F	pValue
-------	----	--------	---	--------

Total	58.9211587783208	900	0.0654679541981343		
Model	58.9211587783208	9	6.54679541981343	Inf	0
Residual	0	891	0		

```

for Q(x) A = 0.400839, B = 0.322975
for D(x) A = 0.709306, B = 0.474699
order Q(x) = 4.000000, order D(x) = 6.000000
Xmin = 1.000000 and Xmax = 10.000000
Ymin = 0.000000 and Ymax = 1.000000
List of powers for Q(x): [0.723814, 0.562327, 0.508498, 0.481583]
List of powers for D(x): [1.184005, 1.658704, 2.133403, 2.608102, 3.082801, 3.557500]
Fitting log(x) in range (1.000000, 2.000000)
MSS of errors squared = 2.833306e-09
Corrected MSS of errors squared = 5.666612e-09
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 5

```

Fitting log(x) in range (1.000000, 10.000000)
Shammas polynomial power for Q(x) is A+B*sqrt(i)
Shammas polynomial power for D(x) is A+B*i

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-3.46366158790905	0	-Inf	0
x1	86.1937058321701	0	Inf	0
x2	0	0	NaN	NaN
x3	-195.361295734653	0	-Inf	0
x4	0	0	NaN	NaN
x5	112.770008413204	0	Inf	0
x6	3.92549347706161	0	Inf	0
x7	-3.67141298647551	0	-Inf	0
x8	0.660746632273289	0	Inf	0
x9	-0.0382895447474868	0	-Inf	0
x10	-0.020685137946028	0	-Inf	0
x11	0.00590703755892678	0	Inf	0
x12	-0.000516295785683355	0	-Inf	0

Number of observations: 901, Error degrees of freedom: 890

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	58.9211587781871	900	0.0654679541979857		
Model	58.9211587781871	10	5.89211587781871	Inf	0
Residual	0	890	0		

for Q(x) A = 0.581913, B = 0.500000

for D(x) A = 0.460185, B = 1.168787

order Q(x) = 5.000000, order D(x) = 7.000000

Xmin = 1.000000 and Xmax = 10.000000

Ymin = 0.000000 and Ymax = 1.000000

List of powers for Q(x): [1.081913, 1.289020, 1.447938, 1.581913, 1.699947]

List of powers for D(x): [1.628973, 2.797760, 3.966547, 5.135335, 6.304122, 7.472909, 8.641696]

Fitting log(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.811449e-09

Corrected MSS of errors squared = 3.622898e-09

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 6

Fitting log(x) in range (1.000000, 10.000000)

Shammas polynomial power for Q(x) is $A+B*i+C*\sqrt{i}$

Shammas polynomial power for D(x) is $A+B*\sqrt{i}+C*i$

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	8.59922050743758	0	Inf	0
x1	-114.427459806089	0	-Inf	0
x2	338.733730412212	0	Inf	0
x3	-412.653728260337	0	-Inf	0
x4	220.600408070832	0	Inf	0
x5	-40.8177824615023	0	-Inf	0
x6	36.3301604529364	0	Inf	0
x7	-140.350718532687	0	-Inf	0
x8	202.54965940055	0	Inf	0
x9	-123.366223652591	0	-Inf	0
x10	25.8027338704153	0	Inf	0

Number of observations: 901, Error degrees of freedom: 890

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	58.9211587782367	900	0.0654679541980407		
Model	58.9211587782367	10	5.89211587782366	Inf	0
Residual	0	890	0		

for Q(x) A = 0.076833, B = 0.390000, C = 0.220799

for D(x) A = 0.000000, B = 0.386093, C = 0.295283

order Q(x) = 5.000000, order D(x) = 5.000000

Xmin = 1.000000 and Xmax = 10.000000

Ymin = 0.000000 and Ymax = 1.000000

List of powers for Q(x): [0.687633, 1.169091, 1.629269, 2.078432, 2.520556]

List of powers for D(x): [0.681376, 1.136584, 1.554582, 1.953319, 2.339746]

Fitting log(x) in range (1.000000, 2.000000)

MSS of errors squared = 8.232714e-11

Corrected MSS of errors squared = 2.016595e-10

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 7

Fitting log(x) in range (1.000000, 10.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-43.0742020709602	0	-Inf	0
x1	-44.4876111538917	0	-Inf	0
x2	646.21963583829	0	Inf	0
x3	-954.172190381227	0	-Inf	0
x4	395.54716257511	0	Inf	0
x5	986.497770506383	0	Inf	0
x6	-1193.6091618601	0	-Inf	0
x7	228.584279906801	0	Inf	0
x8	-20.5056832737874	0	-Inf	0

Number of observations: 901, Error degrees of freedom: 892

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	58.9211587780763	900	0.0654679541978626		
Model	58.9211587780763	8	7.36514484725954	Inf	0
Residual	0	892	0		

for Q(x) A = 0.091591, B = 0.108000

for D(x) A = 1.345454, B = 2.498839

order Q(x) = 4.000000, order D(x) = 4.000000

Xmin = 1.000000 and Xmax = 10.000000

Ymin = 0.000000 and Ymax = 1.000000

List of powers for Q(x): [0.199592, 0.307592, 0.415592, 0.523593]

List of powers for D(x): [1.345454, 1.365974, 1.474949, 1.673774]

Fitting log(x) in range (1.000000, 2.000000)

MSS of errors squared = 6.807933e-09

Corrected MSS of errors squared = 1.361587e-08

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 8

Fitting log(x) in range (1.000000, 10.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.0771724137552619	0	Inf	0
x1	-0.000733021869030927	0	-Inf	0
x2	0.168077345312599	0	Inf	0
x3	-1.92815358260837	0	-Inf	0
x4	4.75023804979267	0	Inf	0
x5	-3.06609055605522	0	-Inf	0
x6	113.5606476626	0	Inf	0
x7	0	0	NaN	NaN
x8	-174.959634875643	0	-Inf	0
x9	62.3984765668966	0	Inf	0

Number of observations: 901, Error degrees of freedom: 892

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	58.921158778176	900	0.0654679541979734		
Model	58.921158778176	8	7.365144847272	Inf	0
Residual	0	892	0		

for Q(x) A = 0.453809, B = 2.756807

for D(x) A = 0.255566, B = 0.299213

order Q(x) = 5.000000, order D(x) = 4.000000

Xmin = 1.000000 and Xmax = 10.000000

Ymin = 0.000000 and Ymax = 1.000000

List of powers for Q(x): [3.210616, 1.832213, 1.372745, 1.143011, 1.005170]

List of powers for D(x): [0.255566, 0.258023, 0.271072, 0.294879]

Fitting log(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.201374e-08

Corrected MSS of errors squared = 2.402747e-08

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 9

Fitting log(x) in range (1.000000, 10.000000)

Shammas polynomial power for Q(x) is $A+B*\log_{10}(i)^4$

Shammas polynomial power for D(x) is $A+B*i$

Linear regression model:

$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-7.40987485600517	0	-Inf	0
x1	659.083430659688	0	Inf	0
x2	0	0	NaN	NaN
x3	0	0	NaN	NaN
x4	0	0	NaN	NaN
x5	-1776.35256649782	0	-Inf	0
x6	1124.72616934701	0	Inf	0
x7	3.08277897738495	0	Inf	0
x8	-2.5458897516525	0	-Inf	0
x9	0.464178423527271	0	Inf	0
x10	-0.0498977953630564	0	-Inf	0
x11	0.00167165917305291	0	Inf	0

Number of observations: 901, Error degrees of freedom: 892

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	58.9211587781356	900	0.0654679541979284		
Model	58.9211587781356	8	7.36514484726694	Inf	0
Residual	0	892	0		

for Q(x) A = 0.663658, B = 0.370104

for D(x) A = 0.149030, B = 0.753730

order Q(x) = 6.000000, order D(x) = 5.000000

Xmin = 1.000000 and Xmax = 10.000000

Ymin = 0.000000 and Ymax = 1.000000

List of powers for Q(x): [0.663658, 0.666697, 0.682837, 0.712285, 0.751998, 0.799358]

List of powers for D(x): [0.902759, 1.656489, 2.410219, 3.163948, 3.917678]

Fitting log(x) in range (1.000000, 2.000000)

MSS of errors squared = 8.988273e-09

Corrected MSS of errors squared = 1.797655e-08

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 10

Fitting log(x) in range (1.000000, 10.000000)

Shammas polynomial power for Q(x) is $A+B*\log_{10}(i)^4$

Shammas polynomial power for D(x) is $A+B/i$

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.0243850130523552	0	Inf	0
x1	-0.239097835018434	0	-Inf	0
x2	0	0	NaN	NaN
x3	0.214724634973782	0	Inf	0
x4	0.0120815496108302	0	Inf	0
x5	-0.792545525193208	0	-Inf	0
x6	4.11520218789486	0	Inf	0
x7	0	0	NaN	NaN
x8	-11.4478944852119	0	-Inf	0
x9	0	0	NaN	NaN

```
x10          9.11314446001092    0    Inf    0
```

Number of observations: 901, Error degrees of freedom: 893

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	58.9211587781908	900	0.0654679541979897		
Model	58.9211587781908	7	8.41730839688439	Inf	0
Residual	0	893	0		

for Q(x) A = 0.152483, B = 0.346923

for D(x) A = 0.000000, B = 0.617514

order Q(x) = 3.000000, order D(x) = 7.000000

Xmin = 1.000000 and Xmax = 10.000000

Ymin = 0.000000 and Ymax = 1.000000

List of powers for Q(x): [0.152483, 0.155332, 0.170461]

List of powers for D(x): [0.617514, 0.308757, 0.205838, 0.154378, 0.123503, 0.102919, 0.088216]

Fitting log(x) in range (1.000000, 2.000000)

MSS of errors squared = 2.871660e-08

Corrected MSS of errors squared = 5.743320e-08

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Comments

The following table shows the summary results for the ten models that fit the common logarithm function. The results are sorted in ascending order using the MSSE values.

<i>Power Q(x)</i>	<i>Power D(x)</i>	<i>MSSE</i>	<i>Q/D Orders</i>
$A+B*\sqrt{i}+C*i$	$A+B*i+C*\sqrt{i}$	5.33150200E-11	6, 6
$A+B*i+C*\sqrt{i}$	$A+B*\sqrt{i}+C*i$	8.23271400E-11	5, 5
$A+B*i$	$A+B/i$	6.32496900E-10	6, 5
$A+B*\sqrt{i}$	$A+B*i$	1.81144900E-09	5, 7
$A+B/i$	$A+B*i$	2.83330600E-09	4, 6
$A+B*i$	$A+B*\sqrt{i}$	2.84879500E-09	4, 5
$A+B*i$	$A+B*\log_{10}(i)^4$	6.80793300E-09	4, 4
$A+B*\log_{10}(i)^4$	$A+B*i$	8.98827300E-09	6, 5
$A+B/i$	$A+B*\log_{10}(i)^4$	1.20137400E-08	5, 4
$A+B*\log_{10}(i)^4$	$A+B/i$	2.87166000E-08	3, 7

RESULTS FOR THE COMMON LOGARITHM OF THE GAMMA FUNCTION

Combo Powers 1

Fitting $\log_{10}\Gamma(x)$ in range (2.000000, 100.000000)

Shammas polynomial power for Q(x) is $A+B*i$

Shammas polynomial power for D(x) is $A+B/i$

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	155.522327358789	0	Inf	0
x1	-2834.1052815679	0	-Inf	0
x2	9378.44807393124	0	Inf	0
x3	-14382.6533899039	0	-Inf	0
x4	12070.1467042551	0	Inf	0
x5	-5409.97248540241	0	-Inf	0
x6	1022.62472354628	0	Inf	0
x7	7.96674639014079	0	Inf	0
x8	-56.7221191317535	0	-Inf	0
x9	49.7447004082236	0	Inf	0

Number of observations: 981, Error degrees of freedom: 971

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	88.4325686544542	980	0.0902373149535247		
Model	88.4325686544542	9	9.82584096160602	Inf	0

	SumSq	DF	MeanSq	F	pValue
Total	88.4325686559094	980	0.0902373149550096		
Model	88.4325686559094	10	8.84325686559094	Inf	0
Residual	0	970	0		

```

for Q(x) A = 0.483537, B = 0.100000
for D(x) A = 2.000000, B = 0.557548
order Q(x) = 7.000000, order D(x) = 7.000000
Xmin = 2.000000 and Xmax = 100.000000
Ymin = 0.000000 and Ymax = 155.970004
List of powers for Q(x): [0.583537, 0.683537, 0.783537, 0.883537, 0.983537, 1.083537,
1.183537]
List of powers for D(x): [2.557548, 2.788492, 2.965701, 3.115096, 3.246715, 3.365708,
3.475133]
Fitting log10Gamma(x) in range (1.000000, 2.000000)
MSS of errors squared = 1.374819e-08
Corrected MSS of errors squared = 2.749638e-08
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 3

```

Fitting log10Gamma(x) in range (2.000000, 100.000000)
Shammas polynomial power for Q(x) is A+B*sqrt(i)+C*i
Shammas polynomial power for D(x) is A+B*i+C*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.502244088594858	0	-Inf	0
x1	38.5312343363265	0	Inf	0
x2	-125.010089535266	0	-Inf	0
x3	135.965249306316	0	Inf	0
x4	-48.9843870189298	0	-Inf	0
x5	28.3428363757211	0	Inf	0
x6	-78.1241396453999	0	-Inf	0
x7	84.6338187945021	0	Inf	0
x8	-41.6972381085199	0	-Inf	0
x9	7.8449595860933	0	Inf	0

Number of observations: 981, Error degrees of freedom: 971

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	88.4325686539686	980	0.0902373149530292		
Model	88.4325686539686	9	9.82584096155207	Inf	0
Residual	0	971	0		

for Q(x) A = 0.412426, B = 0.308665, C = 0.024902

for D(x) A = 0.303397, B = 0.143438, C = 0.051309

order Q(x) = 4.000000, order D(x) = 5.000000

Xmin = 2.000000 and Xmax = 100.000000

Ymin = 0.000000 and Ymax = 155.970004

List of powers for Q(x): [0.745994, 0.898749, 1.021756, 1.129365]

List of powers for D(x): [0.498144, 0.662835, 0.822581, 0.979767, 1.135317]

Fitting log10Gamma(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.678527e-08

Corrected MSS of errors squared = 4.111536e-08

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 4

Fitting log10Gamma(x) in range (2.000000, 100.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	45.2998532816273	0	Inf	0
x1	235.730828803969	0	Inf	0
x2	-2200.33627986093	0	-Inf	0
x3	0	0	NaN	NaN
x4	5859.57778589436	0	Inf	0
x5	0	0	NaN	NaN
x6	0	0	NaN	NaN
x7	-3940.27023596411	0	-Inf	0
x8	3.1863416435293	0	Inf	0
x9	-3.22088496037501	0	-Inf	0
x10	1.19879556336984	0	Inf	0
x11	-0.166204433096684	0	-Inf	0

Number of observations: 981, Error degrees of freedom: 972

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	88.4325686523828	980	0.090237314951411		
Model	88.4325686523828	8	11.0540710815478	Inf	0
Residual	0	972	0		

for Q(x) A = 0.318665, B = 0.385575

for D(x) A = 0.100000, B = 0.455288

order Q(x) = 7.000000, order D(x) = 4.000000

Xmin = 2.000000 and Xmax = 100.000000

Ymin = 0.000000 and Ymax = 155.970004

List of powers for Q(x): [0.704241, 0.511453, 0.447190, 0.415059, 0.395780, 0.382928, 0.373747]

List of powers for D(x): [0.555288, 1.010577, 1.465865, 1.921154]

Fitting log10Gamma(x) in range (1.000000, 2.000000)

MSS of errors squared = 2.870134e-08

Corrected MSS of errors squared = 5.740267e-08

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 5

Fitting log10Gamma(x) in range (2.000000, 100.000000)

Shammas polynomial power for Q(x) is A+B*sqrt(i)

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12 + x13 + x14$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-2.61684113666878	0	-Inf	0
x1	474.530413939366	0	Inf	0
x2	0	0	NaN	NaN
x3	-1987.91950478818	0	-Inf	0
x4	0	0	NaN	NaN
x5	2570.36598003409	0	Inf	0
x6	0	0	NaN	NaN
x7	-1054.37849400769	0	-Inf	0
x8	95.501283100179	0	Inf	0
x9	-322.996177079872	0	-Inf	0

x10	453.493838404707	0	Inf	0
x11	-340.732112729523	0	-Inf	0
x12	146.753156105038	0	Inf	0
x13	-34.4364735656326	0	-Inf	0
x14	3.43493176797904	0	Inf	0

Number of observations: 981, Error degrees of freedom: 969

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	88.4325686530411	980	0.0902373149520827		
Model	88.4325686530411	11	8.03932442300374	Inf	0
Residual	0	969	0		

for Q(x) A = 1.357785, B = 0.386770

for D(x) A = 1.561455, B = 0.549565

order Q(x) = 7.000000, order D(x) = 7.000000

Xmin = 2.000000 and Xmax = 100.000000

Ymin = 0.000000 and Ymax = 155.970004

List of powers for Q(x): [1.744554, 1.904760, 2.027690, 2.131324, 2.222628, 2.305173, 2.381081]

List of powers for D(x): [2.111020, 2.660585, 3.210150, 3.759716, 4.309281, 4.858846, 5.408411]

Fitting log10Gamma(x) in range (1.000000, 2.000000)

MSS of errors squared = 4.378619e-09

Corrected MSS of errors squared = 8.757237e-09

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 6

Fitting log10Gamma(x) in range (2.000000, 100.000000)

Shammas polynomial power for Q(x) is A+B*i+C*sqrt(i)

Shammas polynomial power for D(x) is A+B*sqrt(i)+C*i

Linear regression model:

$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.989464285655904	0	Inf	0
x1	-5.53728932261865	0	-Inf	0
x2	-9.30887045001413	0	-Inf	0

x3	74.413371379715	0	Inf	0
x4	-131.30591106539	0	-Inf	0
x5	98.1675996701532	0	Inf	0
x6	-27.4195219522795	0	-Inf	0
x7	7.16651928449585	0	Inf	0
x8	-11.3542661945479	0	-Inf	0
x9	5.73266593412229	0	Inf	0
x10	-0.543761565807644	0	-Inf	0

Number of observations: 981, Error degrees of freedom: 970

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	88.4325686540238	980	0.0902373149530855		
Model	88.4325686540238	10	8.84325686540238	Inf	0
Residual	0	970	0		

for Q(x) A = 0.067057, B = 0.270795, C = 0.302463

for D(x) A = 0.241908, B = 0.227725, C = 0.365526

order Q(x) = 6.000000, order D(x) = 4.000000

Xmin = 2.000000 and Xmax = 100.000000

Ymin = 0.000000 and Ymax = 155.970004

List of powers for Q(x): [0.640315, 1.036395, 1.403324, 1.755164, 2.097361, 2.432708]

List of powers for D(x): [0.835159, 1.295011, 1.732917, 2.159462]

Fitting log10Gamma(x) in range (1.000000, 2.000000)

MSS of errors squared = 4.913328e-09

Corrected MSS of errors squared = 1.203515e-08

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 7

Fitting log10Gamma(x) in range (2.000000, 100.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	109.444047895037	0	Inf	0
x1	-1835.20927512653	0	-Inf	0

x2	2867.60635300226	0	Inf	0
x3	0	0	NaN	NaN
x4	-2051.57344769569	0	-Inf	0
x5	909.734794833575	0	Inf	0
x6	146249.704198823	0	Inf	0
x7	-184029.038085363	0	-Inf	0
x8	43434.2723390794	0	Inf	0
x9	-5653.94092535169	0	-Inf	0

Number of observations: 981, Error degrees of freedom: 972

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	88.4325686573908	980	0.0902373149565212		
Model	88.4325686573908	8	11.0540710821738	Inf	0
Residual	0	972	0		

for Q(x) A = 0.234543, B = 0.106277

for D(x) A = 1.538336, B = 0.504345

order Q(x) = 5.000000, order D(x) = 4.000000

Xmin = 2.000000 and Xmax = 100.000000

Ymin = 0.000000 and Ymax = 155.970004

List of powers for Q(x): [0.340819, 0.447096, 0.553372, 0.659649, 0.765925]

List of powers for D(x): [1.538336, 1.542478, 1.564472, 1.604602]

Fitting log10Gamma(x) in range (1.000000, 2.000000)

MSS of errors squared = 7.694870e-08

Corrected MSS of errors squared = 1.538974e-07

R-squared = 1.000000000

R-squared Adjusted = 1.000000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 8

Fitting log10Gamma(x) in range (2.000000, 100.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.0049361204216672	0	Inf	0
x1	-0.00128165282710213	0	-Inf	0

x2	-0.451440841824751	0	-Inf	0
x3	6.14674072572543	0	Inf	0
x4	-20.2949802673525	0	-Inf	0
x5	20.4505249970146	0	Inf	0
x6	0	0	NaN	NaN
x7	-5.8545004930209	0	-Inf	0
x8	127.688664417208	0	Inf	0
x9	-145.960011069645	0	-Inf	0
x10	19.27134806431	0	Inf	0

Number of observations: 981, Error degrees of freedom: 971

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	88.4325686540077	980	0.0902373149530691		
Model	88.4325686540077	9	9.82584096155641	Inf	0
Residual	0	971	0		

for Q(x) A = 0.500000, B = 1.267226

for D(x) A = 0.100000, B = 0.500000

order Q(x) = 7.000000, order D(x) = 3.000000

Xmin = 2.000000 and Xmax = 100.000000

Ymin = 0.000000 and Ymax = 155.970004

List of powers for Q(x): [1.767226, 1.133613, 0.922409, 0.816807, 0.753445, 0.711204, 0.681032]

List of powers for D(x): [0.100000, 0.104106, 0.125911]

Fitting log10Gamma(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.137677e-08

Corrected MSS of errors squared = 2.275354e-08

R-squared = 1.000000000

R-squared Adjusted = 1.000000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 9

Fitting log10Gamma(x) in range (2.000000, 100.000000)

Shammas polynomial power for Q(x) is A+B*log10(i)^4

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12$$

Estimated Coefficients:

Estimate	SE	tStat	pValue
----------	----	-------	--------

(Intercept)	0.0296391102268717	0	Inf	0
x1	-107.457606593569	0	-Inf	0
x2	0	0	NaN	NaN
x3	0	0	NaN	NaN
x4	236.825649381543	0	Inf	0
x5	-129.397548949037	0	-Inf	0
x6	11.6204978692631	0	Inf	0
x7	-30.9203614648385	0	-Inf	0
x8	38.9433605157752	0	Inf	0
x9	-28.1180463500293	0	-Inf	0
x10	12.0396162891087	0	Inf	0
x11	-2.85564932299642	0	-Inf	0
x12	0.290449509584069	0	Inf	0

Number of observations: 981, Error degrees of freedom: 970

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	88.4325686540868	980	0.0902373149531498		
Model	88.4325686540868	10	8.84325686540868	Inf	0
Residual	0	970	0		

for Q(x) A = 2.054770, B = 0.259931

for D(x) A = 0.400282, B = 0.522184

order Q(x) = 5.000000, order D(x) = 7.000000

Xmin = 2.000000 and Xmax = 100.000000

Ymin = 0.000000 and Ymax = 155.970004

List of powers for Q(x): [2.054770, 2.056905, 2.068240, 2.088922, 2.116813]

List of powers for D(x): [0.922466, 1.444650, 1.966834, 2.489018, 3.011202, 3.533386, 4.055570]

Fitting log10Gamma(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.011249e-06

Corrected MSS of errors squared = 2.022498e-06

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 10

Fitting log10Gamma(x) in range (2.000000, 100.000000)

Shammas polynomial power for Q(x) is A+B*log10(i)^4

Shammas polynomial power for D(x) is A+B/i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.0212098361702943	0	-Inf	0
x1	1.61679232816521	0	Inf	0
x2	0	0	NaN	NaN
x3	0	0	NaN	NaN
x4	0	0	NaN	NaN
x5	-4.65020628547734	0	-Inf	0
x6	3.05507248259907	0	Inf	0
x7	-0.801808344821705	0	-Inf	0
x8	151.688085422715	0	Inf	0
x9	-2126.43311113898	0	-Inf	0
x10	7991.92681119945	0	Inf	0
x11	-10868.9851811269	0	-Inf	0
x12	4853.60475528863	0	Inf	0

Number of observations: 981, Error degrees of freedom: 971

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	88.432568657669	980	0.0902373149568051		
Model	88.432568657669	9	9.82584096196322	Inf	0
Residual	0	971	0		

for Q(x) A = 0.180343, B = 0.483732

for D(x) A = 0.889628, B = 1.381535

order Q(x) = 6.000000, order D(x) = 6.000000

Xmin = 2.000000 and Xmax = 100.000000

Ymin = 0.000000 and Ymax = 155.970004

List of powers for Q(x): [0.180343, 0.184315, 0.205411, 0.243900, 0.295805, 0.357705]

List of powers for D(x): [2.271163, 1.580395, 1.350140, 1.235012, 1.165935, 1.119884]

Fitting log10Gamma(x) in range (1.000000, 2.000000)

MSS of errors squared = 4.691180e-08

Corrected MSS of errors squared = 9.382360e-08

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Comments

The following table shows the summary results for the ten models that fit the common logarithm of the gamma function. The results are sorted in ascending order using the MSSE values.

<i>Power Q(x)</i>	<i>Power D(x)</i>	<i>MSSE</i>	<i>Q/D Orders</i>
A+B*sqrt(i)	A+B*i	4.37861900E-09	7, 7
A+B*i+C*sqrt(i)	A+B*sqrt(i)+C*i	4.91332800E-09	6, 4
A+B/i	A+B*log10(i)^4	1.13767700E-08	7, 3
A+B*i	A+B*sqrt(i)	1.37481900E-08	7, 7
A+B*sqrt(i)+C*i	A+B*i+C*sqrt(i)	1.67852700E-08	4, 5
A+B*i	A+B/i	1.88071200E-08	6, 3
A+B/i	A+B*i	2.87013400E-08	7, 4
A+B*log10(i)^4	A+B/i	4.69118000E-08	6, 6
A+B*i	A+B*log10(i)^4	7.69487000E-08	5, 4
A+B*log10(i)^4	A+B*i	1.01124900E-06	5, 7

RESULTS FOR THE COMMON EXPONENT (10^X)

Combo Powers 1

Fitting 10^x in range (0.000000, 1.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B/i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.99232278733845	0	Inf	0
x1	-1.50048271624545	0	-Inf	0
x2	2.93582721339009	0	Inf	0
x3	-5.82479088345883	0	-Inf	0
x4	6.07005304495148	0	Inf	0
x5	-3.01413567906093	0	-Inf	0
x6	0.610019323185775	0	Inf	0
x7	0.0731157678320091	0	Inf	0
x8	-1.51970373206296	0	-Inf	0
x9	2.17777487417869	0	Inf	0

Number of observations: 101, Error degrees of freedom: 91

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	7.96302009058558	100	0.0796302009058558		
Model	7.96302009058558	9	0.884780010065065	Inf	0
Residual	0	91	0		

```

for Q(x) A = 0.217427, B = 0.341575
for D(x) A = 0.333140, B = 1.589035
order Q(x) = 6.000000, order D(x) = 3.000000
Xmin = 0.000000 and Xmax = 1.000000
Ymin = 1.000000 and Ymax = 10.000000
List of powers for Q(x): [0.559002, 0.900576, 1.242151, 1.583726, 1.925300, 2.266875]
List of powers for D(x): [1.922174, 1.127657, 0.862818]
Fitting 10^x in range (1.000000, 2.000000)
MSS of errors squared = 1.469034e-11
Corrected MSS of errors squared = 2.938068e-11
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 2

```

Fitting 10^x in range (0.000000, 1.000000)
Shammas polynomial power for Q(x) is A+B*i
Shammas polynomial power for D(x) is A+B*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	1.32388766280501	0	Inf	0
x1	-4.59025877089806	0	-Inf	0
x2	14.5790852843684	0	Inf	0
x3	-21.4778439768105	0	-Inf	0
x4	13.691193391425	0	Inf	0
x5	-3.15085215639446	0	-Inf	0
x6	53.1293628553001	0	Inf	0
x7	-200.308246832187	0	-Inf	0
x8	286.415002620277	0	Inf	0
x9	-181.731632884427	0	-Inf	0
x10	43.120302806572	0	Inf	0

Number of observations: 101, Error degrees of freedom: 90

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	7.96302009060261	100	0.0796302009060261		
Model	7.96302009060261	10	0.796302009060261	Inf	0
Residual	0	90	0		

```

for Q(x) A = 0.121127, B = 0.387628
for D(x) A = 1.823233, B = 0.538949
order Q(x) = 5.000000, order D(x) = 5.000000
Xmin = 0.000000 and Xmax = 1.000000
Ymin = 1.000000 and Ymax = 10.000000
List of powers for Q(x): [0.508755, 0.896383, 1.284012, 1.671640, 2.059268]
List of powers for D(x): [2.362181, 2.585421, 2.756719, 2.901130, 3.028359]
Fitting 10^x in range (1.000000, 2.000000)
MSS of errors squared = 8.304451e-12
Corrected MSS of errors squared = 1.660890e-11
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 3

```

Fitting 10^x in range (0.000000, 1.000000)
Shammas polynomial power for Q(x) is A+B*sqrt(i)+C*i
Shammas polynomial power for D(x) is A+B*i+C*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.921962167491086	0	Inf	0
x1	-2.31242400071847	0	-Inf	0
x2	2.07873699470689	0	Inf	0
x3	-0.572271999745948	0	-Inf	0
x4	4.83731306355854	0	Inf	0
x5	-9.09935162394983	0	-Inf	0
x6	9.33097702799284	0	Inf	0
x7	-6.23194356681661	0	-Inf	0
x8	2.4683050962528	0	Inf	0
x9	-0.421303158781209	0	-Inf	0

Number of observations: 101, Error degrees of freedom: 91

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	7.96302009058566	100	0.0796302009058566		
Model	7.96302009058566	9	0.884780010065073	Inf	0
Residual	0	91	0		

```

for Q(x) A = 0.060214, B = 0.393311, C = 0.207637
for D(x) A = 0.308062, B = 0.239433, C = 0.246003
order Q(x) = 3.000000, order D(x) = 6.000000
Xmin = 0.000000 and Xmax = 1.000000
Ymin = 1.000000 and Ymax = 10.000000
List of powers for Q(x): [0.661162, 1.031713, 1.364359]
List of powers for D(x): [0.793497, 1.134827, 1.452448, 1.757797, 2.055303, 2.347238]
Fitting 10^x in range (1.000000, 2.000000)
MSS of errors squared = 1.121157e-11
Corrected MSS of errors squared = 2.746262e-11
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 4

```

Fitting 10^x in range (0.000000, 1.000000)
Shammas polynomial power for Q(x) is A+B/i
Shammas polynomial power for D(x) is A+B*i

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.636414968767064	0	Inf	0
x1	5.3052207504631	0	Inf	0
x2	-23.1362867528399	0	-Inf	0
x3	17.5709470753009	0	Inf	0
x4	1.65638589213408	0	Inf	0
x5	-1.84269306568811	0	-Inf	0
x6	1.24777706826939	0	Inf	0
x7	-0.594789152350444	0	-Inf	0
x8	0.189615207655734	0	Inf	0
x9	-0.03552057393442	0	-Inf	0
x10	0.00292858216309262	0	Inf	0

Number of observations: 101, Error degrees of freedom: 90

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	7.96302009058525	100	0.0796302009058525		
Model	7.96302009058525	10	0.796302009058525	Inf	0

```

Residual          0      90          0

for Q(x) A = 0.490738, B = 0.479877
for D(x) A = 0.554862, B = 0.801690
order Q(x) = 3.000000, order D(x) = 7.000000
Xmin = 0.000000 and Xmax = 1.000000
Ymin = 1.000000 and Ymax = 10.000000
List of powers for Q(x): [0.970615, 0.730677, 0.650697]
List of powers for D(x): [1.356552, 2.158243, 2.959933, 3.761624, 4.563314, 5.365004,
6.166695]
Fitting 10^x in range (1.000000, 2.000000)
MSS of errors squared = 1.129645e-11
Corrected MSS of errors squared = 2.259290e-11
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Total	7.96302009062148	100	0.0796302009062148		
Model	7.96302009062148	8	0.995377511327685	Inf	0
Residual	0	92	0		

```

for Q(x) A = 0.305119, B = 0.064006
for D(x) A = 1.643525, B = 0.081630
order Q(x) = 6.000000, order D(x) = 6.000000
Xmin = 0.000000 and Xmax = 1.000000
Ymin = 1.000000 and Ymax = 10.000000
List of powers for Q(x): [0.369125, 0.395637, 0.415981, 0.433131, 0.448241, 0.461901]
List of powers for D(x): [1.725155, 1.806785, 1.888415, 1.970045, 2.051675, 2.133305]
Fitting 10^x in range (1.000000, 2.000000)
MSS of errors squared = 5.953929e-10
Corrected MSS of errors squared = 1.190786e-09
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 6

```

Fitting 10^x in range (0.000000, 1.000000)
Shammas polynomial power for Q(x) is A+B*i+C*sqrt(i)
Shammas polynomial power for D(x) is A+B*sqrt(i)+C*i

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.911015687949328	0	Inf	0
x1	-2.57431052348309	0	-Inf	0
x2	2.92388295456244	0	Inf	0
x3	-1.35674199357873	0	-Inf	0
x4	0.245010575673306	0	Inf	0
x5	1.04792909232307	0	Inf	0
x6	6.87793829782759	0	Inf	0
x7	-15.1091506233692	0	-Inf	0
x8	10.4780336334682	0	Inf	0
x9	-2.44360710136858	0	-Inf	0

Number of observations: 101, Error degrees of freedom: 91

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

SumSq	DF	MeanSq	F	pValue
-------	----	--------	---	--------

Total	7.96302009058567	100	0.0796302009058567		
Model	7.96302009058567	9	0.884780010065074	Inf	0
Residual	0	91	0		

```

for Q(x) A = 0.252220, B = 0.351605, C = 0.192133
for D(x) A = 0.362938, B = 0.269347, C = 0.134608
order Q(x) = 4.000000, order D(x) = 5.000000
Xmin = 0.000000 and Xmax = 1.000000
Ymin = 1.000000 and Ymax = 10.000000
List of powers for Q(x): [0.795958, 1.227147, 1.639819, 2.042906]
List of powers for D(x): [0.766893, 1.013068, 1.233285, 1.440064, 1.638257]
Fitting 10^x in range (1.000000, 2.000000)
MSS of errors squared = 2.183044e-12
Corrected MSS of errors squared = 5.347344e-12
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 7

```

Fitting 10^x in range (0.000000, 1.000000)
Shammas polynomial power for Q(x) is A+B*i
Shammas polynomial power for D(x) is A+B*log10(i)^4

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	1.05511985623682	0	Inf	0
x1	-1.94749362270618	0	-Inf	0
x2	6.54660020013621	0	Inf	0
x3	-11.4432416808509	0	-Inf	0
x4	9.59346302793571	0	Inf	0
x5	-4.05916864386589	0	-Inf	0
x6	0.763268002025614	0	Inf	0
x7	66.9906638007614	0	Inf	0
x8	0	0	NaN	NaN
x9	-102.489460535467	0	-Inf	0
x10	35.9902495959383	0	Inf	0

Number of observations: 101, Error degrees of freedom: 91

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	7.96302009058604	100	0.0796302009058604		
Model	7.96302009058604	9	0.884780010065115	Inf	0
Residual	0	91	0		

for Q(x) A = 0.178160, B = 0.390735
 for D(x) A = 1.777074, B = 0.755678
 order Q(x) = 6.000000, order D(x) = 4.000000
 Xmin = 0.000000 and Xmax = 1.000000
 Ymin = 1.000000 and Ymax = 10.000000
 List of powers for Q(x): [0.568895, 0.959631, 1.350366, 1.741102, 2.131837, 2.522572]
 List of powers for D(x): [1.777074, 1.783280, 1.816235, 1.876362]
 Fitting 10^x in range (1.000000, 2.000000)
 MSS of errors squared = 1.967102e-11
 Corrected MSS of errors squared = 3.934205e-11
 R-squared = 1.00000000
 R-squared Adjusted = 1.00000000
 Particle swarm AICc = -1.000000e+99
 AIC = -Inf
 AICc = -Inf

Combo Powers 8

Fitting 10^x in range (0.000000, 1.000000)
 Shammas polynomial power for Q(x) is A+B/i
 Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.757528677194339	0	Inf	0
x1	0.349497449590138	0	Inf	0
x2	-0.304543686516712	0	-Inf	0
x3	-3.28850767913829	0	-Inf	0
x4	2.79410725406133	0	Inf	0
x5	0.113030852484216	0	Inf	0
x6	65.1312227721081	0	Inf	0
x7	0	0	NaN	NaN
x8	0	0	NaN	NaN
x9	-131.278469850935	0	-Inf	0
x10	66.726134209746	0	Inf	0

Number of observations: 101, Error degrees of freedom: 92

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	7.96302009058854	100	0.0796302009058854		
Model	7.96302009058854	8	0.995377511323567	Inf	0
Residual	0	92	0		

for Q(x) A = 0.209269, B = 1.391045

for D(x) A = 1.624460, B = 0.389574

order Q(x) = 5.000000, order D(x) = 5.000000

Xmin = 0.000000 and Xmax = 1.000000

Ymin = 1.000000 and Ymax = 10.000000

List of powers for Q(x): [1.600314, 0.904792, 0.672951, 0.557030, 0.487478]

List of powers for D(x): [1.624460, 1.627659, 1.644648, 1.675645, 1.717447]

Fitting 10^x in range (1.000000, 2.000000)

MSS of errors squared = 2.691391e-10

Corrected MSS of errors squared = 5.382782e-10

R-squared = 1.000000000

R-squared Adjusted = 1.000000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 9

Fitting 10^x in range (0.000000, 1.000000)

Shammas polynomial power for Q(x) is $A+B*\log_{10}(i)^4$

Shammas polynomial power for D(x) is $A+B*i$

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.0691620324222612	0	-Inf	0
x1	1287.33192459429	0	Inf	0
x2	0	0	NaN	NaN
x3	0	0	NaN	NaN
x4	0	0	NaN	NaN
x5	-6722.88314845811	0	-Inf	0
x6	8147.69453123935	0	Inf	0
x7	-2712.09505600291	0	-Inf	0
x8	11.2543868306123	0	Inf	0
x9	-21.8415560062893	0	-Inf	0
x10	15.3407839167333	0	Inf	0
x11	-3.73270408142972	0	-Inf	0

Number of observations: 101, Error degrees of freedom: 92

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	7.96302009052389	100	0.0796302009052389		
Model	7.96302009052389	8	0.995377511315486	Inf	0
Residual	0	92	0		

for Q(x) A = 1.613992, B = 0.090661

for D(x) A = 0.112123, B = 0.103354

order Q(x) = 7.000000, order D(x) = 4.000000

Xmin = 0.000000 and Xmax = 1.000000

Ymin = 1.000000 and Ymax = 10.000000

List of powers for Q(x): [1.613992, 1.614737, 1.618690, 1.625904, 1.635632, 1.647233, 1.660236]

List of powers for D(x): [0.215477, 0.318830, 0.422184, 0.525538]

Fitting 10^x in range (1.000000, 2.000000)

MSS of errors squared = 3.951331e-10

Corrected MSS of errors squared = 7.902661e-10

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 10

Fitting 10^x in range (0.000000, 1.000000)

Shammas polynomial power for Q(x) is A+B*log10(i)^4

Shammas polynomial power for D(x) is A+B/i

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.900856067677215	0	Inf	0
x1	-499.428816674242	0	-Inf	0
x2	0	0	NaN	NaN
x3	0	0	NaN	NaN
x4	1540.69522605572	0	Inf	0
x5	-1419.21814426784	0	-Inf	0
x6	377.144707430659	0	Inf	0
x7	-2.20924546906677	0	-Inf	0
x8	52.9097041653928	0	Inf	0
x9	-152.47927845913	0	-Inf	0
x10	102.684991150979	0	Inf	0

Number of observations: 101, Error degrees of freedom: 92

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	7.96302009058809	100	0.0796302009058809		
Model	7.96302009058809	8	0.995377511323511	Inf	0
Residual	0	92	0		

for Q(x) A = 1.189900, B = 0.408122

for D(x) A = 0.931485, B = 0.724773

order Q(x) = 6.000000, order D(x) = 4.000000

Xmin = 0.000000 and Xmax = 1.000000

Ymin = 1.000000 and Ymax = 10.000000

List of powers for Q(x): [1.189900, 1.193251, 1.211050, 1.243523, 1.287315, 1.339539]

List of powers for D(x): [1.656258, 1.293872, 1.173076, 1.112679]

Fitting 10^x in range (1.000000, 2.000000)

MSS of errors squared = 2.171129e-10

Corrected MSS of errors squared = 4.342258e-10

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Comments

The following table shows the summary results for the ten models that fit the commn exponent function. The results are sorted in ascending order using the MSSE values.

<i>Power Q(x)</i>	<i>Power D(x)</i>	<i>MSSE</i>	<i>Q/D Orders</i>
$A+B*i+C*\sqrt{i}$	$A+B*\sqrt{i}+C*i$	2.18304400E-12	4, 5
$A+B*i$	$A+B*\sqrt{i}$	8.30445100E-12	5, 5
$A+B*\sqrt{i}+C*i$	$A+B*i+C*\sqrt{i}$	1.12115700E-11	3, 6
$A+B/i$	$A+B*i$	1.12964500E-11	3, 7
$A+B*i$	$A+B/i$	1.46903400E-11	6, 3
$A+B*i$	$A+B*\log_{10}(i)^4$	1.96710200E-11	6, 4
$A+B*\log_{10}(i)^4$	$A+B/i$	2.17112900E-10	6, 4
$A+B/i$	$A+B*\log_{10}(i)^4$	2.69139100E-10	5, 5
$A+B*\log_{10}(i)^4$	$A+B*i$	3.95133100E-10	7, 4
$A+B*\sqrt{i}$	$A+B*i$	5.95392900E-10	6, 6

RESULTS FOR THE SINE INTEGRAL

Combo Powers 1

Fitting Si(x) in range (1.300000, 20.000000)

Shammas polynomial power for Q(x) is $A+B*i$

Shammas polynomial power for D(x) is $A+B/i$

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	5.43363379485532e-09	0	Inf	0
x1	-4.69066941892037e-08	0	-Inf	0
x2	5.86304398032247e-08	0	Inf	0
x3	-1.70611538457859e-08	0	-Inf	0
x4	0.00101104421982868	0	Inf	0
x5	-0.494520492458516	0	-Inf	0
x6	15.1866973885457	0	Inf	0
x7	-117.745213405757	0	-Inf	0
x8	342.972888272708	0	Inf	0
x9	-413.678292697037	0	-Inf	0
x10	174.757429889686	0	Inf	0

Number of observations: 188, Error degrees of freedom: 177

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	5.02479642278603	187	0.0268705691058077		
Model	5.02479642278603	10	0.502479642278603	Inf	0
Residual	0	177	0		

```

for Q(x) A = 0.446167, B = 0.100000
for D(x) A = 0.008591, B = 1.267636
order Q(x) = 3.000000, order D(x) = 7.000000
Xmin = 1.300000 and Xmax = 20.000000
Ymin = 1.183958 and Ymax = 1.851659
List of powers for Q(x): [0.546167, 0.646167, 0.746167]
List of powers for D(x): [1.276228, 0.642409, 0.431137, 0.325500, 0.262119, 0.219864,
0.189682]
Fitting Si(x) in range (1.000000, 2.000000)
MSS of errors squared = 8.581168e-03
Corrected MSS of errors squared = 1.716234e-02
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 2

```

Fitting Si(x) in range (1.300000, 20.000000)
Shammas polynomial power for Q(x) is A+B*i
Shammas polynomial power for D(x) is A+B*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.00048238205439272	0	-Inf	0
x1	0.00639299442044581	0	Inf	0
x2	-0.0175769021521365	0	-Inf	0
x3	0.0219645877583974	0	Inf	0
x4	-0.0146735495947928	0	-Inf	0
x5	0.00510528721523432	0	Inf	0
x6	-0.000730023147212959	0	-Inf	0
x7	68.5736623446351	0	Inf	0
x8	-504.637147282443	0	-Inf	0
x9	1542.00561020225	0	Inf	0
x10	-2457.59684175622	0	-Inf	0
x11	2159.51382244444	0	Inf	0
x12	-995.515783761743	0	-Inf	0
x13	188.656677796728	0	Inf	0

Number of observations: 188, Error degrees of freedom: 174

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

SumSq	DF	MeanSq	F	pValue
-------	----	--------	---	--------

Total	5.0247964227718	187	0.0268705691057315		
Model	5.0247964227718	13	0.386522801751677	Inf	0
Residual	0	174	0		

```

for Q(x) A = 0.407291, B = 0.473306
for D(x) A = 0.000000, B = 0.449346
order Q(x) = 6.000000, order D(x) = 7.000000
Xmin = 1.300000 and Xmax = 20.000000
Ymin = 1.183958 and Ymax = 1.851659
List of powers for Q(x): [0.880597, 1.353903, 1.827209, 2.300515, 2.773821, 3.247127]
List of powers for D(x): [0.449346, 0.635472, 0.778291, 0.898693, 1.004769, 1.100669, 1.188859]
Fitting Si(x) in range (1.000000, 2.000000)
MSS of errors squared = 2.538857e-03
Corrected MSS of errors squared = 5.077713e-03
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 3

```

Fitting Si(x) in range (1.300000, 20.000000)
Shammas polynomial power for Q(x) is A+B*sqrt(i)+C*i
Shammas polynomial power for D(x) is A+B*i+C*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	2.57321466114868	0	Inf	0
x1	-111.824127945862	0	-Inf	0
x2	503.633018202943	0	Inf	0
x3	-949.731655516745	0	-Inf	0
x4	905.974347589021	0	Inf	0
x5	-433.861067280716	0	-Inf	0
x6	83.2362702843364	0	Inf	0
x7	8.18321909804447	0	Inf	0
x8	-21.4125134308194	0	-Inf	0
x9	27.4987363694152	0	Inf	0
x10	-19.3171559115088	0	-Inf	0
x11	7.13724866134892	0	Inf	0
x12	-1.08953474136378	0	-Inf	0

Number of observations: 188, Error degrees of freedom: 175

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	5.02479642278744	187	0.0268705691058152		
Model	5.02479642278744	12	0.418733035232287	Inf	0
Residual	0	175	0		

for Q(x) A = 0.173975, B = 0.377480, C = 0.083299

for D(x) A = 0.074466, B = 0.325467, C = 0.063442

order Q(x) = 6.000000, order D(x) = 6.000000

Xmin = 1.300000 and Xmax = 20.000000

Ymin = 1.183958 and Ymax = 1.851659

List of powers for Q(x): [0.634754, 0.874410, 1.077685, 1.262129, 1.434539, 1.598400]

List of powers for D(x): [0.463376, 0.815122, 1.160754, 1.503221, 1.843665, 2.182672]

Fitting Si(x) in range (1.000000, 2.000000)

MSS of errors squared = 3.450218e-01

Corrected MSS of errors squared = 8.451273e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 4

Fitting Si(x) in range (1.300000, 20.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	3.36620544753163e-06	0	Inf	0
x1	0	0	NaN	NaN
x2	-2.65031618054688e-06	0	-Inf	0
x3	0	0	NaN	NaN
x4	0	0	NaN	NaN
x5	8.54588597023265	0	Inf	0
x6	-24.4488923633018	0	-Inf	0
x7	36.2207413125078	0	Inf	0
x8	-31.5584754084343	0	-Inf	0
x9	16.3496084239817	0	Inf	0
x10	-4.68129361211003	0	-Inf	0
x11	0.572424954581375	0	Inf	0

Number of observations: 188, Error degrees of freedom: 179

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	5.02479642278329	187	0.026870569105793		
Model	5.02479642278329	8	0.628099552847912	Inf	0
Residual	0	179	0		

for Q(x) A = 0.483411, B = 0.000000

for D(x) A = 0.100000, B = 0.390085

order Q(x) = 4.000000, order D(x) = 7.000000

Xmin = 1.300000 and Xmax = 20.000000

Ymin = 1.183958 and Ymax = 1.851659

List of powers for Q(x): [0.483411, 0.483411, 0.483411, 0.483411]

List of powers for D(x): [0.490085, 0.880170, 1.270254, 1.660339, 2.050424, 2.440509, 2.830593]

Fitting Si(x) in range (1.000000, 2.000000)

MSS of errors squared = 6.722470e-02

Corrected MSS of errors squared = 1.344494e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 5

Fitting Si(x) in range (1.300000, 20.000000)

Shammas polynomial power for Q(x) is A+B*sqrt(i)

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.000432842338094901	0	Inf	0
x1	-0.296480061373948	0	-Inf	0
x2	1.47984705514363	0	Inf	0
x3	-2.63847482046546	0	-Inf	0
x4	2.02440300929992	0	Inf	0
x5	-0.569727991112	0	-Inf	0
x6	211.67437296419	0	Inf	0
x7	-402.180824091511	0	-Inf	0
x8	191.506451104675	0	Inf	0

Number of observations: 188, Error degrees of freedom: 179

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	5.02479642278341	187	0.0268705691057936		
Model	5.02479642278341	8	0.628099552847926	Inf	0
Residual	0	179	0		

for Q(x) A = 0.798963, B = 0.281515

for D(x) A = 0.028897, B = 0.001590

order Q(x) = 5.000000, order D(x) = 3.000000

Xmin = 1.300000 and Xmax = 20.000000

Ymin = 1.183958 and Ymax = 1.851659

List of powers for Q(x): [1.080477, 1.197085, 1.286561, 1.361992, 1.428449]

List of powers for D(x): [0.030487, 0.032078, 0.033668]

Fitting Si(x) in range (1.000000, 2.000000)

MSS of errors squared = 4.579208e-02

Corrected MSS of errors squared = 9.158415e-02

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 6

Fitting Si(x) in range (1.300000, 20.000000)

Shammas polynomial power for Q(x) is A+B*i+C*sqrt(i)

Shammas polynomial power for D(x) is A+B*sqrt(i)+C*i

Linear regression model:

$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-6.13006600692171e-05	0	-Inf	0
x1	0.00062539940830645	0	Inf	0
x2	-0.0014998611986881	0	-Inf	0
x3	0.00154582819505115	0	Inf	0
x4	-0.000752029502300759	0	-Inf	0
x5	0.000141966792869766	0	Inf	0
x6	36.5978751522529	0	Inf	0
x7	-169.061469826081	0	-Inf	0
x8	330.089665728217	0	Inf	0
x9	-326.553697376023	0	-Inf	0

x10	162.18286870131	0	Inf	0
x11	-32.2552423828002	0	-Inf	0

Number of observations: 188, Error degrees of freedom: 176

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	5.02479642278642	187	0.0268705691058097		
Model	5.02479642278642	11	0.456799674798765	Inf	0
Residual	0	176	0		

for Q(x) A = 0.230659, B = 0.362176, C = 0.279394

for D(x) A = 0.039555, B = 0.163949, C = 0.033226

order Q(x) = 5.000000, order D(x) = 6.000000

Xmin = 1.300000 and Xmax = 20.000000

Ymin = 1.183958 and Ymax = 1.851659

List of powers for Q(x): [0.872229, 1.350133, 1.801111, 2.238150, 2.666282]

List of powers for D(x): [0.236730, 0.337866, 0.423201, 0.500357, 0.572286, 0.640503]

Fitting Si(x) in range (1.000000, 2.000000)

MSS of errors squared = 4.927475e-02

Corrected MSS of errors squared = 1.206980e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 7

Fitting Si(x) in range (1.300000, 20.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12 + x13 + x14

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.0329930860161378	0	-Inf	0
x1	5.0589558156085	0	Inf	0
x2	-12.4192599089647	0	-Inf	0
x3	0	0	NaN	NaN
x4	29.529050952316	0	Inf	0
x5	-38.2437385046272	0	-Inf	0
x6	20.0262299721164	0	Inf	0
x7	-3.918245230915	0	-Inf	0

x8	206.307129670784	0	Inf	0
x9	-261.087413290141	0	-Inf	0
x10	70.0931677783232	0	Inf	0
x11	-17.3645966544393	0	-Inf	0
x12	3.50425798881747	0	Inf	0
x13	-0.486033260288645	0	-Inf	0
x14	0.033487757469358	0	Inf	0

Number of observations: 188, Error degrees of freedom: 174

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	5.02479642278286	187	0.0268705691057907		
Model	5.02479642278286	13	0.386522801752528	Inf	0
Residual	0	174	0		

for Q(x) A = 0.461885, B = 0.100000

for D(x) A = 0.347429, B = 2.948397

order Q(x) = 7.000000, order D(x) = 7.000000

Xmin = 1.300000 and Xmax = 20.000000

Ymin = 1.183958 and Ymax = 1.851659

List of powers for Q(x): [0.561885, 0.661885, 0.761885, 0.861885, 0.961885, 1.061885, 1.161885]

List of powers for D(x): [0.347429, 0.371641, 0.500221, 0.734816, 1.051182, 1.428470, 1.851313]

Fitting Si(x) in range (1.000000, 2.000000)

MSS of errors squared = 7.218736e-04

Corrected MSS of errors squared = 1.443747e-03

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 8

Fitting Si(x) in range (1.300000, 20.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.000478236636412401	0	Inf	0

x1	0.000160912192877347	0	Inf	0
x2	-0.0288842005769855	0	-Inf	0
x3	0.402802245190583	0	Inf	0
x4	-1.51983345565423	0	-Inf	0
x5	2.0785253517055	0	Inf	0
x6	-0.93324908937003	0	-Inf	0
x7	553.987335753676	0	Inf	0
x8	-697.651517404178	0	-Inf	0
x9	173.368685748508	0	Inf	0
x10	-31.6505231170908	0	-Inf	0
x11	2.94601901898517	0	Inf	0

Number of observations: 188, Error degrees of freedom: 176

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	5.024796422781	187	0.0268705691057808		
Model	5.024796422781	11	0.456799674798273	Inf	0
Residual	0	176	0		

for Q(x) A = 0.500000, B = 1.249153

for D(x) A = 0.100000, B = 0.500000

order Q(x) = 6.000000, order D(x) = 5.000000

Xmin = 1.300000 and Xmax = 20.000000

Ymin = 1.183958 and Ymax = 1.851659

List of powers for Q(x): [1.749153, 1.124577, 0.916384, 0.812288, 0.749831, 0.708192]

List of powers for D(x): [0.100000, 0.104106, 0.125911, 0.165695, 0.219345]

Fitting Si(x) in range (1.000000, 2.000000)

MSS of errors squared = 2.634329e-01

Corrected MSS of errors squared = 5.268658e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 9

Fitting Si(x) in range (1.300000, 20.000000)

Shammas polynomial power for Q(x) is $A+B*\log_{10}(i)^4$

Shammas polynomial power for D(x) is $A+B*i$

Linear regression model:

$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9$

Estimated Coefficients:

Estimate	SE	tStat	pValue
----------	----	-------	--------

(Intercept)	-4.40906056695522e-12	0	-Inf	0
x1	8.13436976514173e-07	0	Inf	0
x2	0	0	NaN	NaN
x3	-1.95307570091008e-06	0	-Inf	0
x4	1.7341572357695e-06	0	Inf	0
x5	-7.1370678359639e-07	0	-Inf	0
x6	1.19192679819382e-07	0	Inf	0
x7	1	0	Inf	0
x8	0	0	NaN	NaN
x9	0	0	NaN	NaN

Number of observations: 188, Error degrees of freedom: 181

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	5.02479642278399	187	0.0268705691057967		
Model	5.02479642278399	6	0.837466070463998	Inf	0
Residual	0	181	0		

for Q(x) A = 1.525891, B = 0.433664

for D(x) A = 0.000000, B = 0.000000

order Q(x) = 6.000000, order D(x) = 3.000000

Xmin = 1.300000 and Xmax = 20.000000

Ymin = 1.183958 and Ymax = 1.851659

List of powers for Q(x): [1.525891, 1.529453, 1.548365, 1.582870, 1.629403, 1.684896]

List of powers for D(x): [0.000000, 0.000000, 0.000000]

Fitting Si(x) in range (1.000000, 2.000000)

MSS of errors squared = 3.676159e-02

Corrected MSS of errors squared = 7.352318e-02

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 10

Fitting Si(x) in range (1.300000, 20.000000)

Shammas polynomial power for Q(x) is $A+B*\log_{10}(i)^4$

Shammas polynomial power for D(x) is $A+B/i$

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10$$

Estimated Coefficients:

Estimate	SE	tStat	pValue
----------	----	-------	--------

(Intercept)	-1.53591784686598e-05	0	-Inf	0
x1	0.0961464886798523	0	Inf	0
x2	-0.113596768298106	0	-Inf	0
x3	0.0174656887016919	0	Inf	0
x4	153.794612321419	0	Inf	0
x5	-4756.48178996458	0	-Inf	0
x6	16140.2921732737	0	Inf	0
x7	0	0	NaN	NaN
x8	-26421.3449640003	0	-Inf	0
x9	0	0	NaN	NaN
x10	14884.7399683218	0	Inf	0

Number of observations: 188, Error degrees of freedom: 179

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	5.02479642273022	187	0.0268705691055092		
Model	5.02479642273022	8	0.628099552841278	Inf	0
Residual	0	179	0		

for Q(x) A = 0.899685, B = 0.494043

for D(x) A = 0.244269, B = 0.109711

order Q(x) = 3.000000, order D(x) = 7.000000

Xmin = 1.300000 and Xmax = 20.000000

Ymin = 1.183958 and Ymax = 1.851659

List of powers for Q(x): [0.899685, 0.903741, 0.925287]

List of powers for D(x): [0.353980, 0.299125, 0.280839, 0.271697, 0.266211, 0.262554, 0.259942]

Fitting Si(x) in range (1.000000, 2.000000)

MSS of errors squared = 7.700645e-02

Corrected MSS of errors squared = 1.540129e-01

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Comments

The following table shows the summary results for the ten models that fit the sine Integral. The results are sorted in ascending order using the MSSE values.

<i>Power Q(x)</i>	<i>Power D(x)</i>	<i>MSSE</i>	<i>Q/D Orders</i>
A+B*i	A+B*log10(i)^4	7.21873600E-04	7, 7
A+B*i	A+B*sqrt(i)	2.53885700E-03	6, 7
A+B*i	A+B/i	8.58116800E-03	3, 7
A+B*log10(i)^4	A+B*i	3.67615900E-02	6, 3
A+B*sqrt(i)	A+B*i	4.57920800E-02	5, 3
A+B*i+C*sqrt(i)	A+B*sqrt(i)+C*i	4.92747500E-02	5, 6
A+B/i	A+B*i	6.72247000E-02	4, 7
A+B*log10(i)^4	A+B/i	7.70064500E-02	3, 7
A+B/i	A+B*log10(i)^4	2.63432900E-01	6, 5
A+B*sqrt(i)+C*i	A+B*i+C*sqrt(i)	3.45021800E-01	6, 6

RESULTS FOR THE SINE FUNCTION

Combo Powers 1

Fitting $\sin(x)$ in range (0.000000, 1.570796)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B/i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.107385747519945	0	-Inf	0
x1	0.441813270882395	0	Inf	0
x2	0	0	NaN	NaN
x3	-1.27172015529611	0	-Inf	0
x4	0	0	NaN	NaN
x5	3.5475334418324	0	Inf	0
x6	-3.62362248388672	0	-Inf	0
x7	1.06000028043114	0	Inf	0
x8	-0.202278382098776	0	-Inf	0
x9	5.99270270972775	0	Inf	0
x10	-15.5300319524009	0	-Inf	0
x11	0	0	NaN	NaN
x12	10.6929890183193	0	Inf	0

Number of observations: 501, Error degrees of freedom: 491

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	47.6265936125814	500	0.0952531872251627		

Model	47.6265936125814	9	5.29184373473126	Inf	0
Residual	0	491	0		

for Q(x) A = 0.066555, B = 0.237442
 for D(x) A = 0.283812, B = 0.892248
 order Q(x) = 7.000000, order D(x) = 5.000000
 Xmin = 0.000000 and Xmax = 1.570796
 Ymin = 0.000000 and Ymax = 1.000000
 List of powers for Q(x): [0.303997, 0.541439, 0.778881, 1.016323, 1.253765, 1.491207, 1.728649]
 List of powers for D(x): [1.176059, 0.729936, 0.581228, 0.506874, 0.462261]
 Fitting sin(x) in range (1.000000, 2.000000)
 MSS of errors squared = 3.094588e-12
 Corrected MSS of errors squared = 6.189176e-12
 R-squared = 1.00000000
 R-squared Adjusted = 1.00000000
 Particle swarm AICc = -1.000000e+99
 AIC = -Inf
 AICc = -Inf

Combo Powers 2

Fitting sin(x) in range (0.000000, 1.570796)
 Shammas polynomial power for Q(x) is A+B*i
 Shammas polynomial power for D(x) is A+B*sqrt(i)

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-97.9027128363826	0	-Inf	0
x1	1022.86545094898	0	Inf	0
x2	-1510.24829102949	0	-Inf	0
x3	0	0	NaN	NaN
x4	1052.0723215569	0	Inf	0
x5	-466.195306628085	0	-Inf	0
x6	18.1235370641228	0	Inf	0
x7	-37.8453081549523	0	-Inf	0
x8	0	0	NaN	NaN
x9	38.0038642453702	0	Inf	0
x10	0	0	NaN	NaN
x11	-30.7653785661337	0	-Inf	0
x12	12.8918233966112	0	Inf	0

Number of observations: 501, Error degrees of freedom: 491

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	47.6265936125435	500	0.0952531872250869		
Model	47.6265936125435	9	5.29184373472705	Inf	0
Residual	0	491	0		

```

for Q(x) A = 0.128792, B = 0.100000
for D(x) A = 1.512917, B = 0.534251
order Q(x) = 5.000000, order D(x) = 7.000000
Xmin = 0.000000 and Xmax = 1.570796
Ymin = 0.000000 and Ymax = 1.000000
List of powers for Q(x): [0.228792, 0.328792, 0.428792, 0.528792, 0.628792]
List of powers for D(x): [2.047169, 2.268463, 2.438268, 2.581420, 2.707540, 2.821561,
2.926414]
Fitting sin(x) in range (1.000000, 2.000000)
MSS of errors squared = 6.725578e-11
Corrected MSS of errors squared = 1.345116e-10
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 3

```

Fitting sin(x) in range (0.000000, 1.570796)
Shammas polynomial power for Q(x) is A+B*sqrt(i)+C*i
Shammas polynomial power for D(x) is A+B*i+C*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.0122204384059215	0	-Inf	0
x1	0.184532032372104	0	Inf	0
x2	-0.741725776819975	0	-Inf	0
x3	1.69525804098086	0	Inf	0
x4	0	0	NaN	NaN
x5	-0.729348674162871	0	-Inf	0
x6	0	0	NaN	NaN
x7	0.101131600172364	0	Inf	0
x8	1.38391620555494	0	Inf	0
x9	-1.10223007182707	0	-Inf	0
x10	0	0	NaN	NaN
x11	0.360403090022343	0	Inf	0
x12	-0.15814156005997	0	-Inf	0
x13	0.0184255521653632	0	Inf	0

Number of observations: 501, Error degrees of freedom: 490

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	47.6265936125819	500	0.0952531872251639		
Model	47.6265936125819	10	4.7626593612582	Inf	0
Residual	0	490	0		

for Q(x) A = 0.093429, B = 0.249972, C = 0.400000

for D(x) A = 0.308092, B = 0.315430, C = 0.343511

order Q(x) = 7.000000, order D(x) = 6.000000

Xmin = 0.000000 and Xmax = 1.570796

Ymin = 0.000000 and Ymax = 1.000000

List of powers for Q(x): [0.743401, 1.246943, 1.726393, 2.193373, 2.652384, 3.105733, 3.554793]

List of powers for D(x): [0.967034, 1.424751, 1.849361, 2.256835, 2.653357, 3.042100]

Fitting sin(x) in range (1.000000, 2.000000)

MSS of errors squared = 2.006086e-13

Corrected MSS of errors squared = 4.913886e-13

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 4

Fitting sin(x) in range (0.000000, 1.570796)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.559716336749882	0	Inf	0
x1	-10.2995717465425	0	-Inf	0
x2	47.6205552128028	0	Inf	0
x3	-37.7909503007589	0	-Inf	0
x4	7.95455471028222	0	Inf	0
x5	-13.8773246822669	0	-Inf	0
x6	8.82907790118329	0	Inf	0
x7	-1.99605743178261	0	-Inf	0

Number of observations: 501, Error degrees of freedom: 493

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	47.6265936125837	500	0.0952531872251673		
Model	47.6265936125837	7	6.80379908751195	Inf	0
Residual	0	493	0		

for Q(x) A = 0.378873, B = 0.248041

for D(x) A = 0.435818, B = 0.234608

order Q(x) = 3.000000, order D(x) = 4.000000

Xmin = 0.000000 and Xmax = 1.570796

Ymin = 0.000000 and Ymax = 1.000000

List of powers for Q(x): [0.626914, 0.502894, 0.461554]

List of powers for D(x): [0.670427, 0.905035, 1.139644, 1.374252]

Fitting sin(x) in range (1.000000, 2.000000)

MSS of errors squared = 6.130744e-10

Corrected MSS of errors squared = 1.226149e-09

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 5

Fitting sin(x) in range (0.000000, 1.570796)

Shammas polynomial power for Q(x) is A+B*sqrt(i)

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	1.48946686546271e-13	0	Inf	0
x1	-2.6145226497281e-12	0	-Inf	0
x2	4.7690759301785e-12	0	Inf	0
x3	-2.29701184766025e-12	0	-Inf	0
x4	0.999999999999993	0	Inf	0
x5	0	0	NaN	NaN
x6	0	0	NaN	NaN
x7	0	0	NaN	NaN
x8	0	0	NaN	NaN
x9	0	0	NaN	NaN

Number of observations: 501, Error degrees of freedom: 496

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	47.6265936125817	500	0.0952531872251634		
Model	47.6265936125817	4	11.9066484031454	Inf	0
Residual	0	496	0		

for Q(x) A = 0.469497, B = 0.500000

for D(x) A = 0.000000, B = 0.000000

order Q(x) = 3.000000, order D(x) = 6.000000

Xmin = 0.000000 and Xmax = 1.570796

Ymin = 0.000000 and Ymax = 1.000000

List of powers for Q(x): [0.969497, 1.176604, 1.335523]

List of powers for D(x): [0.000000, 0.000000, 0.000000, 0.000000, 0.000000, 0.000000]

Fitting sin(x) in range (1.000000, 2.000000)

MSS of errors squared = 3.238282e-03

Corrected MSS of errors squared = 6.476564e-03

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 6

Fitting sin(x) in range (0.000000, 1.570796)

Shammas polynomial power for Q(x) is A+B*i+C*sqrt(i)

Shammas polynomial power for D(x) is A+B*sqrt(i)+C*i

Linear regression model:

$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.0243076025814993	0	Inf	0
x1	-0.540524304324528	0	-Inf	0
x2	1.84307488611813	0	Inf	0
x3	-1.60053251748063	0	-Inf	0
x4	0.416984143796114	0	Inf	0
x5	6.63354173399786	0	Inf	0
x6	-9.61813430172408	0	-Inf	0
x7	0	0	NaN	NaN
x8	8.81143598452119	0	Inf	0
x9	-6.4173789322052	0	-Inf	0
x10	1.44722570483216	0	Inf	0

Number of observations: 501, Error degrees of freedom: 491

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	47.6265936125815	500	0.0952531872251629		
Model	47.6265936125815	9	5.29184373473127	Inf	0
Residual	0	491	0		

for Q(x) A = 0.258566, B = 0.349712, C = 0.296207

for D(x) A = 0.377431, B = 0.302118, C = 0.165124

order Q(x) = 4.000000, order D(x) = 6.000000

Xmin = 0.000000 and Xmax = 1.570796

Ymin = 0.000000 and Ymax = 1.000000

List of powers for Q(x): [0.904484, 1.376889, 1.820747, 2.249827]

List of powers for D(x): [0.844673, 1.134938, 1.396086, 1.642162, 1.878606, 2.108208]

Fitting sin(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.246598e-11

Corrected MSS of errors squared = 3.053528e-11

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 7

Fitting sin(x) in range (0.000000, 1.570796)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.73975976289005	0	Inf	0
x1	-3.69724443657645	0	-Inf	0
x2	5.31814980879517	0	Inf	0
x3	-2.04590206086991	0	-Inf	0
x4	9010.45019095918	0	Inf	0
x5	-11518.7035316178	0	-Inf	0
x6	3030.61928282593	0	Inf	0
x7	-572.985092455231	0	-Inf	0
x8	51.3043872226848	0	Inf	0

Number of observations: 501, Error degrees of freedom: 492

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	47.6265936120818	500	0.0952531872241636		
Model	47.6265936120818	8	5.95332420151023	Inf	0
Residual	0	492	0		

for Q(x) A = 0.071931, B = 0.366031

for D(x) A = 1.720393, B = 1.648784

order Q(x) = 3.000000, order D(x) = 5.000000

Xmin = 0.000000 and Xmax = 1.570796

Ymin = 0.000000 and Ymax = 1.000000

List of powers for Q(x): [0.437962, 0.803993, 1.170024]

List of powers for D(x): [1.720393, 1.733932, 1.805836, 1.937025, 2.113941]

Fitting sin(x) in range (1.000000, 2.000000)

MSS of errors squared = 4.256965e-10

Corrected MSS of errors squared = 8.513929e-10

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 8

Fitting sin(x) in range (0.000000, 1.570796)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-1.72420999720367	0	-Inf	0
x1	-29.1554570473904	0	-Inf	0
x2	375.125015272005	0	Inf	0
x3	-879.023425380094	0	-Inf	0
x4	535.358624519111	0	Inf	0
x5	129.519993042449	0	Inf	0
x6	0	0	NaN	NaN
x7	0	0	NaN	NaN
x8	0	0	NaN	NaN
x9	-593.720183581281	0	-Inf	0
x10	672.771253641353	0	Inf	0

x11 -208.151610460143 0 -Inf 0

Number of observations: 501, Error degrees of freedom: 492

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	47.6265936123086	500	0.0952531872246172		
Model	47.6265936123086	8	5.95332420153858	Inf	0
Residual	0	492	0		

for Q(x) A = 0.500000, B = 0.632967

for D(x) A = 1.980445, B = 0.486101

order Q(x) = 4.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 1.570796

Ymin = 0.000000 and Ymax = 1.000000

List of powers for Q(x): [1.132967, 0.816483, 0.710989, 0.658242]

List of powers for D(x): [1.980445, 1.984437, 2.005636, 2.044313, 2.096472, 2.158676, 2.228389]

Fitting sin(x) in range (1.000000, 2.000000)

MSS of errors squared = 2.354361e-10

Corrected MSS of errors squared = 4.708722e-10

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 9

Fitting sin(x) in range (0.000000, 1.570796)

Shammas polynomial power for Q(x) is A+B*log10(i)^4

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	2.40681940311961e-13	0	Inf	0
x1	-9.68814356978417e-09	0	-Inf	0
x2	1.14956014997417e-08	0	Inf	0
x3	-1.80726337336522e-09	0	-Inf	0
x4	0.999999999999566	0	Inf	0
x5	0	0	NaN	NaN
x6	0	0	NaN	NaN
x7	0	0	NaN	NaN

x8	0	0	NaN	NaN
x9	0	0	NaN	NaN
x10	0	0	NaN	NaN

Number of observations: 501, Error degrees of freedom: 496

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	47.6265936125815	500	0.095253187225163		
Model	47.6265936125815	4	11.9066484031454	Inf	0
Residual	0	496	0		

for Q(x) A = 1.607560, B = 0.500000

for D(x) A = 0.000000, B = 0.000000

order Q(x) = 3.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 1.570796

Ymin = 0.000000 and Ymax = 1.000000

List of powers for Q(x): [1.607560, 1.611665, 1.633471]

List of powers for D(x): [0.000000, 0.000000, 0.000000, 0.000000, 0.000000, 0.000000, 0.000000]

Fitting sin(x) in range (1.000000, 2.000000)

MSS of errors squared = 6.454006e-05

Corrected MSS of errors squared = 1.290801e-04

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 10

Fitting sin(x) in range (0.000000, 1.570796)

Shammas polynomial power for Q(x) is A+B*log10(i)^4

Shammas polynomial power for D(x) is A+B/i

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.00891687250798855	0	-Inf	0
x1	22.1811336519767	0	Inf	0
x2	0	0	NaN	NaN
x3	0	0	NaN	NaN
x4	-48.430604137765	0	-Inf	0
x5	26.2561391586202	0	Inf	0

x6	0.0388979190594302	0	Inf	0
x7	-3.70140544315966	0	-Inf	0
x8	31.5277893702675	0	Inf	0
x9	-68.3918008984543	0	-Inf	0
x10	41.5287672516101	0	Inf	0

Number of observations: 501, Error degrees of freedom: 492

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	47.6265936125861	500	0.0952531872251722		
Model	47.6265936125861	8	5.95332420157326	Inf	0
Residual	0	492	0		

for Q(x) A = 1.085004, B = 0.176701

for D(x) A = 0.158630, B = 1.612432

order Q(x) = 5.000000, order D(x) = 5.000000

Xmin = 0.000000 and Xmax = 1.570796

Ymin = 0.000000 and Ymax = 1.000000

List of powers for Q(x): [1.085004, 1.086455, 1.094161, 1.108221, 1.127181]

List of powers for D(x): [1.771061, 0.964845, 0.696107, 0.561738, 0.481116]

Fitting sin(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.071574e-09

Corrected MSS of errors squared = 2.143149e-09

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Comments

The following table shows the summary results for the ten models that fit the sine function. The results are sorted in ascending order using the MSSE values.

<i>Power Q(x)</i>	<i>Power D(x)</i>	<i>MSSE</i>	<i>Q/D Orders</i>
$A+B*\sqrt{i}+C*i$	$A+B*i+C*\sqrt{i}$	2.00608600E-13	7, 6
$A+B*i$	$A+B/i$	3.09458800E-12	7, 5
$A+B*i+C*\sqrt{i}$	$A+B*\sqrt{i}+C*i$	1.24659800E-11	4, 6
$A+B*i$	$A+B*\sqrt{i}$	6.72557800E-11	5, 7
$A+B/i$	$A+B*\log_{10}(i)^4$	2.35436100E-10	4, 7
$A+B*i$	$A+B*\log_{10}(i)^4$	4.25696500E-10	3, 5
$A+B/i$	$A+B*i$	6.13074400E-10	3, 4
$A+B*\log_{10}(i)^4$	$A+B/i$	1.07157400E-09	5, 5
$A+B*\log_{10}(i)^4$	$A+B*i$	6.45400600E-05	3, 7
$A+B*\sqrt{i}$	$A+B*i$	3.23828200E-03	3, 6

RESULTS FOR THE HYPERBOLIC SINE FUNCTION

Combo Powers 1

Fitting $\sinh(x)$ in range (0.000000, 5.000000)

Shammas polynomial power for Q(x) is $A+B*i$

Shammas polynomial power for D(x) is $A+B/i$

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.0715623344407216	0	-Inf	0
x1	0.167029724942576	0	Inf	0
x2	-0.130106895337251	0	-Inf	0
x3	0.0338809549442365	0	Inf	0
x4	-102.138024805489	0	-Inf	0
x5	1152.13302504533	0	Inf	0
x6	-1808.35453644918	0	-Inf	0
x7	0	0	NaN	NaN
x8	0	0	NaN	NaN
x9	759.360294746841	0	Inf	0

Number of observations: 501, Error degrees of freedom: 493

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	30.8337526138898	500	0.0616675052277797		
Model	30.8337526138898	7	4.40482180198426	Inf	0
Residual	0	493	0		

```

for Q(x) A = 0.000000, B = 0.394664
for D(x) A = 0.226071, B = 0.100000
order Q(x) = 3.000000, order D(x) = 6.000000
Xmin = 0.000000 and Xmax = 5.000000
Ymin = 0.000000 and Ymax = 74.203211
List of powers for Q(x): [0.394664, 0.789327, 1.183991]
List of powers for D(x): [0.326071, 0.276071, 0.259404, 0.251071, 0.246071, 0.242737]
Fitting sinh(x) in range (1.000000, 2.000000)
MSS of errors squared = 9.128091e-06
Corrected MSS of errors squared = 1.825618e-05
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 2

```

Fitting sinh(x) in range (0.000000, 5.000000)
Shammas polynomial power for Q(x) is A+B*i
Shammas polynomial power for D(x) is A+B*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.191650268556251	0	Inf	0
x1	-1.81904369596363	0	-Inf	0
x2	4.28912409376615	0	Inf	0
x3	-5.31145677386694	0	-Inf	0
x4	4.04722172500317	0	Inf	0
x5	-1.9243804843895	0	-Inf	0
x6	0.530451028298521	0	Inf	0
x7	-0.065959139103704	0	-Inf	0
x8	23.0201100395161	0	Inf	0
x9	0	0	NaN	NaN
x10	-34.599113422496	0	-Inf	0
x11	0	0	NaN	NaN
x12	0	0	NaN	NaN
x13	0	0	NaN	NaN
x14	12.6413963617183	0	Inf	0

Number of observations: 501, Error degrees of freedom: 490

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

SumSq	DF	MeanSq	F	pValue

Total	30.8337526139326	500	0.0616675052278653		
Model	30.8337526139326	10	3.08337526139326	Inf	0
Residual	0	490	0		

```

for Q(x) A = 0.092191, B = 0.722871
for D(x) A = 0.267133, B = 0.100000
order Q(x) = 7.000000, order D(x) = 7.000000
Xmin = 0.000000 and Xmax = 5.000000
Ymin = 0.000000 and Ymax = 74.203211
List of powers for Q(x): [0.815061, 1.537932, 2.260802, 2.983673, 3.706543, 4.429414,
5.152284]
List of powers for D(x): [0.367133, 0.408555, 0.440338, 0.467133, 0.490740, 0.512082,
0.531708]
Fitting sinh(x) in range (1.000000, 2.000000)
MSS of errors squared = 5.129974e-10
Corrected MSS of errors squared = 1.025995e-09
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 3

```

Fitting sinh(x) in range (0.000000, 5.000000)
Shammas polynomial power for Q(x) is A+B*sqrt(i)+C*i
Shammas polynomial power for D(x) is A+B*i+C*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.32211492430936	0	-Inf	0
x1	9.17911504590739	0	Inf	0
x2	-15.2068837831075	0	-Inf	0
x3	0	0	NaN	NaN
x4	8.61983457877623	0	Inf	0
x5	0	0	NaN	NaN
x6	-2.23197977443248	0	-Inf	0
x7	14.9896777028149	0	Inf	0
x8	-46.9122372358411	0	-Inf	0
x9	66.8655872366678	0	Inf	0
x10	-50.4960036448329	0	-Inf	0
x11	19.596399203887	0	Inf	0
x12	-3.08139440552509	0	-Inf	0

Number of observations: 501, Error degrees of freedom: 490

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	30.8337526139298	500	0.0616675052278596		
Model	30.8337526139298	10	3.08337526139298	Inf	0
Residual	0	490	0		

for Q(x) A = 0.237955, B = 0.276068, C = 0.070002

for D(x) A = 0.333314, B = 0.305078, C = 0.070046

order Q(x) = 6.000000, order D(x) = 6.000000

Xmin = 0.000000 and Xmax = 5.000000

Ymin = 0.000000 and Ymax = 74.203211

List of powers for Q(x): [0.584025, 0.768378, 0.926125, 1.070099, 1.205271, 1.334192]

List of powers for D(x): [0.708438, 1.042530, 1.369871, 1.693718, 2.015332, 2.335359]

Fitting sinh(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.393185e-10

Corrected MSS of errors squared = 3.412593e-10

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 4

Fitting sinh(x) in range (0.000000, 5.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12 + x13$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-26.3872061063459	0	-Inf	0
x1	231.522647623755	0	Inf	0
x2	-10428.7888114481	0	-Inf	0
x3	55118.5973644001	0	Inf	0
x4	-71219.4923843095	0	-Inf	0
x5	0	0	NaN	NaN
x6	26324.7838986788	0	Inf	0
x7	870.582290182585	0	Inf	0
x8	0	0	NaN	NaN
x9	0	0	NaN	NaN
x10	-1693.13566409131	0	-Inf	0
x11	0	0	NaN	NaN

x12	0	0	NaN	NaN
x13	823.317865021418	0	Inf	0

Number of observations: 501, Error degrees of freedom: 492

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	30.83375262163	500	0.06166750524326		
Model	30.83375262163	8	3.85421907770375	Inf	0
Residual	0	492	0		

for Q(x) A = 0.500000, B = 0.500000

for D(x) A = 1.588974, B = 0.005786

order Q(x) = 6.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 5.000000

Ymin = 0.000000 and Ymax = 74.203211

List of powers for Q(x): [1.000000, 0.750000, 0.666667, 0.625000, 0.600000, 0.583333]

List of powers for D(x): [1.594760, 1.600546, 1.606332, 1.612117, 1.617903, 1.623689, 1.629475]

Fitting sinh(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.129928e-08

Corrected MSS of errors squared = 2.259855e-08

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 5

Fitting sinh(x) in range (0.000000, 5.000000)

Shammas polynomial power for Q(x) is A+B*sqrt(i)

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12 + x13 + x14

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.3623920824322	0	Inf	0
x1	7.8824994248359	0	Inf	0
x2	0	0	NaN	NaN
x3	0	0	NaN	NaN
x4	-14.317126232497	0	-Inf	0
x5	0	0	NaN	NaN
x6	0	0	NaN	NaN

x7	7.01276249056609	0	Inf	0
x8	-2.81575927361312	0	-Inf	0
x9	7.29071699438088	0	Inf	0
x10	-7.43436333644082	0	-Inf	0
x11	4.18472734947278	0	Inf	0
x12	-1.41785585434811	0	-Inf	0
x13	0.275797519569494	0	Inf	0
x14	-0.023791151022402	0	-Inf	0

Number of observations: 501, Error degrees of freedom: 490

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	30.833752613937	500	0.0616675052278739		
Model	30.833752613937	10	3.0833752613937	Inf	0
Residual	0	490	0		

for Q(x) A = 0.928116, B = 0.500000

for D(x) A = 1.574434, B = 0.886026

order Q(x) = 7.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 5.000000

Ymin = 0.000000 and Ymax = 74.203211

List of powers for Q(x): [1.428116, 1.635223, 1.794142, 1.928116, 2.046150, 2.152861, 2.250992]

List of powers for D(x): [2.460461, 3.346487, 4.232513, 5.118540, 6.004566, 6.890593, 7.776619]

Fitting sinh(x) in range (1.000000, 2.000000)

MSS of errors squared = 3.389196e-10

Corrected MSS of errors squared = 6.778391e-10

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 6

Fitting sinh(x) in range (0.000000, 5.000000)

Shammas polynomial power for Q(x) is A+B*i+C*sqrt(i)

Shammas polynomial power for D(x) is A+B*sqrt(i)+C*i

Linear regression model:

$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13}$

Estimated Coefficients:

Estimate	SE	tStat	pValue

(Intercept)	0.192778949730845	0	Inf	0
x1	-1.18602625205699	0	-Inf	0
x2	1.92308358862757	0	Inf	0
x3	0	0	NaN	NaN
x4	-2.85869565271331	0	-Inf	0
x5	3.13494208859257	0	Inf	0
x6	-1.42977755310446	0	-Inf	0
x7	0.250396408082422	0	Inf	0
x8	12.2982064454061	0	Inf	0
x9	-21.4655564382892	0	-Inf	0
x10	0	0	NaN	NaN
x11	26.2455466831266	0	Inf	0
x12	-21.4277160768862	0	-Inf	0
x13	5.32281780951617	0	Inf	0

Number of observations: 501, Error degrees of freedom: 489

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	30.8337526139327	500	0.0616675052278655		
Model	30.8337526139327	11	2.80306841944843	Inf	0
Residual	0	489	0		

for Q(x) A = 0.340000, B = 0.390000, C = 0.290722

for D(x) A = 0.263790, B = 0.195231, C = 0.160650

order Q(x) = 7.000000, order D(x) = 6.000000

Xmin = 0.000000 and Xmax = 5.000000

Ymin = 0.000000 and Ymax = 74.203211

List of powers for Q(x): [1.020722, 1.531143, 2.013545, 2.481444, 2.940074, 3.392120, 3.839177]

List of powers for D(x): [0.619671, 0.861189, 1.083890, 1.296853, 1.503590, 1.705907]

Fitting sinh(x) in range (1.000000, 2.000000)

MSS of errors squared = 2.976918e-11

Corrected MSS of errors squared = 7.291930e-11

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 7

Fitting sinh(x) in range (0.000000, 5.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.0167838727321532	0	Inf	0
x1	-0.0450192296102452	0	-Inf	0
x2	0.0384709290302174	0	Inf	0
x3	-0.0103556057861961	0	-Inf	0
x4	87.3874066704843	0	Inf	0
x5	0	0	NaN	NaN
x6	-195.959890607933	0	-Inf	0
x7	169.489307492856	0	Inf	0
x8	-77.4666966039831	0	-Inf	0
x9	19.7695349686606	0	Inf	0
x10	-2.21954188655078	0	-Inf	0

Number of observations: 501, Error degrees of freedom: 491

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	30.8337526139267	500	0.0616675052278534		
Model	30.8337526139267	9	3.42597251265852	Inf	0
Residual	0	491	0		

for Q(x) A = 0.089653, B = 0.500000

for D(x) A = 0.470592, B = 1.805423

order Q(x) = 3.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 5.000000

Ymin = 0.000000 and Ymax = 74.203211

List of powers for Q(x): [0.589653, 1.089653, 1.589653]

List of powers for D(x): [0.470592, 0.485417, 0.564152, 0.707804, 0.901528, 1.132556, 1.391480]

Fitting sinh(x) in range (1.000000, 2.000000)

MSS of errors squared = 5.243939e-09

Corrected MSS of errors squared = 1.048788e-08

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 8

Fitting sinh(x) in range (0.000000, 5.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.0115383616207217	0	-Inf	0
x1	0.0100845051379129	0	Inf	0
x2	-0.0811664195206555	0	-Inf	0
x3	0.0825388494355278	0	Inf	0
x4	128.211679409988	0	Inf	0
x5	0	0	NaN	NaN
x6	0	0	NaN	NaN
x7	-393.314617047987	0	-Inf	0
x8	365.032489225505	0	Inf	0
x9	-98.9294701620813	0	-Inf	0

Number of observations: 501, Error degrees of freedom: 493

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	30.83375261393	500	0.0616675052278599		
Model	30.83375261393	7	4.40482180198999	Inf	0
Residual	0	493	0		

for Q(x) A = 0.205751, B = 0.673711

for D(x) A = 0.128718, B = 0.157321

order Q(x) = 3.000000, order D(x) = 6.000000

Xmin = 0.000000 and Xmax = 5.000000

Ymin = 0.000000 and Ymax = 74.203211

List of powers for Q(x): [0.879462, 0.542607, 0.430322]

List of powers for D(x): [0.128718, 0.130010, 0.136871, 0.149388, 0.166269, 0.186400]

Fitting sinh(x) in range (1.000000, 2.000000)

MSS of errors squared = 5.657953e-05

Corrected MSS of errors squared = 1.131591e-04

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 9

Fitting sinh(x) in range (0.000000, 5.000000)

Shammas polynomial power for Q(x) is A+B*log10(i)^4

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12 + x13$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-7.90824528906348	0	-Inf	0
x1	511.976714862304	0	Inf	0
x2	0	0	NaN	NaN
x3	0	0	NaN	NaN
x4	0	0	NaN	NaN
x5	-872.644157488882	0	-Inf	0
x6	0	0	NaN	NaN
x7	368.930020360107	0	Inf	0
x8	0.244395130193204	0	Inf	0
x9	0.929003960538255	0	Inf	0
x10	-0.630501420400187	0	-Inf	0
x11	0.0767747576499455	0	Inf	0
x12	0.0335577802403413	0	Inf	0
x13	-0.00756265423045537	0	-Inf	0

Number of observations: 501, Error degrees of freedom: 491

R-squared: 1, Adjusted R-squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	30.8337526138879	500	0.0616675052277758		
Model	30.8337526138879	9	3.42597251265421	Inf	0
Residual	0	491	0		

for Q(x) A = 0.489259, B = 0.226582

for D(x) A = 0.000000, B = 0.840582

order Q(x) = 7.000000, order D(x) = 6.000000

Xmin = 0.000000 and Xmax = 5.000000

Ymin = 0.000000 and Ymax = 74.203211

List of powers for Q(x): [0.489259, 0.491119, 0.501001, 0.519029, 0.543341, 0.572336, 0.604831]

List of powers for D(x): [0.840582, 1.681165, 2.521747, 3.362330, 4.202912, 5.043495]

Fitting sinh(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.054239e-10

Corrected MSS of errors squared = 2.108478e-10

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 10

Fitting $\sinh(x)$ in range (0.000000, 5.000000)

Shammas polynomial power for $Q(x)$ is $A+B*\log_{10}(i)^4$

Shammas polynomial power for $D(x)$ is $A+B/i$

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-16.4173547977815	0	-Inf	0
x1	177.012322472881	0	Inf	0
x2	0	0	NaN	NaN
x3	0	0	NaN	NaN
x4	0	0	NaN	NaN
x5	-252.862048959424	0	-Inf	0
x6	0	0	NaN	NaN
x7	92.6449009044389	0	Inf	0
x8	-0.0878356122561635	0	-Inf	0
x9	3.62219228318573	0	Inf	0
x10	-6.8787325913666	0	-Inf	0
x11	-9.93109910857772	0	-Inf	0
x12	13.8976553417422	0	Inf	0

Number of observations: 501, Error degrees of freedom: 492

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	30.8337526139548	500	0.0616675052279097		
Model	30.8337526139548	8	3.85421907674436	Inf	0
Residual	0	492	0		

for $Q(x)$ $A = 0.346369$, $B = 0.486574$

for $D(x)$ $A = 1.787341$, $B = 1.940627$

order $Q(x) = 7.000000$, order $D(x) = 5.000000$

Xmin = 0.000000 and Xmax = 5.000000

Ymin = 0.000000 and Ymax = 74.203211

List of powers for $Q(x)$: [0.346369, 0.350364, 0.371584, 0.410299, 0.462509, 0.524773, 0.594555]

List of powers for $D(x)$: [3.727968, 2.757654, 2.434216, 2.272497, 2.175466]

Fitting $\sinh(x)$ in range (1.000000, 2.000000)

MSS of errors squared = 7.685861e-09

Corrected MSS of errors squared = 1.537172e-08

R-squared = 1.00000000

```
R-squared Adjusted = 1.00000000  
Particle swarm AICc = -1.000000e+99  
AIC = -Inf  
AICc = -Inf
```

Comments

The following table shows the summary results for the ten models that fit the hyperbolic sine function. The results are sorted in ascending order using the MSSE values.

<i>Power Q(x)</i>	<i>Power D(x)</i>	<i>MSSE</i>	<i>Q/D Orders</i>
$A+B*i+C*\sqrt{i}$	$A+B*\sqrt{i}+C*i$	2.97691800E-11	7, 6
$A+B*\log_{10}(i)^4$	$A+B*i$	1.05423900E-10	7, 6
$A+B*\sqrt{i}+C*i$	$A+B*i+C*\sqrt{i}$	1.39318500E-10	6, 6
$A+B*\sqrt{i}$	$A+B*i$	3.38919600E-10	7, 7
$A+B*i$	$A+B*\sqrt{i}$	5.12997400E-10	7, 7
$A+B*i$	$A+B*\log_{10}(i)^4$	5.24393900E-09	3, 7
$A+B*\log_{10}(i)^4$	$A+B/i$	7.68586100E-09	7, 5
$A+B/i$	$A+B*i$	1.12992800E-08	6, 7
$A+B*i$	$A+B/i$	9.12809100E-06	3, 6
$A+B/i$	$A+B*\log_{10}(i)^4$	5.65795300E-05	3, 6

RESULTS FOR THE TANGENT FUNCTION

Combo Powers 1

Fitting $\tan(x)$ in range (0.000000, 1.000000)

Shammas polynomial power for $Q(x)$ is $A+B*i$

Shammas polynomial power for $D(x)$ is $A+B/i$

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00163072098614923	0	Inf	0
x1	0.434376634013369	0	Inf	0
x2	-0.590899399785197	0	-Inf	0
x3	0.225766660594216	0	Inf	0
x4	-0.0182891323438976	0	-Inf	0
x5	-67.4157452108172	0	-Inf	0
x6	758.916300349482	0	Inf	0
x7	-1132.04967541524	0	-Inf	0
x8	0	0	NaN	NaN
x9	0	0	NaN	NaN
x10	0	0	NaN	NaN
x11	441.496534793118	0	Inf	0

Number of observations: 101, Error degrees of freedom: 92

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	7.61910551705274	100	0.0761910551705274		
Model	7.61910551705274	8	0.952388189631592	Inf	0

```

Residual          0      92          0

for Q(x) A = 0.460770, B = 0.295008
for D(x) A = 0.469916, B = 0.175561
order Q(x) = 4.000000, order D(x) = 7.000000
Xmin = 0.000000 and Xmax = 1.000000
Ymin = 0.000000 and Ymax = 1.557408
List of powers for Q(x): [0.755778, 1.050786, 1.345794, 1.640802]
List of powers for D(x): [0.645477, 0.557697, 0.528437, 0.513806, 0.505028, 0.499176,
0.494996]
Fitting tan(x) in range (1.000000, 2.000000)
MSS of errors squared = 1.664514e-11
Corrected MSS of errors squared = 3.329028e-11
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 2

```
Fitting tan(x) in range (0.000000, 1.000000)
Shammas polynomial power for Q(x) is A+B*i
Shammas polynomial power for D(x) is A+B*sqrt(i)
```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.940443828884959	0	Inf	0
x1	-23.7001171435478	0	-Inf	0
x2	33.4431513348207	0	Inf	0
x3	0	0	NaN	NaN
x4	0	0	NaN	NaN
x5	-13.841394558014	0	-Inf	0
x6	0	0	NaN	NaN
x7	3.82381427316874	0	Inf	0
x8	0.86014255343435	0	Inf	0
x9	-2.07628590339565	0	-Inf	0
x10	4.59292596146555	0	Inf	0
x11	-5.54695960646805	0	-Inf	0
x12	3.23622537288052	0	Inf	0
x13	-0.731946113225744	0	-Inf	0

Number of observations: 101, Error degrees of freedom: 90
R-squared: 1, Adjusted R-Squared: 1
F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	7.61910551705061	100	0.0761910551705061		
Model	7.61910551705061	10	0.761910551705061	Inf	0
Residual	0	90	0		

```

for Q(x) A = 0.331042, B = 0.115291
for D(x) A = 0.610111, B = 1.000000
order Q(x) = 7.000000, order D(x) = 6.000000
Xmin = 0.000000 and Xmax = 1.000000
Ymin = 0.000000 and Ymax = 1.557408
List of powers for Q(x): [0.446332, 0.561623, 0.676913, 0.792204, 0.907495, 1.022785,
1.138076]
List of powers for D(x): [1.610111, 2.024324, 2.342161, 2.610111, 2.846179, 3.059600]
Fitting tan(x) in range (1.000000, 2.000000)
MSS of errors squared = 3.697792e-13
Corrected MSS of errors squared = 7.395584e-13
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 3

```

Fitting tan(x) in range (0.000000, 1.000000)
Shammas polynomial power for Q(x) is A+B*sqrt(i)+C*i
Shammas polynomial power for D(x) is A+B*i+C*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.0858296255998112	0	Inf	0
x1	1.0388413318524	0	Inf	0
x2	0.286901477428923	0	Inf	0
x3	-1.18205383926576	0	-Inf	0
x4	0.607084227268289	0	Inf	0
x5	-0.117457391138361	0	-Inf	0
x6	7.18775490876574	0	Inf	0
x7	-13.4220719907361	0	-Inf	0
x8	6.51517165231825	0	Inf	0

Number of observations: 101, Error degrees of freedom: 92

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	7.61910551705119	100	0.0761910551705119		
Model	7.61910551705119	8	0.952388189631399	Inf	0
Residual	0	92	0		

```

for Q(x) A = 0.550000, B = 0.361935, C = 0.400000
for D(x) A = 0.340000, B = 0.209577, C = 0.094344
order Q(x) = 5.000000, order D(x) = 3.000000
Xmin = 0.000000 and Xmax = 1.000000
Ymin = 0.000000 and Ymax = 1.557408
List of powers for Q(x): [1.311935, 1.861854, 2.376890, 2.873870, 3.359312]
List of powers for D(x): [0.643920, 0.892575, 1.132138]
Fitting tan(x) in range (1.000000, 2.000000)
MSS of errors squared = 1.287456e-10
Corrected MSS of errors squared = 3.153611e-10
R-squared = 1.000000000
R-squared Adjusted = 1.000000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 4

```

Fitting tan(x) in range (0.000000, 1.000000)
Shammas polynomial power for Q(x) is A+B/i
Shammas polynomial power for D(x) is A+B*i

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.755340031522343	0	-Inf	0
x1	0.392391715511399	0	Inf	0
x2	0	0	NaN	NaN
x3	1.33084502388051	0	Inf	0
x4	0	0	NaN	NaN
x5	0	0	NaN	NaN
x6	0.0400606796719027	0	Inf	0
x7	-0.0127807122990588	0	-Inf	0
x8	0.00659757349913064	0	Inf	0
x9	-0.00228864059980707	0	-Inf	0
x10	0.00059906884881165	0	Inf	0
x11	-9.15848909407914e-05	0	-Inf	0
x12	6.89910193336643e-06	0	Inf	0

Number of observations: 101, Error degrees of freedom: 91

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	7.61910551705132	100	0.0761910551705132		
Model	7.61910551705132	9	0.846567279672369	Inf	0
Residual	0	91	0		

for Q(x) A = 0.102166, B = 0.413028

for D(x) A = 1.738606, B = 1.603786

order Q(x) = 5.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 1.000000

Ymin = 0.000000 and Ymax = 1.557408

List of powers for Q(x): [0.515194, 0.308680, 0.239842, 0.205423, 0.184772]

List of powers for D(x): [3.342391, 4.946177, 6.549963, 8.153749, 9.757535, 11.361321, 12.965107]

Fitting tan(x) in range (1.000000, 2.000000)

MSS of errors squared = 3.984460e-10

Corrected MSS of errors squared = 7.968920e-10

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 5

Fitting tan(x) in range (0.000000, 1.000000)

Shammas polynomial power for Q(x) is A+B*sqrt(i)

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.0532718929603989	0	Inf	0
x1	56.8567406903135	0	Inf	0
x2	-138.588825253567	0	-Inf	0
x3	0	0	NaN	NaN
x4	181.410697028317	0	Inf	0
x5	0	0	NaN	NaN
x6	-186.146989756477	0	-Inf	0
x7	87.3290803652764	0	Inf	0
x8	0.0898392527230112	0	Inf	0
x9	-0.00389357522401812	0	-Inf	0
x10	7.93595498326693e-05	0	Inf	0

Number of observations: 101, Error degrees of freedom: 92

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	7.61910551704761	100	0.0761910551704761		
Model	7.61910551704761	8	0.952388189630952	Inf	0
Residual	0	92	0		

for Q(x) A = 1.119944, B = 0.500000

for D(x) A = 0.853515, B = 2.000000

order Q(x) = 7.000000, order D(x) = 3.000000

Xmin = 0.000000 and Xmax = 1.000000

Ymin = 0.000000 and Ymax = 1.557408

List of powers for Q(x): [1.619944, 1.827051, 1.985970, 2.119944, 2.237978, 2.344689, 2.442820]

List of powers for D(x): [2.853515, 4.853515, 6.853515]

Fitting tan(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.884001e-10

Corrected MSS of errors squared = 3.768002e-10

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 6

Fitting tan(x) in range (0.000000, 1.000000)

Shammas polynomial power for Q(x) is $A+B*i+C*\sqrt{i}$

Shammas polynomial power for D(x) is $A+B*\sqrt{i}+C*i$

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.202647193332149	0	-Inf	0
x1	2.13400074586963	0	Inf	0
x2	-2.77258150576193	0	-Inf	0
x3	0.948348330466615	0	Inf	0
x4	45.0389495211989	0	Inf	0
x5	-164.673352375726	0	-Inf	0
x6	233.641337087264	0	Inf	0
x7	-148.53793882902	0	-Inf	0
x8	35.4238842187145	0	Inf	0

Number of observations: 101, Error degrees of freedom: 92

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	7.61910551704899	100	0.0761910551704899		
Model	7.61910551704899	8	0.952388189631124	Inf	0
Residual	0	92	0		

for Q(x) A = 0.217916, B = 0.333680, C = 0.000000

for D(x) A = 0.547584, B = 0.264337, C = 0.062791

order Q(x) = 3.000000, order D(x) = 5.000000

Xmin = 0.000000 and Xmax = 1.000000

Ymin = 0.000000 and Ymax = 1.557408

List of powers for Q(x): [0.551596, 0.885276, 1.218956]

List of powers for D(x): [0.874713, 1.046996, 1.193803, 1.327423, 1.452615]

Fitting tan(x) in range (1.000000, 2.000000)

MSS of errors squared = 7.328234e-10

Corrected MSS of errors squared = 1.795043e-09

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 7

Fitting tan(x) in range (0.000000, 1.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	1.03381503123299	0	Inf	0
x1	-5.65200656587238	0	-Inf	0
x2	8.53349073574006	0	Inf	0
x3	0	0	NaN	NaN
x4	-4.77429619118983	0	-Inf	0
x5	1.55436478589395	0	Inf	0
x6	1.67455593027363	0	Inf	0
x7	0	0	NaN	NaN
x8	-1.54299452512444	0	-Inf	0
x9	0.219339713650807	0	Inf	0

x10	-0.131741655736387	0	-Inf	0
x11	0.105320323163476	0	Inf	0
x12	-0.0198475820283583	0	-Inf	0

Number of observations: 101, Error degrees of freedom: 90

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	7.61910551705129	100	0.0761910551705129		
Model	7.61910551705129	10	0.761910551705129	Inf	0
Residual	0	90	0		

for Q(x) A = 0.033056, B = 0.267105

for D(x) A = 1.982212, B = 3.144519

order Q(x) = 5.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 1.000000

Ymin = 0.000000 and Ymax = 1.557408

List of powers for Q(x): [0.300161, 0.567266, 0.834372, 1.101477, 1.368582]

List of powers for D(x): [1.982212, 2.008035, 2.145168, 2.395368, 2.732778, 3.135162, 3.586132]

Fitting tan(x) in range (1.000000, 2.000000)

MSS of errors squared = 3.382968e-13

Corrected MSS of errors squared = 6.765936e-13

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 8

Fitting tan(x) in range (0.000000, 1.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.153594808238249	0	-Inf	0
x1	-0.0765998746838564	0	-Inf	0
x2	2.17161591495281	0	Inf	0
x3	-7.29762248435441	0	-Inf	0
x4	5.41302142103507	0	Inf	0
x5	241.799698397475	0	Inf	0

x6		0	0	NaN	NaN
x7	-381.960624010775		0	-Inf	0
x8	141.104105443487		0	Inf	0

Number of observations: 101, Error degrees of freedom: 93

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	7.61910551705115	100	0.0761910551705115		
Model	7.61910551705115	7	1.08844364529302	Inf	0
Residual	0	93	0		

for Q(x) A = 0.266728, B = 1.150543

for D(x) A = 0.319650, B = 0.191714

order Q(x) = 4.000000, order D(x) = 4.000000

Xmin = 0.000000 and Xmax = 1.000000

Ymin = 0.000000 and Ymax = 1.557408

List of powers for Q(x): [1.417271, 0.841999, 0.650242, 0.554364]

List of powers for D(x): [0.319650, 0.321224, 0.329585, 0.344839]

Fitting tan(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.584091e-09

Corrected MSS of errors squared = 3.168182e-09

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 9

Fitting tan(x) in range (0.000000, 1.000000)

Shammas polynomial power for Q(x) is A+B*log10(i)^4

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.103646313479766	0	Inf	0
x1	1032.36968700471	0	Inf	0
x2	0	0	NaN	NaN
x3	-1628.66948110546	0	-Inf	0
x4	598.542298585008	0	Inf	0
x5	-4.74047024099886	0	-Inf	0
x6	1.55256571733319	0	Inf	0

x7	3.05698733105099	0	Inf	0
x8	-1.21523360454554	0	-Inf	0

Number of observations: 101, Error degrees of freedom: 93

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	7.61910551707512	100	0.0761910551707512		
Model	7.61910551707512	7	1.08844364529645	Inf	0
Residual	0	93	0		

for Q(x) A = 1.884900, B = 0.487930

for D(x) A = 0.765481, B = 0.418773

order Q(x) = 4.000000, order D(x) = 4.000000

Xmin = 0.000000 and Xmax = 1.000000

Ymin = 0.000000 and Ymax = 1.557408

List of powers for Q(x): [1.884900, 1.888907, 1.910186, 1.949009]

List of powers for D(x): [1.184255, 1.603028, 2.021802, 2.440575]

Fitting tan(x) in range (1.000000, 2.000000)

MSS of errors squared = 6.592336e-11

Corrected MSS of errors squared = 1.318467e-10

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 10

Fitting tan(x) in range (0.000000, 1.000000)

Shammas polynomial power for Q(x) is A+B*log10(i)^4

Shammas polynomial power for D(x) is A+B/i

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	42.0842812756598	0	Inf	0
x1	-845.810703135754	0	-Inf	0
x2	0	0	NaN	NaN
x3	0	0	NaN	NaN
x4	1585.96258029051	0	Inf	0
x5	-781.609458206612	0	-Inf	0
x6	-3.60242268422843	0	-Inf	0
x7	145.421141308943	0	Inf	0

x8	-785.366176700158	0	-Inf	0
x9	1272.95072792194	0	Inf	0
x10	-629.029970069544	0	-Inf	0

Number of observations: 101, Error degrees of freedom: 92

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	7.61910551707407	100	0.0761910551707407		
Model	7.61910551707407	8	0.952388189634259	Inf	0
Residual	0	92	0		

for Q(x) A = 0.168823, B = 0.319315

for D(x) A = 1.536616, B = 0.555778

order Q(x) = 5.000000, order D(x) = 5.000000

Xmin = 0.000000 and Xmax = 1.000000

Ymin = 0.000000 and Ymax = 1.557408

List of powers for Q(x): [0.168823, 0.171445, 0.185370, 0.210777, 0.245040]

List of powers for D(x): [2.092394, 1.814505, 1.721876, 1.675561, 1.647772]

Fitting tan(x) in range (1.000000, 2.000000)

MSS of errors squared = 7.939400e-11

Corrected MSS of errors squared = 1.587880e-10

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Comments

The following table shows the summary results for the ten models that fit the tangent function. The results are sorted in ascending order using the MSSE values.

<i>Power Q(x)</i>	<i>Power D(x)</i>	<i>MSSE</i>	<i>Q/D Orders</i>
A+B*i	A+B*log10(i)^4	3.38296800E-13	5, 7
A+B*i	A+B*sqrt(i)	3.69779200E-13	7, 6
A+B*i	A+B/i	1.66451400E-11	4, 7
A+B*log10(i)^4	A+B*i	6.59233600E-11	4, 4
A+B*log10(i)^4	A+B/i	7.93940000E-11	5, 5
A+B*sqrt(i)+C*i	A+B*i+C*sqrt(i)	1.28745600E-10	5, 3
A+B*sqrt(i)	A+B*i	1.88400100E-10	7, 3
A+B/i	A+B*i	3.98446000E-10	5, 7
A+B*i+C*sqrt(i)	A+B*sqrt(i)+C*i	7.32823400E-10	3, 5
A+B/i	A+B*log10(i)^4	1.58409100E-09	4, 4

RESULTS FOR THE HYPERBOLIC TANGENT FUNCTION

Combo Powers 1

Fitting $\tanh(x)$ in range (0.000000, 3.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B/i

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	84.5667988763925	0	Inf	0
x1	-1165.94052929176	0	-Inf	0
x2	3725.25583662955	0	Inf	0
x3	-5619.51824859246	0	-Inf	0
x4	4613.77819382102	0	Inf	0
x5	-1997.08807694956	0	-Inf	0
x6	358.975253904351	0	Inf	0
x7	-2.8160190949287	0	-Inf	0
x8	53.2841795862238	0	Inf	0
x9	-145.990802521985	0	-Inf	0
x10	96.4934136333089	0	Inf	0

Number of observations: 301, Error degrees of freedom: 290

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	23.2802959868014	300	0.0776009866226714		
Model	23.2802959868014	10	2.32802959868014	Inf	0
Residual	0	290	0		

```

for Q(x) A = 0.122215, B = 0.209900
for D(x) A = 0.828257, B = 1.095893
order Q(x) = 6.000000, order D(x) = 4.000000
Xmin = 0.000000 and Xmax = 3.000000
Ymin = 0.000000 and Ymax = 0.995055
List of powers for Q(x): [0.332115, 0.542014, 0.751914, 0.961814, 1.171713, 1.381613]
List of powers for D(x): [1.924150, 1.376204, 1.193555, 1.102230]
Fitting tanh(x) in range (1.000000, 2.000000)
MSS of errors squared = 5.535522e-11
Corrected MSS of errors squared = 1.107104e-10
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 2

```

Fitting tanh(x) in range (0.000000, 3.000000)
Shammas polynomial power for Q(x) is A+B*i
Shammas polynomial power for D(x) is A+B*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.0125999149145116	0	-Inf	0
x1	0.121683377226359	0	Inf	0
x2	-0.186384387722576	0	-Inf	0
x3	0.117852805591129	0	Inf	0
x4	1078.20137131513	0	Inf	0
x5	-6877.65771414314	0	-Inf	0
x6	16498.2768204179	0	Inf	0
x7	-19046.9257655636	0	-Inf	0
x8	10717.9834417076	0	Inf	0
x9	-2368.91870561647	0	-Inf	0

Number of observations: 301, Error degrees of freedom: 291

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	23.280295987947	300	0.0776009866264901		
Model	23.280295987947	9	2.58669955421634	Inf	0
Residual	0	291	0		

```

for Q(x) A = 0.339660, B = 0.569068
for D(x) A = 1.383924, B = 0.461817
order Q(x) = 3.000000, order D(x) = 6.000000
Xmin = 0.000000 and Xmax = 3.000000
Ymin = 0.000000 and Ymax = 0.995055
List of powers for Q(x): [0.908728, 1.477795, 2.046863]
List of powers for D(x): [1.845741, 2.037031, 2.183814, 2.307557, 2.416577, 2.515139]
Fitting tanh(x) in range (1.000000, 2.000000)
MSS of errors squared = 6.992960e-10
Corrected MSS of errors squared = 1.398592e-09
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 3

```

Fitting tanh(x) in range (0.000000, 3.000000)
Shammas polynomial power for Q(x) is A+B*sqrt(i)+C*i
Shammas polynomial power for D(x) is A+B*i+C*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00770738389861521	0	Inf	0
x1	-0.172077069298587	0	-Inf	0
x2	0.612056821478965	0	Inf	0
x3	-0.874061521199272	0	-Inf	0
x4	0.584333301512775	0	Inf	0
x5	-0.150117003968077	0	-Inf	0
x6	9.76766918246748	0	Inf	0
x7	-23.2116777355016	0	-Inf	0
x8	24.0214615995034	0	Inf	0
x9	-11.9157690484852	0	-Inf	0
x10	2.33047408966958	0	Inf	0

Number of observations: 301, Error degrees of freedom: 290

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	23.2802959868213	300	0.0776009866227375		
Model	23.2802959868213	10	2.32802959868213	Inf	0

	SumSq	DF	MeanSq	F	pValue
Total	23.2802959868197	300	0.0776009866227323		
Model	23.2802959868197	10	2.32802959868197	Inf	0
Residual	0	290	0		

```

for Q(x) A = 0.445023, B = 0.068774
for D(x) A = 0.272093, B = 0.525616
order Q(x) = 7.000000, order D(x) = 7.000000
Xmin = 0.000000 and Xmax = 3.000000
Ymin = 0.000000 and Ymax = 0.995055
List of powers for Q(x): [0.513797, 0.479410, 0.467947, 0.462216, 0.458778, 0.456485,
0.454848]
List of powers for D(x): [0.797709, 1.323325, 1.848941, 2.374557, 2.900172, 3.425788,
3.951404]
Fitting tanh(x) in range (1.000000, 2.000000)
MSS of errors squared = 1.271207e-10
Corrected MSS of errors squared = 2.542413e-10
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 5

```

Fitting tanh(x) in range (0.000000, 3.000000)
Shammas polynomial power for Q(x) is A+B*sqrt(i)
Shammas polynomial power for D(x) is A+B*i

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.0346688334306645	0	-Inf	0
x1	0.0498347568453249	0	Inf	0
x2	0	0	NaN	NaN
x3	0	0	NaN	NaN
x4	3.76407267640167	0	Inf	0
x5	-6.22055979997314	0	-Inf	0
x6	5.79861424014839	0	Inf	0
x7	-3.29089089108163	0	-Inf	0
x8	1.13509948889158	0	Inf	0
x9	-0.219940366161457	0	-Inf	0
x10	0.0184386631957547	0	Inf	0

Number of observations: 301, Error degrees of freedom: 292

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	23.2802959868193	300	0.077600986622731		
Model	23.2802959868193	8	2.91003699835241	Inf	0
Residual	0	292	0		

for Q(x) A = 0.218063, B = 0.000000

for D(x) A = 0.000000, B = 0.868150

order Q(x) = 3.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 3.000000

Ymin = 0.000000 and Ymax = 0.995055

List of powers for Q(x): [0.218063, 0.218063, 0.218063]

List of powers for D(x): [0.868150, 1.736300, 2.604451, 3.472601, 4.340751, 5.208901, 6.077051]

Fitting tanh(x) in range (1.000000, 2.000000)

MSS of errors squared = 8.339155e-08

Corrected MSS of errors squared = 1.667831e-07

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 6

Fitting tanh(x) in range (0.000000, 3.000000)

Shammas polynomial power for Q(x) is A+B*i+C*sqrt(i)

Shammas polynomial power for D(x) is A+B*sqrt(i)+C*i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	22.992931120495	0	Inf	0
x1	-888.991262468872	0	-Inf	0
x2	3545.69265399345	0	Inf	0
x3	-6142.0293672094	0	-Inf	0
x4	5560.10719591312	0	Inf	0
x5	-2591.6633393817	0	-Inf	0
x6	493.923501894669	0	Inf	0
x7	14.9122122731597	0	Inf	0
x8	-34.0329036131045	0	-Inf	0
x9	28.3250624124515	0	Inf	0
x10	-8.23668493426559	0	-Inf	0

Number of observations: 301, Error degrees of freedom: 290

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	23.2802959866935	300	0.0776009866223117		
Model	23.2802959866935	10	2.32802959866935	Inf	0
Residual	0	290	0		

for Q(x) A = 0.234694, B = 0.165833, C = 0.082833

for D(x) A = 0.451033, B = 0.359044, C = 0.167504

order Q(x) = 6.000000, order D(x) = 4.000000

Xmin = 0.000000 and Xmax = 3.000000

Ymin = 0.000000 and Ymax = 0.995055

List of powers for Q(x): [0.483361, 0.683505, 0.875666, 1.063694, 1.249082, 1.432594]

List of powers for D(x): [0.977581, 1.293806, 1.575427, 1.839137]

Fitting tanh(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.034184e-11

Corrected MSS of errors squared = 2.533224e-11

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 7

Fitting tanh(x) in range (0.000000, 3.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.0225201777984887	0	Inf	0
x1	-0.128733427774013	0	-Inf	0
x2	0.249153372691605	0	Inf	0
x3	-0.207335000601024	0	-Inf	0
x4	0.0675782844617378	0	Inf	0
x5	1163.32266287438	0	Inf	0
x6	0	0	NaN	NaN
x7	0	0	NaN	NaN
x8	-3774.67443079635	0	-Inf	0
x9	3627.59314820933	0	Inf	0

x10 -1015.24456369362 0 -Inf 0

Number of observations: 301, Error degrees of freedom: 292

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	23.2802959868341	300	0.0776009866227802		
Model	23.2802959868341	8	2.91003699835426	Inf	0
Residual	0	292	0		

for Q(x) A = 0.091326, B = 0.437172

for D(x) A = 0.638692, B = 0.272437

order Q(x) = 4.000000, order D(x) = 6.000000

Xmin = 0.000000 and Xmax = 3.000000

Ymin = 0.000000 and Ymax = 0.995055

List of powers for Q(x): [0.528498, 0.965671, 1.402843, 1.840016]

List of powers for D(x): [0.638692, 0.640929, 0.652810, 0.674487, 0.703720, 0.738582]

Fitting tanh(x) in range (1.000000, 2.000000)

MSS of errors squared = 9.699769e-10

Corrected MSS of errors squared = 1.939954e-09

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 8

Fitting tanh(x) in range (0.000000, 3.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	210.171472495715	0	Inf	0
x1	-38.6556811681987	0	-Inf	0
x2	-4544.80064388343	0	-Inf	0
x3	85281.1587434925	0	Inf	0
x4	-320422.462479894	0	-Inf	0
x5	347432.211965443	0	Inf	0
x6	0	0	NaN	NaN
x7	-107917.528425919	0	-Inf	0
x8	3860.15760110866	0	Inf	0

x9		0	0	NaN	NaN
x10		0	0	NaN	NaN
x11	-8478.50416864507	0	-Inf	0	
x12	4619.25161711853	0	Inf	0	

Number of observations: 301, Error degrees of freedom: 291

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	23.2802959968288	300	0.0776009866560961		
Model	23.2802959968288	9	2.5866995552032	Inf	0
Residual	0	291	0		

for Q(x) A = 0.500000, B = 1.313651

for D(x) A = 2.000000, B = 0.142136

order Q(x) = 7.000000, order D(x) = 5.000000

Xmin = 0.000000 and Xmax = 3.000000

Ymin = 0.000000 and Ymax = 0.995055

List of powers for Q(x): [1.813651, 1.156826, 0.937884, 0.828413, 0.762730, 0.718942, 0.687664]

List of powers for D(x): [2.000000, 2.001167, 2.007366, 2.018675, 2.033926]

Fitting tanh(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.062499e-07

Corrected MSS of errors squared = 2.124998e-07

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 9

Fitting tanh(x) in range (0.000000, 3.000000)

Shammas polynomial power for Q(x) is A+B*log10(i)^4

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.000341726254612981	0	Inf	0
x1	29.1897318394055	0	Inf	0
x2	0	0	NaN	NaN
x3	0	0	NaN	NaN
x4	-101.061933842911	0	-Inf	0

x5	102.206361234279	0	Inf	0
x6	-30.3337825201261	0	-Inf	0
x7	503.006587132712	0	Inf	0
x8	-1750.18012251788	0	-Inf	0
x9	2309.23300531834	0	Inf	0
x10	-1367.0053241298	0	-Inf	0
x11	305.945135759655	0	Inf	0

Number of observations: 301, Error degrees of freedom: 291

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	23.2802959868948	300	0.0776009866229826		
Model	23.2802959868948	9	2.58669955409942	Inf	0
Residual	0	291	0		

for Q(x) A = 2.179593, B = 0.269904

for D(x) A = 0.691284, B = 0.086557

order Q(x) = 6.000000, order D(x) = 5.000000

Xmin = 0.000000 and Xmax = 3.000000

Ymin = 0.000000 and Ymax = 0.995055

List of powers for Q(x): [2.179593, 2.181810, 2.193580, 2.215056, 2.244017, 2.278554]

List of powers for D(x): [0.777840, 0.864397, 0.950954, 1.037511, 1.124068]

Fitting tanh(x) in range (1.000000, 2.000000)

MSS of errors squared = 5.345008e-08

Corrected MSS of errors squared = 1.069002e-07

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 10

Fitting tanh(x) in range (0.000000, 3.000000)

Shammas polynomial power for Q(x) is A+B*log10(i)^4

Shammas polynomial power for D(x) is A+B/i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12 + x13 + x14$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.473435464313204	0	Inf	0
x1	-78.9247541669364	0	-Inf	0
x2	0	0	NaN	NaN

x3	0	0	NaN	NaN
x4	0	0	NaN	NaN
x5	382.773808158155	0	Inf	0
x6	-450.226099756132	0	-Inf	0
x7	145.918984303012	0	Inf	0
x8	90.7603230433163	0	Inf	0
x9	-3222.63499015277	0	-Inf	0
x10	11416.3095162968	0	Inf	0
x11	0	0	NaN	NaN
x12	-19321.8862380484	0	-Inf	0
x13	0	0	NaN	NaN
x14	11038.43601486	0	Inf	0

Number of observations: 301, Error degrees of freedom: 291

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	23.280295986204	300	0.0776009866206801		
Model	23.280295986204	9	2.58669955402267	Inf	0
Residual	0	291	0		

for Q(x) A = 0.712355, B = 0.500000

for D(x) A = 1.094828, B = 0.412363

order Q(x) = 7.000000, order D(x) = 7.000000

Xmin = 0.000000 and Xmax = 3.000000

Ymin = 0.000000 and Ymax = 0.995055

List of powers for Q(x): [0.712355, 0.716461, 0.738266, 0.778050, 0.831700, 0.895682, 0.967389]

List of powers for D(x): [1.507191, 1.301010, 1.232283, 1.197919, 1.177301, 1.163556, 1.153737]

Fitting tanh(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.055000e-09

Corrected MSS of errors squared = 2.110000e-09

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Comments

The following table shows the summary results for the ten models that fit the hyperbolic tangent function. The results are sorted in ascending order using the MSSE values.

<i>Power Q(x)</i>	<i>Power D(x)</i>	<i>MSSE</i>	<i>Q/D Orders</i>
A+B*i+C*sqrt(i)	A+B*sqrt(i)+C*i	1.03418400E-11	6, 4
A+B*i	A+B/i	5.53552200E-11	6, 4
A+B/i	A+B*i	1.27120700E-10	7, 7
A+B*sqrt(i)+C*i	A+B*i+C*sqrt(i)	1.62756500E-10	5, 5
A+B*i	A+B*sqrt(i)	6.99296000E-10	3, 6
A+B*i	A+B*log10(i)^4	9.69976900E-10	4, 6
A+B*log10(i)^4	A+B/i	1.05500000E-09	7, 7
A+B*log10(i)^4	A+B*i	5.34500800E-08	6, 5
A+B*sqrt(i)	A+B*i	8.33915500E-08	3, 7
A+B/i	A+B*log10(i)^4	1.06249900E-07	7, 5

RESULTS FOR THE TWO-SIDED T-INVERSE DISTRIBUTION

Combo Powers 1

Fitting $\text{tinv}(0.975, x)$ in range (2.000000, 100.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B/i

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	2.97591715134405	0	Inf	0
x1	-185.518025843871	0	-Inf	0
x2	422.679368526138	0	Inf	0
x3	0	0	NaN	NaN
x4	-766.918376141583	0	-Inf	0
x5	741.520843235758	0	Inf	0
x6	-214.739499230293	0	-Inf	0
x7	-10.4706439718306	0	-Inf	0
x8	198.138097296168	0	Inf	0
x9	-362.962985732829	0	-Inf	0
x10	0	0	NaN	NaN
x11	0	0	NaN	NaN
x12	176.295418553465	0	Inf	0

Number of observations: 981, Error degrees of freedom: 971

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	8.91039026617502	980	0.00909223496548472		

Model	8.91039026617502	9	0.990043362908336	Inf	0
Residual	0	971	0		

for Q(x) A = 0.427134, B = 0.114000
 for D(x) A = 1.350718, B = 0.454344
 order Q(x) = 6.000000, order D(x) = 6.000000
 Xmin = 2.000000 and Xmax = 100.000000
 Ymin = 1.983972 and Ymax = 4.302653
 List of powers for Q(x): [0.541134, 0.655134, 0.769134, 0.883133, 0.997133, 1.111133]
 List of powers for D(x): [1.805062, 1.577890, 1.502166, 1.464304, 1.441587, 1.426442]
 Fitting tinv(0.975,x) in range (1.000000, 2.000000)
 MSS of errors squared = 1.398905e-07
 Corrected MSS of errors squared = 2.797810e-07
 R-squared = 1.00000000
 R-squared Adjusted = 1.00000000
 Particle swarm AICc = -1.000000e+99
 AIC = -Inf
 AICc = -Inf

Combo Powers 2

Fitting tinv(0.975,x) in range (2.000000, 100.000000)
 Shammas polynomial power for Q(x) is A+B*i
 Shammas polynomial power for D(x) is A+B*sqrt(i)

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12 + x13$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	3.67676427460922	0	Inf	0
x1	-36.5750684061287	0	-Inf	0
x2	0	0	NaN	NaN
x3	0	0	NaN	NaN
x4	163.747210571768	0	Inf	0
x5	0	0	NaN	NaN
x6	-310.72820895451	0	-Inf	0
x7	179.879824911175	0	Inf	0
x8	35.757247285316	0	Inf	0
x9	-92.1510163960406	0	-Inf	0
x10	75.4888614433089	0	Inf	0
x11	0	0	NaN	NaN
x12	-28.0791538138318	0	-Inf	0
x13	9.98380026623981	0	Inf	0

Number of observations: 981, Error degrees of freedom: 971

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	8.91039026623116	980	0.009092234965542		
Model	8.91039026623116	9	0.990043362914573	Inf	0
Residual	0	971	0		

```

for Q(x) A = 0.345898, B = 0.100000
for D(x) A = 1.256316, B = 0.602167
order Q(x) = 7.000000, order D(x) = 6.000000
Xmin = 2.000000 and Xmax = 100.000000
Ymin = 1.983972 and Ymax = 4.302653
List of powers for Q(x): [0.445898, 0.545898, 0.645898, 0.745898, 0.845898, 0.945898,
1.045898]
List of powers for D(x): [1.858483, 2.107908, 2.299299, 2.460649, 2.602802, 2.731317]
Fitting tinv(0.975,x) in range (1.000000, 2.000000)
MSS of errors squared = 1.590964e-07
Corrected MSS of errors squared = 3.181928e-07
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 3

```

Fitting tinv(0.975,x) in range (2.000000, 100.000000)
Shammas polynomial power for Q(x) is A+B*sqrt(i)+C*i
Shammas polynomial power for D(x) is A+B*i+C*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	9.18871806939147	0	Inf	0
x1	-22.6990251220727	0	-Inf	0
x2	0	0	NaN	NaN
x3	13.5103450864774	0	Inf	0
x4	7.82587300986287	0	Inf	0
x5	-18.3128924827687	0	-Inf	0
x6	21.9122827777814	0	Inf	0
x7	-15.9008134842491	0	-Inf	0
x8	7.04623058484561	0	Inf	0
x9	-1.76194519869096	0	-Inf	0
x10	0.191245775750286	0	Inf	0

Number of observations: 981, Error degrees of freedom: 971

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	8.91039026622925	980	0.00909223496554005		
Model	8.91039026622925	9	0.990043362914361	Inf	0
Residual	0	971	0		

for Q(x) A = 0.007257, B = 0.100000, C = 0.000000

for D(x) A = 0.256851, B = 0.390000, C = 0.045918

order Q(x) = 3.000000, order D(x) = 7.000000

Xmin = 2.000000 and Xmax = 100.000000

Ymin = 1.983972 and Ymax = 4.302653

List of powers for Q(x): [0.107257, 0.148678, 0.180462]

List of powers for D(x): [0.692769, 1.101789, 1.506384, 1.908688, 2.309527, 2.709327, 3.108339]

Fitting tinv(0.975,x) in range (1.000000, 2.000000)

MSS of errors squared = 7.981721e-08

Corrected MSS of errors squared = 1.955114e-07

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 4

Fitting tinv(0.975,x) in range (2.000000, 100.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12 + x13$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	2.54159934841641	0	Inf	0
x1	-0.425772717659024	0	-Inf	0
x2	6.61722167609748	0	Inf	0
x3	0	0	NaN	NaN
x4	0	0	NaN	NaN
x5	0	0	NaN	NaN
x6	-8.73299787245948	0	-Inf	0
x7	4.09289753583777	0	Inf	0
x8	-6.71458882777865	0	-Inf	0
x9	6.2950803432727	0	Inf	0
x10	-3.88806103495356	0	-Inf	0
x11	1.52529062751997	0	Inf	0

x12	-0.344941554877203	0	-Inf	0
x13	0.034297692790729	0	Inf	0

Number of observations: 981, Error degrees of freedom: 970

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	8.91039026622905	980	0.00909223496553984		
Model	8.91039026622905	10	0.891039026622905	Inf	0
Residual	0	970	0		

for Q(x) A = 0.203680, B = 0.365922

for D(x) A = 0.100000, B = 0.501034

order Q(x) = 6.000000, order D(x) = 7.000000

Xmin = 2.000000 and Xmax = 100.000000

Ymin = 1.983972 and Ymax = 4.302653

List of powers for Q(x): [0.569602, 0.386641, 0.325654, 0.295160, 0.276864, 0.264667]

List of powers for D(x): [0.601034, 1.102069, 1.603103, 2.104138, 2.605172, 3.106207, 3.607241]

Fitting tinv(0.975,x) in range (1.000000, 2.000000)

MSS of errors squared = 1.011473e-07

Corrected MSS of errors squared = 2.022945e-07

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 5

Fitting tinv(0.975,x) in range (2.000000, 100.000000)

Shammas polynomial power for Q(x) is A+B*sqrt(i)

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.985789605331644	0	Inf	0
x1	-11.6152884789766	0	-Inf	0
x2	69.2349847038183	0	Inf	0
x3	-125.84148458941	0	-Inf	0
x4	67.23689735952	0	Inf	0
x5	-360.656957123825	0	-Inf	0
x6	0	0	NaN	NaN

x7		0	0	NaN	NaN
x8	1763.69277380566		0	Inf	0
x9		0	0	NaN	NaN
x10	-3003.84590867344		0	-Inf	0
x11	1601.8096426416		0	Inf	0

Number of observations: 981, Error degrees of freedom: 972

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	8.91039026590533	980	0.00909223496520952		
Model	8.91039026590533	8	1.11379878323817	Inf	0
Residual	0	972	0		

for Q(x) A = 1.106698, B = 0.418477

for D(x) A = 1.955054, B = 0.017337

order Q(x) = 4.000000, order D(x) = 7.000000

Xmin = 2.000000 and Xmax = 100.000000

Ymin = 1.983972 and Ymax = 4.302653

List of powers for Q(x): [1.525175, 1.698514, 1.831522, 1.943653]

List of powers for D(x): [1.972391, 1.989728, 2.007065, 2.024402, 2.041739, 2.059076, 2.076413]

Fitting tinv(0.975,x) in range (1.000000, 2.000000)

MSS of errors squared = 2.746330e-07

Corrected MSS of errors squared = 5.492660e-07

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 6

Fitting tinv(0.975,x) in range (2.000000, 100.000000)

Shammas polynomial power for Q(x) is A+B*i+C*sqrt(i)

Shammas polynomial power for D(x) is A+B*sqrt(i)+C*i

Linear regression model:

$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13}$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.972795193280601	0	Inf	0
x1	-4.5648427790028	0	-Inf	0
x2	4.532097528829	0	Inf	0
x3	0.525416244143019	0	Inf	0

x4	-1.68146665282917	0	-Inf	0
x5	0.243013020116278	0	Inf	0
x6	-0.0270131910848342	0	-Inf	0
x7	2.43002797033534	0	Inf	0
x8	4.32466802188707	0	Inf	0
x9	-7.31048843415421	0	-Inf	0
x10	0	0	NaN	NaN
x11	0	0	NaN	NaN
x12	0	0	NaN	NaN
x13	1.55579276020584	0	Inf	0

Number of observations: 981, Error degrees of freedom: 970

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	8.91039026622821	980	0.00909223496553898		
Model	8.91039026622821	10	0.891039026622821	Inf	0
Residual	0	970	0		

for Q(x) A = 0.000000, B = 0.109601, C = 0.337168

for D(x) A = 0.000000, B = 0.400000, C = 0.000000

order Q(x) = 6.000000, order D(x) = 7.000000

Xmin = 2.000000 and Xmax = 100.000000

Ymin = 1.983972 and Ymax = 4.302653

List of powers for Q(x): [0.446769, 0.696030, 0.912796, 1.112741, 1.301938, 1.483498]

List of powers for D(x): [0.400000, 0.565685, 0.692820, 0.800000, 0.894427, 0.979796, 1.058301]

Fitting tinv(0.975,x) in range (1.000000, 2.000000)

MSS of errors squared = 1.027260e-09

Corrected MSS of errors squared = 2.516263e-09

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 7

Fitting tinv(0.975,x) in range (2.000000, 100.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12 + x13 + x14

Estimated Coefficients:

Estimate	SE	tStat	pValue
----------	----	-------	--------

(Intercept)	5.60596427802622e-06	0	Inf	0
x1	-2.95394131344704e-05	0	-Inf	0
x2	0	0	NaN	NaN
x3	1.86119005691394e-05	0	Inf	0
x4	0	0	NaN	NaN
x5	2.03715177809646e-05	0	Inf	0
x6	0	0	NaN	NaN
x7	-1.50499584230986e-05	0	-Inf	0
x8	0.999995345124004	0	Inf	0
x9	0	0	NaN	NaN
x10	-4.71590411022593e-06	0	-Inf	0
x11	2.70914827260211e-05	0	Inf	0
x12	-2.49310759779923e-05	0	-Inf	0
x13	7.26794051716312e-06	0	Inf	0
x14	-5.75727008055961e-08	0	-Inf	0

Number of observations: 981, Error degrees of freedom: 970

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	8.91039026622865	980	0.00909223496553944		
Model	8.91039026622865	10	0.891039026622865	Inf	0
Residual	0	970	0		

for Q(x) A = 0.406625, B = 0.100000

for D(x) A = 0.000000, B = 3.262021

order Q(x) = 7.000000, order D(x) = 7.000000

Xmin = 2.000000 and Xmax = 100.000000

Ymin = 1.983972 and Ymax = 4.302653

List of powers for Q(x): [0.506625, 0.606625, 0.706625, 0.806625, 0.906625, 1.006625, 1.106625]

List of powers for D(x): [0.000000, 0.026787, 0.169045, 0.428594, 0.778612, 1.196032, 1.663854]

Fitting tinv(0.975,x) in range (1.000000, 2.000000)

MSS of errors squared = 3.371891e-06

Corrected MSS of errors squared = 6.743782e-06

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 8

Fitting tinv(0.975,x) in range (2.000000, 100.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	2.12500080216692	0	Inf	0
x1	32.4519671926863	0	Inf	0
x2	-286.04296249697	0	-Inf	0
x3	465.276179212571	0	Inf	0
x4	0	0	NaN	NaN
x5	-213.809920012229	0	-Inf	0
x6	212.496924864373	0	Inf	0
x7	0	0	NaN	NaN
x8	0	0	NaN	NaN
x9	-580.101541596	0	-Inf	0
x10	481.927301836316	0	Inf	0
x11	-113.322817465117	0	-Inf	0

Number of observations: 981, Error degrees of freedom: 972

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	8.91039026622462	980	0.00909223496553532		
Model	8.91039026622462	8	1.11379878327808	Inf	0
Residual	0	972	0		

for Q(x) A = 0.391105, B = 0.486063

for D(x) A = 1.350142, B = 0.499669

order Q(x) = 5.000000, order D(x) = 6.000000

Xmin = 2.000000 and Xmax = 100.000000

Ymin = 1.983972 and Ymax = 4.302653

List of powers for Q(x): [0.877168, 0.634137, 0.553126, 0.512621, 0.488318]

List of powers for D(x): [1.350142, 1.354245, 1.376036, 1.415793, 1.469408, 1.533347]

Fitting tinv(0.975,x) in range (1.000000, 2.000000)

MSS of errors squared = 2.131323e-07

Corrected MSS of errors squared = 4.262647e-07

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 9

Fitting tinv(0.975,x) in range (2.000000, 100.000000)

Shammas polynomial power for $Q(x)$ is $A+B*\log_{10}(i)^4$

Shammas polynomial power for $D(x)$ is $A+B*i$

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.875902099074903	0	Inf	0
x1	308.604001312268	0	Inf	0
x2	0	0	NaN	NaN
x3	0	0	NaN	NaN
x4	0	0	NaN	NaN
x5	-607.20367550607	0	-Inf	0
x6	0	0	NaN	NaN
x7	297.725420001783	0	Inf	0
x8	4.23482092274143	0	Inf	0
x9	-4.63919823499375	0	-Inf	0
x10	1.90725067743633	0	Inf	0
x11	-0.628117069735703	0	-Inf	0
x12	0.142755240045068	0	Inf	0
x13	-0.0195428150769177	0	-Inf	0
x14	0.00120720090729859	0	Inf	0

Number of observations: 981, Error degrees of freedom: 970

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	8.91039026618725	980	0.00909223496549719		
Model	8.91039026618725	10	0.891039026618725	Inf	0
Residual	0	970	0		

for $Q(x)$ $A = 2.200000$, $B = 0.300628$

for $D(x)$ $A = 1.400000$, $B = 1.231706$

order $Q(x) = 7.000000$, order $D(x) = 7.000000$

Xmin = 2.000000 and Xmax = 100.000000

Ymin = 1.983972 and Ymax = 4.302653

List of powers for $Q(x)$: [2.200000, 2.202469, 2.215579, 2.239499, 2.271757, 2.310226, 2.353341]

List of powers for $D(x)$: [2.631706, 3.863413, 5.095119, 6.326825, 7.558531, 8.790238, 10.021944]

Fitting $\text{tinv}(0.975, x)$ in range (1.000000, 2.000000)

MSS of errors squared = 3.797782e-07

Corrected MSS of errors squared = 7.595565e-07

R-squared = 1.00000000

R-squared Adjusted = 1.00000000
 Particle swarm AICc = -1.000000e+99
 AIC = -Inf
 AICc = -Inf

Combo Powers 10

Fitting $\text{tinv}(0.975, x)$ in range (2.000000, 100.000000)
 Shammas polynomial power for $Q(x)$ is $A+B*\log_{10}(i)^4$
 Shammas polynomial power for $D(x)$ is $A+B/i$

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.00017808764942247	0	-Inf	0
x1	1.89087523149569	0	Inf	0
x2	0	0	NaN	NaN
x3	-2.98592872145637	0	-Inf	0
x4	0	0	NaN	NaN
x5	2.27663915706269	0	Inf	0
x6	-1.4875851315592	0	-Inf	0
x7	0.306177529682063	0	Inf	0
x8	0.224686554536905	0	Inf	0
x9	-1.06167160213116	0	-Inf	0
x10	0	0	NaN	NaN
x11	0	0	NaN	NaN
x12	0	0	NaN	NaN
x13	1.83698505880558	0	Inf	0

Number of observations: 981, Error degrees of freedom: 972

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	8.91039026622868	980	0.00909223496553947		
Model	8.91039026622868	8	1.11379878327859	Inf	0
Residual	0	972	0		

for $Q(x)$ $A = 1.400000$, $B = 0.430799$

for $D(x)$ $A = 0.000000$, $B = 0.100000$

order $Q(x) = 7.000000$, order $D(x) = 6.000000$

Xmin = 2.000000 and Xmax = 100.000000

Ymin = 1.983972 and Ymax = 4.302653

List of powers for $Q(x)$: [1.400000, 1.403538, 1.422325, 1.456602, 1.502827, 1.557954, 1.619737]

```
List of powers for D(x): [0.100000, 0.050000, 0.033333, 0.025000, 0.020000, 0.016667]
Fitting tinv(0.975,x) in range (1.000000, 2.000000)
MSS of errors squared = 2.331573e-07
Corrected MSS of errors squared = 4.663145e-07
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf
```

Comments

The following table shows the summary results for the ten models that fit the two-sided t-inverse function. The results are sorted in ascending order using the MSSE values.

<i>Power Q(x)</i>	<i>Power D(x)</i>	<i>MSSE</i>	<i>Q/D Orders</i>
A+B*i+C*sqrt(i)	A+B*sqrt(i)+C*i	1.02726000E-09	6, 7
A+B*sqrt(i)+C*i	A+B*i+C*sqrt(i)	7.98172100E-08	3, 7
A+B/i	A+B*i	1.01147300E-07	6, 7
A+B*i	A+B/i	1.39890500E-07	6, 6
A+B*i	A+B*sqrt(i)	1.59096400E-07	7, 6
A+B/i	A+B*log10(i)^4	2.13132300E-07	5, 6
A+B*log10(i)^4	A+B/i	2.33157300E-07	7, 6
A+B*sqrt(i)	A+B*i	2.74633000E-07	4, 7
A+B*log10(i)^4	A+B*i	3.79778200E-07	7, 7
A+B*i	A+B*log10(i)^4	3.37189100E-06	7, 7

RESULTS FOR THE ONE-SIDED T-INVERSE DISTRIBUTION

Combo Powers 1

Fitting $\text{tinv}(0.95, x)$ in range (2.000000, 100.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B/i

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.986872281573302	0	Inf	0
x1	-0.274965470363551	0	-Inf	0
x2	-3.07015266850329	0	-Inf	0
x3	2.35871735795976	0	Inf	0
x4	0.0710779322929276	0	Inf	0
x5	-15.5628259444981	0	-Inf	0
x6	67.3669502974413	0	Inf	0
x7	-83.1310907994361	0	-Inf	0
x8	0	0	NaN	NaN
x9	0	0	NaN	NaN
x10	32.255652756932	0	Inf	0

Number of observations: 981, Error degrees of freedom: 972

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	9.94348494606381	980	0.0101464132102692		
Model	9.94348494606381	8	1.24293561825798	Inf	0
Residual	0	972	0		

```

for Q(x) A = 0.269509, B = 0.465292
for D(x) A = 1.150159, B = 0.825348
order Q(x) = 3.000000, order D(x) = 7.000000
Xmin = 2.000000 and Xmax = 100.000000
Ymin = 1.660234 and Ymax = 2.919986
List of powers for Q(x): [0.734800, 1.200092, 1.665384]
List of powers for D(x): [1.975507, 1.562833, 1.425275, 1.356496, 1.315229, 1.287717,
1.268066]
Fitting tinva(0.95,x) in range (1.000000, 2.000000)
MSS of errors squared = 7.402490e-08
Corrected MSS of errors squared = 1.480498e-07
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 2

```

Fitting tinva(0.95,x) in range (2.000000, 100.000000)
Shammas polynomial power for Q(x) is A+B*i
Shammas polynomial power for D(x) is A+B*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.996262116513427	0	Inf	0
x1	-3.5415003508922	0	-Inf	0
x2	1.74461265991112	0	Inf	0
x3	2.47467998668062	0	Inf	0
x4	-1.71431131421716	0	-Inf	0
x5	0.0402583541741376	0	Inf	0
x6	10.2176092129937	0	Inf	0
x7	-20.6756773991662	0	-Inf	0
x8	19.4190613352868	0	Inf	0
x9	0	0	NaN	NaN
x10	-20.8916014101977	0	-Inf	0
x11	12.9306075349403	0	Inf	0

Number of observations: 981, Error degrees of freedom: 970

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

SumSq	DF	MeanSq	F	pValue

Total	9.9434849460636	980	0.010146413210269		
Model	9.9434849460636	10	0.99434849460636	Inf	0
Residual	0	970	0		

for Q(x) A = 0.402399, B = 0.559702

for D(x) A = 0.348478, B = 0.811952

order Q(x) = 5.000000, order D(x) = 6.000000

Xmin = 2.000000 and Xmax = 100.000000

Ymin = 1.660234 and Ymax = 2.919986

List of powers for Q(x): [0.962101, 1.521803, 2.081506, 2.641208, 3.200910]

List of powers for D(x): [1.160430, 1.496751, 1.754820, 1.972382, 2.164058, 2.337346]

Fitting tinv(0.95,x) in range (1.000000, 2.000000)

MSS of errors squared = 9.815188e-07

Corrected MSS of errors squared = 1.963038e-06

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 3

Fitting tinv(0.95,x) in range (2.000000, 100.000000)

Shammas polynomial power for Q(x) is A+B*sqrt(i)+C*i

Shammas polynomial power for D(x) is A+B*i+C*sqrt(i)

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.989270445292237	0	Inf	0
x1	-0.808185323245527	0	-Inf	0
x2	-3.10113135575329	0	-Inf	0
x3	2.92026725919858	0	Inf	0
x4	3.4493240886089	0	Inf	0
x5	-2.56141600743179	0	-Inf	0
x6	0.197651763055177	0	Inf	0
x7	-0.105256878750681	0	-Inf	0
x8	0.0195865185192197	0	Inf	0

Number of observations: 981, Error degrees of freedom: 972

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	9.94348494606323	980	0.0101464132102686		

Model	9.94348494606323	8	1.2429356182579	Inf	0
Residual	0	972	0		

```

for Q(x) A = 0.221251, B = 0.248435, C = 0.155413
for D(x) A = 0.256126, B = 0.193061, C = 0.344667
order Q(x) = 3.000000, order D(x) = 5.000000
Xmin = 2.000000 and Xmax = 100.000000
Ymin = 1.660234 and Ymax = 2.919986
List of powers for Q(x): [0.625099, 0.883418, 1.117793]
List of powers for D(x): [0.793854, 1.129680, 1.432289, 1.717703, 1.992129]
Fitting tinv(0.95,x) in range (1.000000, 2.000000)
MSS of errors squared = 7.411581e-08
Corrected MSS of errors squared = 1.815459e-07
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 4

```

Fitting tinv(0.95,x) in range (2.000000, 100.000000)
Shammas polynomial power for Q(x) is A+B/i
Shammas polynomial power for D(x) is A+B*i

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-3.38207425189845	0	-Inf	0
x1	66.9100890050936	0	Inf	0
x2	-220.906009004982	0	-Inf	0
x3	0	0	NaN	NaN
x4	157.37853384401	0	Inf	0
x5	2.70671719039816	0	Inf	0
x6	-2.51249262195898	0	-Inf	0
x7	1.09919704132215	0	Inf	0
x8	-0.366452939234948	0	-Inf	0
x9	0.0834872969830307	0	Inf	0
x10	-0.0114319807497809	0	-Inf	0
x11	0.000706188665433473	0	Inf	0

Number of observations: 981, Error degrees of freedom: 970

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

SumSq	DF	MeanSq	F	pValue
-------	----	--------	---	--------

Total	9.94348494605569	980	0.0101464132102609		
Model	9.94348494605569	10	0.994348494605569	Inf	0
Residual	0	970	0		

```

for Q(x) A = 0.495163, B = 0.387039
for D(x) A = 0.128198, B = 1.232940
order Q(x) = 4.000000, order D(x) = 7.000000
Xmin = 2.000000 and Xmax = 100.000000
Ymin = 1.660234 and Ymax = 2.919986
List of powers for Q(x): [0.882203, 0.688683, 0.624176, 0.591923]
List of powers for D(x): [1.361138, 2.594078, 3.827019, 5.059959, 6.292899, 7.525839,
8.758779]
Fitting tinv(0.95,x) in range (1.000000, 2.000000)
MSS of errors squared = 2.692281e-07
Corrected MSS of errors squared = 5.384562e-07
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 5

```

Fitting tinv(0.95,x) in range (2.000000, 100.000000)
Shammas polynomial power for Q(x) is A+B*sqrt(i)
Shammas polynomial power for D(x) is A+B*i

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.950368832801145	0	Inf	0
x1	-0.469712411571455	0	-Inf	0
x2	-5.52405059773851	0	-Inf	0
x3	5.04346553687291	0	Inf	0
x4	2.32915929411591	0	Inf	0
x5	-1.43027985857575	0	-Inf	0
x6	0.110589385835246	0	Inf	0
x7	-0.00950450343462329	0	-Inf	0

Number of observations: 981, Error degrees of freedom: 973

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

SumSq	DF	MeanSq	F	pValue
-------	----	--------	---	--------

Total	9.9434849460639	980	0.0101464132102693		
Model	9.9434849460639	7	1.4204978494377	Inf	0
Residual	0	973	0		

for Q(x) A = 0.125868, B = 0.277585
 for D(x) A = 0.075999, B = 0.338869
 order Q(x) = 3.000000, order D(x) = 4.000000
 Xmin = 2.000000 and Xmax = 100.000000
 Ymin = 1.660234 and Ymax = 2.919986
 List of powers for Q(x): [0.403453, 0.518432, 0.606659]
 List of powers for D(x): [0.414868, 0.753737, 1.092607, 1.431476]
 Fitting tinv(0.95,x) in range (1.000000, 2.000000)
 MSS of errors squared = 1.320353e-07
 Corrected MSS of errors squared = 2.640706e-07
 R-squared = 1.00000000
 R-squared Adjusted = 1.00000000
 Particle swarm AICc = -1.000000e+99
 AIC = -Inf
 AICc = -Inf

Combo Powers 6

Fitting tinv(0.95,x) in range (2.000000, 100.000000)
 Shammas polynomial power for Q(x) is A+B*i+C*sqrt(i)
 Shammas polynomial power for D(x) is A+B*sqrt(i)+C*i

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.998565204681078	0	Inf	0
x1	-2.23778296896078	0	-Inf	0
x2	1.27906513644634	0	Inf	0
x3	-0.0397964067676183	0	-Inf	0
x4	0.6941896475169	0	Inf	0
x5	3.88389344783483	0	Inf	0
x6	-3.57810857695114	0	-Inf	0

Number of observations: 981, Error degrees of freedom: 974

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	9.94348494606311	980	0.0101464132102685		
Model	9.94348494606311	6	1.65724749101052	Inf	0
Residual	0	974	0		

```

for Q(x) A = 0.028010, B = 0.263325, C = 0.041275
for D(x) A = 0.000000, B = 0.300075, C = 0.000000
order Q(x) = 3.000000, order D(x) = 3.000000
Xmin = 2.000000 and Xmax = 100.000000
Ymin = 1.660234 and Ymax = 2.919986
List of powers for Q(x): [0.332610, 0.613032, 0.889476]
List of powers for D(x): [0.300075, 0.424371, 0.519746]
Fitting tinv(0.95,x) in range (1.000000, 2.000000)
MSS of errors squared = 1.256988e-07
Corrected MSS of errors squared = 3.078980e-07
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 7

```

Fitting tinv(0.95,x) in range (2.000000, 100.000000)
Shammas polynomial power for Q(x) is A+B*i
Shammas polynomial power for D(x) is A+B*log10(i)^4

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.960388385779285	0	Inf	0
x1	1.25177765025874	0	Inf	0
x2	-6.5690282080029	0	-Inf	0
x3	-8.43204300196059	0	-Inf	0
x4	24.9013632735002	0	Inf	0
x5	-12.1124703345904	0	-Inf	0
x6	45.485994066938	0	Inf	0
x7	0	0	NaN	NaN
x8	-62.9551706542679	0	-Inf	0
x9	15.3973875403379	0	Inf	0
x10	3.07179516453825	0	Inf	0

Number of observations: 981, Error degrees of freedom: 971

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	9.94348494606641	980	0.0101464132102719		
Model	9.94348494606641	9	1.10483166067405	Inf	0

Residual 0 971 0

for Q(x) A = 0.416978, B = 0.214565
for D(x) A = 1.081377, B = 2.134533
order Q(x) = 5.000000, order D(x) = 5.000000
Xmin = 2.000000 and Xmax = 100.000000
Ymin = 1.660234 and Ymax = 2.919986
List of powers for Q(x): [0.631543, 0.846108, 1.060673, 1.275238, 1.489803]
List of powers for D(x): [1.081377, 1.098905, 1.191993, 1.361831, 1.590868]
Fitting tinv(0.95,x) in range (1.000000, 2.000000)
MSS of errors squared = 1.619511e-08
Corrected MSS of errors squared = 3.239022e-08
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

Combo Powers 8

Fitting tinv(0.95,x) in range (2.000000, 100.000000)
Shammas polynomial power for Q(x) is A+B/i
Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.399315948489481	0	Inf	0
x1	3.84078565273746	0	Inf	0
x2	-34.8890667006388	0	-Inf	0
x3	113.280858234095	0	Inf	0
x4	-123.822839949113	0	-Inf	0
x5	0	0	NaN	NaN
x6	41.1911641817952	0	Inf	0
x7	0	0	NaN	NaN
x8	3341.36698095255	0	Inf	0
x9	-3899.12695208203	0	-Inf	0
x10	558.759862442799	0	Inf	0

Number of observations: 981, Error degrees of freedom: 972

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	9.94348494613076	980	0.0101464132103375		

Model	9.94348494613076	8	1.24293561826635	Inf	0
Residual	0	972	0		

```

for Q(x) A = 0.100000, B = 0.894143
for D(x) A = 1.007672, B = 0.333464
order Q(x) = 7.000000, order D(x) = 3.000000
Xmin = 2.000000 and Xmax = 100.000000
Ymin = 1.660234 and Ymax = 2.919986
List of powers for Q(x): [0.994143, 0.547072, 0.398048, 0.323536, 0.278829, 0.249024,
0.227735]
List of powers for D(x): [1.007672, 1.010411, 1.024953]
Fitting tinv(0.95,x) in range (1.000000, 2.000000)
MSS of errors squared = 6.956695e-08
Corrected MSS of errors squared = 1.391339e-07
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 9

```

Fitting tinv(0.95,x) in range (2.000000, 100.000000)
Shammas polynomial power for Q(x) is A+B*log10(i)^4
Shammas polynomial power for D(x) is A+B*i

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.948522055930655	0	Inf	0
x1	-36.787402773646	0	-Inf	0
x2	0	0	NaN	NaN
x3	0	0	NaN	NaN
x4	44.1321719609123	0	Inf	0
x5	0	0	NaN	NaN
x6	-8.2928289432831	0	-Inf	0
x7	2.21118606886506	0	Inf	0
x8	-1.45186629156646	0	-Inf	0
x9	0.312763250848325	0	Inf	0
x10	-0.0900887467111559	0	-Inf	0
x11	0.0205584706207213	0	Inf	0
x12	-0.0029846738784506	0	-Inf	0
x13	0.000200767645889949	0	Inf	0

Number of observations: 981, Error degrees of freedom: 970

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	9.94348494606754	980	0.010146413210273		
Model	9.94348494606754	10	0.994348494606754	Inf	0
Residual	0	970	0		

for Q(x) A = 1.411617, B = 0.500000

for D(x) A = 0.000000, B = 1.033492

order Q(x) = 6.000000, order D(x) = 7.000000

Xmin = 2.000000 and Xmax = 100.000000

Ymin = 1.660234 and Ymax = 2.919986

List of powers for Q(x): [1.411617, 1.415723, 1.437528, 1.477312, 1.530962, 1.594944]

List of powers for D(x): [1.033492, 2.066983, 3.100475, 4.133966, 5.167458, 6.200950, 7.234441]

Fitting tinva(0.95,x) in range (1.000000, 2.000000)

MSS of errors squared = 4.637813e-08

Corrected MSS of errors squared = 9.275625e-08

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 10

Fitting tinva(0.95,x) in range (2.000000, 100.000000)

Shammas polynomial power for Q(x) is A+B*log10(i)^4

Shammas polynomial power for D(x) is A+B/i

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-1.29068620384015	0	-Inf	0
x1	5250.5882086684	0	Inf	0
x2	0	0	NaN	NaN
x3	-8627.52974354861	0	-Inf	0
x4	3378.23253960672	0	Inf	0
x5	0.0923237239796167	0	Inf	0
x6	-13.3220778760253	0	-Inf	0
x7	122.866888198333	0	Inf	0
x8	-267.165878742898	0	-Inf	0
x9	158.528585391962	0	Inf	0

Number of observations: 981, Error degrees of freedom: 972

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	9.94348494580402	980	0.0101464132100041		
Model	9.94348494580402	8	1.2429356182255	Inf	0
Residual	0	972	0		

for Q(x) A = 0.622406, B = 0.238916

for D(x) A = 1.307704, B = 1.901814

order Q(x) = 4.000000, order D(x) = 5.000000

Xmin = 2.000000 and Xmax = 100.000000

Ymin = 1.660234 and Ymax = 2.919986

List of powers for Q(x): [0.622406, 0.624368, 0.634787, 0.653797]

List of powers for D(x): [3.209518, 2.258611, 1.941642, 1.783157, 1.688067]

Fitting tinv(0.95,x) in range (1.000000, 2.000000)

MSS of errors squared = 6.879452e-07

Corrected MSS of errors squared = 1.375890e-06

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Comments

The following table shows the summary results for the ten models that fit the one-sided t-inverse function. The results are sorted in ascending order using the MSSE values.

<i>Power Q(x)</i>	<i>Power D(x)</i>	<i>MSSE</i>	<i>Q/D Orders</i>
A+B*i	A+B*log10(i)^4	1.61951100E-08	5, 5
A+B*log10(i)^4	A+B*i	4.63781300E-08	6, 7
A+B/i	A+B*log10(i)^4	6.95669500E-08	7, 3
A+B*i	A+B/i	7.40249000E-08	3, 7
A+B*sqrt(i)+C*i	A+B*i+C*sqrt(i)	7.41158100E-08	3, 5
A+B*i+C*sqrt(i)	A+B*sqrt(i)+C*i	1.25698800E-07	3, 3
A+B*sqrt(i)	A+B*i	1.32035300E-07	3, 4
A+B/i	A+B*i	2.69228100E-07	4, 7
A+B*log10(i)^4	A+B/i	6.87945200E-07	4, 5
A+B*i	A+B*sqrt(i)	9.81518800E-07	5, 6

RESULTS FOR THE TRIGAMMA FUNCTION

Combo Powers 1

Fitting $\text{tigamma}(x)$ in range (1.000000, 100.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B/i

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	9.00173957592561	0	Inf	0
x1	-138.435610966254	0	-Inf	0
x2	281.110440697937	0	Inf	0
x3	0	0	NaN	NaN
x4	-499.466862524533	0	-Inf	0
x5	501.597721142106	0	Inf	0
x6	-153.807420896217	0	-Inf	0
x7	-683.495302195431	0	-Inf	0
x8	7878.63812126995	0	Inf	0
x9	-11745.9094514654	0	-Inf	0
x10	0	0	NaN	NaN
x11	0	0	NaN	NaN
x12	0	0	NaN	NaN
x13	4551.76662887561	0	Inf	0

Number of observations: 991, Error degrees of freedom: 981

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

SumSq	DF	MeanSq	F	pValue

Total	5.90929128643641	990	0.00596898109741051		
Model	5.90929128643641	9	0.656587920715156	Inf	0
Residual	0	981	0		

```

for Q(x) A = 0.122754, B = 0.100000
for D(x) A = 0.921856, B = 0.100000
order Q(x) = 6.000000, order D(x) = 7.000000
Xmin = 1.000000 and Xmax = 100.000000
Ymin = 0.010050 and Ymax = 1.644935
List of powers for Q(x): [0.222754, 0.322754, 0.422754, 0.522754, 0.622754, 0.722754]
List of powers for D(x): [1.021856, 0.971856, 0.955189, 0.946856, 0.941856, 0.938522,
0.936141]
Fitting tgamma(x) in range (1.000000, 2.000000)
MSS of errors squared = 5.807903e-07
Corrected MSS of errors squared = 1.161581e-06
R-squared = 1.00000000
R-squared Adjusted = 1.00000000
Particle swarm AICc = -1.000000e+99
AIC = -Inf
AICc = -Inf

```

Combo Powers 2

```

Fitting trigamma(x) in range (1.000000, 100.000000)
Shammas polynomial power for Q(x) is A+B*i
Shammas polynomial power for D(x) is A+B*sqrt(i)

```

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.0231257480691091	0	Inf	0
x1	-0.063911199943711	0	-Inf	0
x2	0.0526992674113239	0	Inf	0
x3	-0.0146590550073695	0	-Inf	0
x4	0.00319900726806379	0	Inf	0
x5	-0.00049046343788714	0	-Inf	0
x6	4.60734210990127e-05	0	Inf	0
x7	-1.97761042482469e-06	0	-Inf	0
x8	12.4336868243254	0	Inf	0
x9	-19.7642604649886	0	-Inf	0
x10	0	0	NaN	NaN
x11	0	0	NaN	NaN
x12	15.92802250611	0	Inf	0
x13	0	0	NaN	NaN
x14	-7.59745256896343	0	-Inf	0

Number of observations: 991, Error degrees of freedom: 979

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	5.9092912862758	990	0.00596898109724828		
Model	5.9092912862758	11	0.537208298752346	Inf	0
Residual	0	979	0		

for Q(x) A = 0.500000, B = 1.600830

for D(x) A = 0.000000, B = 0.484149

order Q(x) = 7.000000, order D(x) = 7.000000

Xmin = 1.000000 and Xmax = 100.000000

Ymin = 0.010050 and Ymax = 1.644935

List of powers for Q(x): [2.100830, 3.701659, 5.302489, 6.903319, 8.504148, 10.104978, 11.705808]

List of powers for D(x): [0.484149, 0.684690, 0.838571, 0.968298, 1.082591, 1.185919, 1.280938]

Fitting trigamma(x) in range (1.000000, 2.000000)

MSS of errors squared = 2.156274e-06

Corrected MSS of errors squared = 4.312547e-06

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 3

Fitting trigamma(x) in range (1.000000, 100.000000)

Shammas polynomial power for Q(x) is A+B*sqrt(i)+C*i

Shammas polynomial power for D(x) is A+B*i+C*sqrt(i)

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	2.22347826603905	0	Inf	0
x1	-10.1473139346774	0	-Inf	0
x2	14.8558524735061	0	Inf	0
x3	-9.50431439508678	0	-Inf	0
x4	3.17935434623022	0	Inf	0
x5	-0.672421473651718	0	-Inf	0
x6	0.0653784598253031	0	Inf	0
x7	-19.8546034563198	0	-Inf	0
x8	68.8344917428945	0	Inf	0
x9	-72.8526758083388	0	-Inf	0

x10 24.8727806485736 0 Inf 0

Number of observations: 991, Error degrees of freedom: 980

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	5.90929128627558	990	0.00596898109724806		
Model	5.90929128627558	10	0.590929128627558	Inf	0
Residual	0	980	0		

for Q(x) A = 0.068944, B = 0.183467, C = 0.329033

for D(x) A = 0.084192, B = 0.154244, C = 0.101921

order Q(x) = 6.000000, order D(x) = 4.000000

Xmin = 1.000000 and Xmax = 100.000000

Ymin = 0.010050 and Ymax = 1.644935

List of powers for Q(x): [0.581444, 0.986471, 1.373816, 1.752009, 2.124352, 2.492541]

List of powers for D(x): [0.340358, 0.536819, 0.723458, 0.905012]

Fitting trigamma(x) in range (1.000000, 2.000000)

MSS of errors squared = 7.304097e-07

Corrected MSS of errors squared = 1.789131e-06

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 4

Fitting trigamma(x) in range (1.000000, 100.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	1.59383515369183	0	Inf	0
x1	-5.92843180165813	0	-Inf	0
x2	123.408199037458	0	Inf	0
x3	-559.152103954806	0	-Inf	0
x4	865.120247221063	0	Inf	0
x5	-425.041743633599	0	-Inf	0
x6	9.38237272059293	0	Inf	0
x7	-13.7162983047818	0	-Inf	0
x8	5.33392457293897	0	Inf	0

Number of observations: 991, Error degrees of freedom: 982

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	5.90929128629177	990	0.00596898109726442		
Model	5.90929128629177	8	0.738661410786472	Inf	0
Residual	0	982	0		

for Q(x) A = 0.196655, B = 0.473020

for D(x) A = 0.282351, B = 0.137955

order Q(x) = 5.000000, order D(x) = 3.000000

Xmin = 1.000000 and Xmax = 100.000000

Ymin = 0.010050 and Ymax = 1.644935

List of powers for Q(x): [0.669675, 0.433165, 0.354328, 0.314910, 0.291259]

List of powers for D(x): [0.420306, 0.558261, 0.696216]

Fitting trigamma(x) in range (1.000000, 2.000000)

MSS of errors squared = 4.779978e-07

Corrected MSS of errors squared = 9.559955e-07

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 5

Fitting trigamma(x) in range (1.000000, 100.000000)

Shammas polynomial power for Q(x) is A+B*sqrt(i)

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	1.04657790861413	0	Inf	0
x1	-40.7423366194352	0	-Inf	0
x2	286.001676671499	0	Inf	0
x3	-697.341319708381	0	-Inf	0
x4	676.969193127606	0	Inf	0
x5	-210.596244211421	0	-Inf	0
x6	-15.3374868497034	0	-Inf	0
x7	254.596651083035	0	Inf	0
x8	-589.299140828067	0	-Inf	0
x9	436.611284709089	0	Inf	0

x10 -100.908825131474 0 -Inf 0

Number of observations: 991, Error degrees of freedom: 980

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	5.90929128619125	990	0.00596898109716287		
Model	5.90929128619125	10	0.590929128619125	Inf	0
Residual	0	980	0		

for Q(x) A = 0.638201, B = 0.500000

for D(x) A = 1.713388, B = 0.096650

order Q(x) = 6.000000, order D(x) = 4.000000

Xmin = 1.000000 and Xmax = 100.000000

Ymin = 0.010050 and Ymax = 1.644935

List of powers for Q(x): [1.138201, 1.345307, 1.504226, 1.638201, 1.756235, 1.862945]

List of powers for D(x): [1.810038, 1.906688, 2.003338, 2.099988]

Fitting trigamma(x) in range (1.000000, 2.000000)

MSS of errors squared = 6.425700e-07

Corrected MSS of errors squared = 1.285140e-06

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 6

Fitting trigamma(x) in range (1.000000, 100.000000)

Shammas polynomial power for Q(x) is A+B*i+C*sqrt(i)

Shammas polynomial power for D(x) is A+B*sqrt(i)+C*i

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.293779053260072	0	Inf	0
x1	-2.2132165888173	0	-Inf	0
x2	3.10165717783176	0	Inf	0
x3	-1.18221837218055	0	-Inf	0
x4	8.37262137012943	0	Inf	0
x5	-26.0441035127663	0	-Inf	0
x6	39.4614370678361	0	Inf	0
x7	-28.8386871934522	0	-Inf	0
x8	8.04873162894087	0	Inf	0

Number of observations: 991, Error degrees of freedom: 982

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	5.90929128627493	990	0.0059689810972474		
Model	5.90929128627493	8	0.738661410784366	Inf	0
Residual	0	982	0		

for Q(x) A = 0.340000, B = 0.188156, C = 0.134060

for D(x) A = 0.038964, B = 0.100000, C = 0.136088

order Q(x) = 3.000000, order D(x) = 5.000000

Xmin = 1.000000 and Xmax = 100.000000

Ymin = 0.010050 and Ymax = 1.644935

List of powers for Q(x): [0.662215, 0.905900, 1.136665]

List of powers for D(x): [0.275052, 0.452561, 0.620433, 0.783316, 0.943010]

Fitting trigamma(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.459799e-05

Corrected MSS of errors squared = 3.575764e-05

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 7

Fitting tigamma(x) in range (1.000000, 100.000000)

Shammas polynomial power for Q(x) is A+B*i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.452085508348813	0	Inf	0
x1	-1.50778965502243	0	-Inf	0
x2	1.22232262088866	0	Inf	0
x3	-0.166603513305423	0	-Inf	0
x4	492.639068154709	0	Inf	0
x5	0	0	NaN	NaN
x6	-1138.03676659134	0	-Inf	0
x7	961.360542653038	0	Inf	0
x8	-371.281092231491	0	-Inf	0
x9	56.3182405207787	0	Inf	0

Number of observations: 991, Error degrees of freedom: 982

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	5.90929128629055	990	0.00596898109726318		
Model	5.90929128629055	8	0.738661410786318	Inf	0
Residual	0	982	0		

for Q(x) A = 0.355467, B = 0.374642

for D(x) A = 0.540824, B = 0.854883

order Q(x) = 3.000000, order D(x) = 6.000000

Xmin = 1.000000 and Xmax = 100.000000

Ymin = 0.010050 and Ymax = 1.644935

List of powers for Q(x): [0.730108, 1.104750, 1.479391]

List of powers for D(x): [0.540824, 0.547844, 0.585126, 0.653146, 0.744876, 0.854270]

Fitting tgamma(x) in range (1.000000, 2.000000)

MSS of errors squared = 4.192906e-06

Corrected MSS of errors squared = 8.385813e-06

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 8

Fitting tgamma(x) in range (1.000000, 100.000000)

Shammas polynomial power for Q(x) is A+B/i

Shammas polynomial power for D(x) is A+B*log10(i)^4

Linear regression model:

$$y \sim 1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	1.02021441701546	0	Inf	0
x1	-0.000395677064541671	0	-Inf	0
x2	2.34540845233304	0	Inf	0
x3	-3.36514107293486	0	-Inf	0
x4	-1312.13856603448	0	-Inf	0
x5	0	0	NaN	NaN
x6	2210.49991076137	0	Inf	0
x7	0	0	NaN	NaN
x8	-2163.44446560421	0	-Inf	0
x9	1716.61608271327	0	Inf	0

x10 -450.533004916393 0 -Inf 0

Number of observations: 991, Error degrees of freedom: 982

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	5.90929128632926	990	0.00596898109730229		
Model	5.90929128632926	8	0.738661410791158	Inf	0
Residual	0	982	0		

for Q(x) A = 0.115157, B = 2.621637

for D(x) A = 0.909010, B = 0.410801

order Q(x) = 3.000000, order D(x) = 7.000000

Xmin = 1.000000 and Xmax = 100.000000

Ymin = 0.010050 and Ymax = 1.644935

List of powers for Q(x): [2.736794, 1.425976, 0.989036]

List of powers for D(x): [0.909010, 0.912383, 0.930298, 0.962985, 1.007064, 1.059632, 1.118547]

Fitting tgamma(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.118507e-06

Corrected MSS of errors squared = 2.237014e-06

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 9

Fitting tgamma(x) in range (1.000000, 100.000000)

Shammas polynomial power for Q(x) is A+B*log10(i)^4

Shammas polynomial power for D(x) is A+B*i

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	1.22186216886544e-15	0	Inf	0
x1	4.08362227941629e-16	0	Inf	0
x2	0	0	NaN	NaN
x3	0	0	NaN	NaN
x4	0.999999999999998	0	Inf	0
x5	0	0	NaN	NaN
x6	0	0	NaN	NaN

Number of observations: 991, Error degrees of freedom: 988

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	5.90929128627539	990	0.00596898109724787		
Model	5.90929128627539	2	2.95464564313769	Inf	0
Residual	0	988	0		

for Q(x) A = 2.200000, B = 0.000000

for D(x) A = 0.000000, B = 0.000000

order Q(x) = 3.000000, order D(x) = 3.000000

Xmin = 1.000000 and Xmax = 100.000000

Ymin = 0.010050 and Ymax = 1.644935

List of powers for Q(x): [2.200000, 2.200000, 2.200000]

List of powers for D(x): [0.000000, 0.000000, 0.000000]

Fitting tgamma(x) in range (1.000000, 2.000000)

MSS of errors squared = 1.696607e-02

Corrected MSS of errors squared = 3.393215e-02

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Combo Powers 10

Fitting trigamma(x) in range (1.000000, 100.000000)

Shammas polynomial power for Q(x) is $A+B*\log_{10}(i)^4$

Shammas polynomial power for D(x) is $A+B/i$

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.938865043204161	0	Inf	0
x1	-11128.7906617607	0	-Inf	0
x2	0	0	NaN	NaN
x3	17651.9640490543	0	Inf	0
x4	0	0	NaN	NaN
x5	-13709.1618897392	0	-Inf	0
x6	9085.23509234276	0	Inf	0
x7	-1900.18539195025	0	-Inf	0
x8	-0.78565474829003	0	-Inf	0
x9	-1.16087360626085	0	-Inf	0
x10	2.9464968204997	0	Inf	0

Number of observations: 991, Error degrees of freedom: 982

R-squared: 1, Adjusted R-Squared: 1

F-statistic vs. constant model: Inf, p-value = 0

	SumSq	DF	MeanSq	F	pValue
Total	5.90929128580366	990	0.00596898109677137		
Model	5.90929128580366	8	0.738661410725457	Inf	0
Residual	0	982	0		

for Q(x) A = 1.373559, B = 0.293574

for D(x) A = 0.442941, B = 1.175159

order Q(x) = 7.000000, order D(x) = 3.000000

Xmin = 1.000000 and Xmax = 100.000000

Ymin = 0.010050 and Ymax = 1.644935

List of powers for Q(x): [1.373559, 1.375970, 1.388773, 1.412132, 1.443632, 1.481199, 1.523302]

List of powers for D(x): [1.618100, 1.030520, 0.834660]

Fitting trigamma(x) in range (1.000000, 2.000000)

MSS of errors squared = 2.815769e-06

Corrected MSS of errors squared = 5.631538e-06

R-squared = 1.00000000

R-squared Adjusted = 1.00000000

Particle swarm AICc = -1.000000e+99

AIC = -Inf

AICc = -Inf

Comments

The following table shows the summary results for the ten models that fit the trigamma function. The results are sorted in ascending order using the MSSE values.

<i>Power Q(x)</i>	<i>Power D(x)</i>	<i>MSSE</i>	<i>Q/D Orders</i>
$A+B/i$	$A+B*i$	4.77997800E-07	5, 3
$A+B*i$	$A+B/i$	5.80790300E-07	6, 7
$A+B*\sqrt{i}$	$A+B*i$	6.42570000E-07	6, 4
$A+B*\sqrt{i}+C*i$	$A+B*i+C*\sqrt{i}$	7.30409700E-07	6, 4
$A+B/i$	$A+B*\log_{10}(i)^4$	1.11850700E-06	3, 7
$A+B*i$	$A+B*\sqrt{i}$	2.15627400E-06	7, 7
$A+B*\log_{10}(i)^4$	$A+B/i$	2.81576900E-06	7, 3
$A+B*i$	$A+B*\log_{10}(i)^4$	4.19290600E-06	3, 6
$A+B*i+C*\sqrt{i}$	$A+B*\sqrt{i}+C*i$	1.45979900E-05	3, 5
$A+B*\log_{10}(i)^4$	$A+B*i$	1.69660700E-02	3, 3

CONCLUSIONS

Overall, the Pade-Shammas polynomials did well in fitting many tested equations. The Pade-Shammas polynomials struggled in fitting the following equations:

- The cosine integral.
- The Bessel functions.
- The Fresnel cosine and sine integrals.

It is interesting to noticed that the more flexible scheme of the Pade-Shammas polynomials, presented in this report, did not do better in fitting the same difficult functions that appear in Part I and Part IB.

So how do the various Shammas Polynomial power combinations compare with each other. The next table shows the frequency of these powers in attaining the top 3 spots. The counts colored in red indicate the top of their placement.

<i>Power Q(x)</i>	<i>Power D(x)</i>	<i>Top 1</i>	<i>Top 2</i>	<i>Top 3</i>
$A+B*i$	$A+B/i$	1	2	3
$A+B*i$	$A+B*\sqrt{i}$	2	0	0
$A+B*\sqrt{i}+C*i$	$A+B*i+C*\sqrt{i}$	2	7	4
$A+B/i$	$A+B*i$	3	5	4
$A+B*\sqrt{i}$	$A+B*i$	10	3	4
$A+B*i+C*\sqrt{i}$	$A+B*\sqrt{i}+C*i$	2	3	3
$A+B*i$	$A+B*\log_{10}(i)^4$	1	2	4
$A+B/i$	$A+B*\log_{10}(i)^4$	6	8	6
$A+B*\log_{10}(i)^4$	$A+B*i$	4	0	1
$A+B*\log_{10}(i)^4$	$A+B/i$	1	1	4

The top rank goes to the power expressions $A+B*\sqrt{i}$ and $A+B*i$. The second, third, and general best ranks go to the oiwers $A+B/I$ and $A+B*\log_{10}(i)^4$.

Personally, I do recommend the scheme of the Pade-Shammas polynomials presented in this study over those in Part I and Part IB. It is a personal preference based on the notion that in this study the Pade-Shammas polynomials have more degrees of freedom in fitting functions.

DOCUMENT HISTORY

<i>Date</i>	<i>Version</i>	<i>Comments</i>
10/4/2020	1.00.00	Initial release.