Fourier-Shammas Series Approximations
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INTRODUCTION

This study looks at the collaboration of two mathematical frameworks—the Fourier series and the Shammas sequences that I have used in the Shammas polynomial approximations. The study combines both frameworks to approximate several common functions..

The Fourier Series

Fourier polynomials are polynomial ratios defined as:

$$y = a_0 + \sum_{i=1}^{n} a_i \sin(i \cdot \pi \cdot x) + \sum_{i=1}^{n} b_i \cos(i \cdot \pi \cdot x) \quad (1)$$

The multiplier i in the sine and cosine terms is an integer.

The Shammas Sequence

In the HHC 2008 conference in Corvallis, Oregon, I introduced the Shammas polynomials as polynomials with non-integer powers. I explained that the powers of such polynomial change using some math expressions that involves the polynomial term sequence number, possible transformations of the term sequence number, and constants. The sequences that I use in this study are:

Power = A + B*i (2)

Power = A + B/i (3)

Power = A+B*sqrt(i) (4)

Power = A + B * $[\log_{10}(i)]^4$ (5)

Where i is the number of term (1, 2, 3, and so on). A, B, and C are parameters that allow the above power expression to progress in smaller and non-integer increments.

> The sequences of regular polynomial powers and Fourier series are integers that are generally defined as:
>
> gx(i, A) = A*i
>
> Where A is typically 1 or 2 for polynomials and π for Fourier series. The Shammas sequences support progressions of smaller values that are not necessarily integers. The *general* forms of the Shammas sequences are:
>
> gx(i, A, B) < i, for i > 0 and gx(i, A, B) < gx(i+|Δi|, A, B)
>
> gx(i, A, B, C) < i, for i > 0 and gx(i, A, B, C) < gx(i+|Δi|, A, B, C)
>
> Where A is a constant that appears as the first term. The second term is constant B multiplied by a function of the sequence number i (as shown above). The third term, if used, is constant C multiplied by another function of i, or better yet i–1. These rules hold true except for A+B/i (or any other form that divides by i) where the sequence values decrease as i increases. Thus gx() can be:
>
> gx(i, A, B) = A + B*g(i)
>
> gx(i, A, B, C) = A + B*g_1(i) + C*g_2(i)
>
> gx(i, A, B, C) = A + B*g_1(i) + C*g_2(i–1)
>
> Where g(), g_1(), and g_2() are functions such as the square root, natural logarithm, common logarithm, reciprocal, and so on. The simplest Shammas sequence is:
>
> gx(i, A, B) = A + B* √i with A = 0 and B = 1.
> = √i

The Fourier-Shammas Series

The Fourier-Shammas polynomials combine the Fourier series and Shammas sequence concepts. The study uses the following multiple regression general form:

$$y = a_0 + \sum_{i=1}^{n} a_i \sin(S_i * gx_i(i, A_i, B_i) * x + Os_i) \qquad (6)$$

Equation 6 shows the Fourier-Shammas Sine Series.

$$y = a_0 + \sum_{i=1}^{n} a_i \cos(C_i * gx_i(i, A_i, B_i) * x + Oc_i) \quad (7)$$

Equation 7 shows the Fourier-Shammas Cosine Series.

$$y = a_0 + \sum_{i=1}^{n} a_i \sin(S_i * gx_i(i, As_i, Bs_i) * x + Os_i) + \sum_{i=1}^{m} a_i \cos(C_i * gx_i(i, Ac_i, Bc_i) * x + Oc_i) \quad (8)$$

Equation 8 shows the Fourier-Shammas Sine/Cosine Series.

OPTIMIZED PARAMETER SCHEMES

The next table lists the various Fourier-Shammas series.

Part	*Models*
Sine series for n=3 to 7	y = a(0) + a(1)*sin(C1*x*gx(1,A1,B1)+Os(1)) + … + a(n)*sin(Cn*x*gx(n,An,Bn)+Os(n))
Cosine Series for n=3 to 7	y = a(0) + a(1)*cos(C1*x*gx(1,A1,B1)+Os(1)) + … + a(n)*cos(Cn*x*gx(n,An,Bn)+Os(n))
Alternating Sine and cosine Series for n=3 to 7	y = a(0) + a(1)*sin(C1*x*gx(1,A1,B1)+Os(1)) + a(2)*cos(C2*x*gx(2,A2,B2)+Os(2)) + a(3)*sin(C3*x*gx(n,A3,B3)+Os(3)) +... + a(n)*sin(Cn*x*gx(n,An,Bn)+Os(n))
Alternating Cosine and sine Series for n = 7 ONLY	y = a(0) + a(1)*cos(C1*x*gx(1,A1,B1)+Os(1)) + a(2)*sin(C2*x*gx(2,A2,B2)+Os(2)) + a(3)*cos(C3*x*gx(n,A3,B3)+Os(3)) +... + a(7)*cos(Cn*x*gx(7,A7,B7)+Os(7))

In the above tables, the coefficients a(0) through a(n) are calculated as multiple regression coefficients. The rest of the coefficients (Cs, As, Bs, and the offset values Os are different for each term and are calculated based on optimization). The addition of an offset term inside each sine and cosine plus having different multipliers C(i) and gx parameters A(i) and B(i) gives the models in the above table more flexibility.

This study looks at Fourier-Shammas series used to approximate common functions that include:

- Trigonometric functions and their inverses.
- Hyperbolic functions.
- Logarithmic functions.
- Exponential functions.

- Bessel functions $J_0(x)$ to $J_5(x)$.
- The sine and cosine integrals.
- The Fresnel sine and cosine.
- Inverse student-t functions.
- The common logarithm of the gamma function.
- The digamma function.
- The trigamma function.

The digamma function is defined as:

$$\psi(x) = \Gamma'(x) / \Gamma(x) = \frac{d \ln(\Gamma(x))}{dx} \tag{9}$$

The following Matlab function implements the code for the digamma function:

```
function y = digamma(x)
%DIGAMMA Summary of this function goes here
%   Detailed explanation goes here
  h = 0.001;
  fp = gammaln(x+h);
  fm = gammaln(x-h);
  y = (fp -fm)/2/h;
end
```

The above implementation of the digamma function was suggested by Albert Chan, a member of the hp museum web site, in a post he wrote on that site. The above code gives slightly more accurate results than the expression (gamma(x+h)-gamma(x-h)/(2*h)/gamma(x).

The trigamma function is defined as:

$$\psi_1(x) = \frac{d^2}{dx^2} \ln(\Gamma(x)) \tag{10}$$

The following Matlab function implements the code for the trigamma function:

```
function y = trigamma(x)
%DIGAMMA Summary of this function goes here
%   Detailed explanation goes here
  h = 0.001;
  fp = gammaln(x+h);
  fm = gammaln(x-h);
  f0 = gammaln(x);
  y = (fp -2*f0 + fm)/h/h;
end
```

There are several choices that we can use to determine the best Fourier-Shammas series parameters C, A, B and O. The best one is to use an optimization function to determine the best values for parameters C, A, B, and Os, for a given Fourier-Shammas series order. The study uses the Matlab particle swarm optimization function to select the best values for Cs, As, Bs, and Os.

The approximations that I obtain are typically for a defined and suitable interval. It is your responsibility to implement expanded versions of the approximation functions that take wider ranges if arguments and map them onto the interval used. For example, given that my approximation for $\log_{10}(x)$ uses the range (1, 10), to calculate $\log_{10}(235)$ use:

$\log_{10}(235) =$

$\log_{10}(2.35 * 100) =$

$\log_{10}(2.35) + \log_{10}(100) =$

$\log_{10}(2.35) + 2$

The argument of the $\log_{10}(x)$ function in the last line falls in the interval (1, 10).

MATLAB CODE

The Sine Series

The algorithm in this study uses particle swarm optimization to obtain the best values for parameters Cs, As, Bs, and Os in a prespecified range of Fourier-Shammas series. The order of the series used is set in the caller routine using the global variable order.

This study uses the function FourierShammasSeries2() to perform various Fourier-Shammas series curve fitting:

```
function
FourierShammasSeries2(fx,gx,xRange,Lb,Ub,runNum,sFxName,diaryFilename)
% FourierShammasSeries2 implements the Fourier-Shammas Sine Series
% Model:
% y = a(0) + a(1)*sin(S(1)*gx(1,A1,B1)*x+Os1) +...+
%            a(n)*sin(S(n)*gx(n,An,Bn)*x+Osn)
clc
global bDeleteIfExists
global bUseDiary
global xdata
global ydata
global order
global ggx

warning('off','all')
if isempty(sFxName)
```

```
  sFxName = getFuncName(fx);
end
xdata = xRange';
ydata = xdata;
for i=1:length(xdata)
  ydata(i)=fx(xdata(i));
end
ggx = gx;

fprintf('Fitting %s in range (%f, %f)\n', sFxName, min(xdata),max(xdata));

options = optimoptions('particleswarm', 'Display', 'iter');
[x,psAICc] = particleswarm(@optimFunc,length(Lb),Lb,Ub,options);
if isinf(psAICc), psAICc = -1e+99; end
S = zeros(order,1);
As = zeros(order,1);
Bs = zeros(order,1);
Os = zeros(order,1);
S = x(1:order);
As = x(order+1:2*order);
Bs = x(2*order+1:3*order);
Os = x(3*order+1:4*order);

if bUseDiary
  diaryFilename = strrep(diaryFilename, ".txt", strcat("_",
num2str(order),"_sin_run", num2str(runNum),".txt"));
  if exist(diaryFilename, 'file')==2
    if bDeleteIfExists
      delete(diaryFilename);
    else
      return;
    end
  end
end
X = [];
for i=1:order
  g = S(i)*gx(i,As(i),Bs(i));
  xs = sin(xdata.*g + Os(i));
  X = [X;xs'];
end
X = X';
lm = fitlm(X,ydata);
if bUseDiary
  diary(diaryFilename)
end
fprintf('Fitting %s in range (%f, %f)\n', sFxName, min(xdata),max(xdata));
sp = getFuncName(gx);
fprintf('Fourier Shammas Series factor is %s\n', sp);
format long
disp(lm);
anva = anova(lm,'summary');
disp(anva);
format short
fprintf('Model is ----------------------------------\n')
fprintf("y = %e +\n", lm.Coefficients{1,1});
k=2;
for i=1:order
```

```
  g = S(i)*gx(i,As(i),Bs(i));
  if g >= 0 && Os(i) > 0
    fprintf("    %e * sin(%e * x + %f) + \n", lm.Coefficients{k,1}, g, Os(i));
  elseif g >= 0 && Os(i) < 0
    fprintf("    %e * sin(%e * x - %f) + \n", lm.Coefficients{k,1}, g, abs(Os(i)));
  elseif g >= 0 && Os(i) == 0
    fprintf("    %e * sin(%e * x) + \n", lm.Coefficients{k,1}, g);
  elseif g < 0 && Os(i) > 0
    fprintf("    %e * sin((%e) * x + %f) + \n", lm.Coefficients{k,1}, g, Os(i));
  elseif g < 0 && Os(i) == 0
    fprintf("    %e * sin((%e) * x) + \n", lm.Coefficients{k,1}, g);
  else % both g and Os(i) are negative
    fprintf("    %e * sin((%e) * x - %f) + \n", lm.Coefficients{k,1}, g, abs(Os(i)));
  end
  k = k + 1;
end
fprintf("\n");

lstFactor = [];
lstOffset = [];
for i=1:order
  g = S(i)*gx(i,As(i),Bs(i));
  lstFactor = [lstFactor,g];
  lstOffset = [lstOffset, Os(i)];
end
fprintf('List of factors: [');
for i=1:order-1
 fprintf('%f, ', lstFactor(i));
end
fprintf('%f]\n', lstFactor(order));
fprintf('List of offsets: [');
for i=1:order-1
 fprintf('%f, ', lstOffset(i));
end
fprintf('%f]\n', lstOffset(order));
fprintf('Fitting %s in range (%f, %f)\n', sFxName, min(xdata),max(xdata));
n = length(xdata);
sumsqr = 0;
for i=1:n
  yc = lm.Coefficients{1,1};
  for j=2:order+1 % length(lm.Coefficients{:,1})
    yc = yc + lm.Coefficients{j,1} * sin(xdata(i)*lstFactor(j-1)+Os(j-1));
  end
  sumsqr = sumsqr + (ydata(i) - yc)^2;
end
k = order + 1;
fprintf('MSS of errors squared = %e\n', sqrt(sumsqr)/n);
fprintf("R-Squared = %12.8f\n", lm.Rsquared.Ordinary);
fprintf("R-Squared Adjusted = %12.8f\n", lm.Rsquared.Adjusted);
AIC = lm.ModelCriterion.AIC;
AICc = AIC + 2*k*(1 + (k+1)/(n-k-1));
fprintf('Particle swarm AICc =  %e\n', psAICc);
fprintf('AIC =  %e\n', AIC);
fprintf('AICc =  %e\n', AICc);

if bUseDiary
  diary off
end
```

```
end

function AICc = optimFunc(x)
  global xdata
  global ydata
  global order
  global ggx

  S = zeros(order,1);
  As = zeros(order,1);
  Bs = zeros(order,1);
  Os = zeros(order,1);
  S = x(1:order);
  As = x(order+1:2*order);
  Bs = x(2*order+1:3*order);
  Os = x(3*order+1:4*order);
  X = [];
  for i=1:order
      g = S(i)*ggx(i,As(i),Bs(i));
    xs = sin(xdata*g + Os(i));
    X = [X;xs'];
  end
  X = X';
  lm = fitlm(X,ydata);

  n = length(xdata);
  k = order + 1;
  AIC = lm.ModelCriterion.AIC;
  AICc = AIC + 2*k*(1 + (k+1)/(n-k-1));
  if isinf(AICc), AICc = -1e+99; end
end

function sFx = getFuncName(fx)
  sFx = func2str(fx);
  if sFx(1:2)=="@("
    i = strfind(sFx,")");
    sFx = sFx(i(1)+1:end);
  elseif sFx(1)=="@"
    sFx = strcat(sFx(2:end),".m");
  else
    % return sFx as is
  end
end
```

The parameters of function FourierShammasSeries2() are:

- The parameter fx is the handle (or inline function) for the function being approximated. An example is @(x)cos(x) which also shows the ***recommended format*** for the argument of parameter fx.
- The parameter gx is the handle (or inline function) for the function that calculates the powers of the Shammas polynomial. An example is @(i,A,B)A+B*sqrt(i) which also shows the ***recommended format*** for the argument of parameter gx.

- The parameter xRange is the array that specifies the minimum value, increment value, and maximum value for the range of approximation.
- The parameter Lb is the array of lower limits for the parameters S, As, Bs, and Os.
- The parameter Ub is the array of upper limits for the parameters S, As, Bs, and Os.
- The parameter runNum specifies the run number. The arguments for this parameter have nothing to do with the calculations and serve in fine tuning the name of the diary files, when used.
- The optional parameter sFxName is the name of the approximated function. An example is "cos(x)".
- The parameter diaryFilename is the name of the diary file. An example is "cos_1.txt".

The above listing performs the following tasks:

1. Pass the order of the Fourier-Shammas series using the global variable order specified by the caller routine.
2. Initialize the data for the curve fitting. The function uses the global variables xdata and ydata to store the data for the series fitting.
3. Store the handle of function gx in the global handle ggx.
4. Set the optimization options and then call the Matlab function particleswarm(). The function call returns the optimized values of S, As, Bs, Os, and the optimum value for *corrected* Akaike information criterion (AICc). The arguments for this function call are:
    a. The handle to the local function optimFunc() that calculates the root mean sum of errors squared.
    b. The number of optimized variables which is equal to the number of elements in parameter Lb.
    c. The lower and upper bounds arrays, Lb and Ub, respectively,
    d. The optimization parameters for function particleswarm().
5. Retrieve the optimum values and perform a Fourier-Shammas seriesfit for the best values of S, As, Bs, and Os.
6. Assemble the data for the multiple regression of the model.
7. Perform the curve fitting of the nest model by calling function fitlm and passing it the arguments for the data matrix X and vector ydata.
8. Display the results of the regression and its associated ANOVA table.

9. Display the Fourier-Shammas series model. This is the form that you can use in predicting other values.
10. Calculate and display the list of Fourier-Shammas series factor.
11. Display the range of the approximated function.
12. Calculate and display the value of the mean square root of the sum of squared errors.
13. Display the coefficient of determination and its adjusted value. The latter statistic serves as a measure of goodness of model fitting.
14. Calculate and display the *corrected* Akaike information criterion. This statistic is calculated using:

AIC = n * ln(SSE/n) + 2*k (8)

AICc = AIC + 2*k*(k+1)/(n-k-1) (9)

Where n is the number of observations, k is the total number of regression coefficients (including the intercept), and SSE is the sum of squared errors. The program obtains the value of AIC using lm.ModelCriterion.AIC. The program uses equation (3) to calculate the value for AICc.

15. Close the diary file, if one is used.

The function optimFunc() obtains the array x containing the current values of the parameters for the Fourier-Shammas series. The function calculates the transformed variables needed to perform a curve fit for a Fourier-Shammas series. This task calls the Matlab function fitlm(). The optimFunc() function returns the AICc as its result. I am using this statistic since the optimization is dealing with different Shammas polynomial orders and thus a varying number of polynomial coefficients. One last thing to keep in mind. The optimization function uses an implementation of the Particle Swarm Optimization algorithm. This method uses random numbers to search for the optimum values. As such, the results can vary between different runs.

The function getFuncName() returns a string-type function name given a handle of a function. The best way to take advantage of this function is to supply arguments like @(x)cos(x) and @(x,A,B)A+B*sqrt(x). Such arguments allow the function to discard the part that declares the variable(s) and return the part that comes after the first closed parenthesis (e.g. cos(x) and A+B*sqrt(x) for the above examples). If you supply an argument like @fx1 which refers to the file fx1.m that defines the function fx1() then the function getFuncName() returns *fx1.m*. This string value indicates that you a rereferencing a separate Matlab file that implements the code for fx1.

	Programming Notes
	The above listing uses the variables S, As, Bs, and Os to represent the variables C, A, B, and Os that I showed in an earlier table of equations. I have renamed C as S and used the trailing letter s to signal that we are dealing with sine terms. The next listing which deals with cosine terms uses the variables C, Ac, Bc, and Oc to represent the variables C, A, B, and Os that I showed in an earlier table of equations. I use the trailing letter c to signal that we are dealing with cosine terms. The upcoming listing which deals with sine and cosine terms uses the variables S, As, Bs, and Os for sine terms and C, Ac, Bc, and Oc cosine terms. The conclusion to draw from the above comments that I have simplified the mathematical represetations of the series in the tables and equations that I presented earlier. In coding the various trigonometric series, I used a different naming scheme for the variables related to the sine and to the cosine terms. This scheme makes is easier to implement and read the code, especially one that involves both sine and cosine calculations.

The Cosine Series

The next listing shows a version of the FourierShamSeries2() coded to model Fourier-Shammas Cosine series.

```
function
FourierShammasSeries2(fx,gx,xRange,Lb,Ub,runNum,sFxName,diaryFilename)
% FourierShammasSeries2 implements the Fourier-Shammas Cosine Series
% Model:
% y = a(0) + a(1)*cos(C(1)*gx(1,A1,B1)*x+Os1) +...+
%            a(n)*cos(C(n)*gx(n,An,Bn)*x+Osn)
clc
global bDeleteIfExists
global bUseDiary
global xdata
global ydata
global order
global ggx

warning('off','all')
if isempty(sFxName)
```



```
  sFxName = getFuncName(fx);
end
xdata = xRange';
ydata = xdata;
for i=1:length(xdata)
  ydata(i)=fx(xdata(i));
end
ggx = gx;

fprintf('Fitting %s in range (%f, %f)\n', sFxName, min(xdata),max(xdata));

options = optimoptions('particleswarm', 'Display', 'iter');
[x,psAICc] = particleswarm(@optimFunc,length(Lb),Lb,Ub,options);
if isinf(psAICc), psAICc = -1e+99; end
C = zeros(order,1);
Ac = zeros(order,1);
Bc = zeros(order,1);
Oc = zeros(order,1);
C = x(1:order);
Ac = x(order+1:2*order);
Bc = x(2*order+1:3*order);
Oc = x(3*order+1:4*order);

if bUseDiary
  diaryFilename = strrep(diaryFilename, ".txt", strcat("_",
num2str(order),"_cos_run", num2str(runNum),".txt"));
  if exist(diaryFilename, 'file')==2
    if bDeleteIfExists
      delete(diaryFilename);
    else
      return;
    end
  end
end
X = [];
for i=1:order
  g = C(i)*gx(i,Ac(i),Bc(i));
  xs = cos(xdata.*g + Oc(i));
  X = [X;xs'];
end
X = X';
lm = fitlm(X,ydata);
if bUseDiary
  diary(diaryFilename)
end
fprintf('Fitting %s in range (%f, %f)\n', sFxName, min(xdata),max(xdata));
sp = getFuncName(gx);
fprintf('Fourier Shammas Series factor is %s\n', sp);
format long
disp(lm);
anva = anova(lm,'summary');
disp(anva);
format short
fprintf('Model is ----------------------------------\n')
fprintf("y = %e +\n", lm.Coefficients{1,1});
k=2;
for i=1:order
```

```
  g = C(i)*gx(i,Ac(i),Bc(i));
  if g >= 0 && Oc(i) > 0
    fprintf("    %e * cos(%e * x + %f) + \n", lm.Coefficients{k,1}, g, Oc(i));
  elseif g >= 0 && Oc(i) < 0
    fprintf("    %e * cos(%e * x - %f) + \n", lm.Coefficients{k,1}, g, abs(Oc(i)));
  elseif g >= 0 && Oc(i) == 0
    fprintf("    %e * cos(%e * x) + \n", lm.Coefficients{k,1}, g);
  elseif g < 0 && Oc(i) > 0
    fprintf("    %e * cos((%e) * x + %f) + \n", lm.Coefficients{k,1}, g, Oc(i));
  elseif g < 0 && Oc(i) == 0
    fprintf("    %e * cos((%e) * x) + \n", lm.Coefficients{k,1}, g);
  else % both g and Oc(i) are negative
    fprintf("    %e * cos((%e) * x - %f) + \n", lm.Coefficients{k,1}, g, abs(Oc(i)));
  end
  k = k + 1;
end
fprintf("\n");

lstFactor = [];
lstOffset = [];
for i=1:order
  g = C(i)*gx(i,Ac(i),Bc(i));
  lstFactor = [lstFactor,g];
  lstOffset = [lstOffset, Oc(i)];
end
fprintf('List of factors: [');
for i=1:order-1
 fprintf('%f, ', lstFactor(i));
end
fprintf('%f]\n', lstFactor(order));
fprintf('List of offsets: [');
for i=1:order-1
 fprintf('%f, ', lstOffset(i));
end
fprintf('%f]\n', lstOffset(order));
fprintf('Fitting %s in range (%f, %f)\n', sFxName, min(xdata),max(xdata));
n = length(xdata);
sumsqr = 0;
for i=1:n
  yc = lm.Coefficients{1,1};
  for j=2:order+1 % length(lm.Coefficients{:,1})
    yc = yc + lm.Coefficients{j,1} * cos(xdata(i)*lstFactor(j-1)+Oc(j-1));
  end
  sumsqr = sumsqr + (ydata(i) - yc)^2;
end
k = order + 1;
fprintf('MSS of errors squared = %e\n', sqrt(sumsqr)/n);
fprintf("R-Squared = %12.8f\n", lm.Rsquared.Ordinary);
fprintf("R-Squared Adjusted = %12.8f\n", lm.Rsquared.Adjusted);
AIC = lm.ModelCriterion.AIC;
AICc = AIC + 2*k*(1 + (k+1)/(n-k-1));
fprintf('Particle swarm AICc =  %e\n', psAICc);
fprintf('AIC =  %e\n', AIC);
fprintf('AICc =  %e\n', AICc);

if bUseDiary
  diary off
end
```

```
end

function AICc = optimFunc(x)
  global xdata
  global ydata
  global order
  global ggx

  C = zeros(order,1);
  Ac = zeros(order,1);
  Bc = zeros(order,1);
  Oc = zeros(order,1);
  C = x(1:order);
  Ac = x(order+1:2*order);
  Bc = x(2*order+1:3*order);
  Oc = x(3*order+1:4*order);
  X = [];
  for i=1:order
      g = C(i)*ggx(i,Ac(i),Bc(i));
    xs = cos(xdata*g + Oc(i));
    X = [X;xs'];
  end
  X = X';
  lm = fitlm(X,ydata);

  n = length(xdata);
  k = order + 1;
  AIC = lm.ModelCriterion.AIC;
  AICc = AIC + 2*k*(1 + (k+1)/(n-k-1));
  if isinf(AICc), AICc = -1e+99; end
end

function sFx = getFuncName(fx)
  sFx = func2str(fx);
  if sFx(1:2)=="@("
    i = strfind(sFx,")");
    sFx = sFx(i(1)+1:end);
  elseif sFx(1)=="@"
    sFx = strcat(sFx(2:end),".m");
  else
    % return sFx as is
  end
end
```

The above listing is very similar to the one before it. It simply replaces reference of sine function with the cosine function and renames the variables S, As, Bs, and Os.

The Alternating Sine/Cosine Series

The next listing shows the version of FourierShammasSeries2() for the Fourier-Shammas alternating sine/cosine series.

```
function
FourierShammasSeries2(fx,gx,xRange,Lb,Ub,runNum,sFxName,diaryFilename)
%FourierShammasSeries2 implements the Fourier-Shammas Sine/Cosine series
% Model:
```

```
% y = a(0) + a(1)*sin(S1*gx(1,A1,B1)+Os1) + b(2)*cos(C1*gx(2,A2,B2)*x+Oc1) +
%        ... +  a(n)*sin(Sn*gx(n,An,Bn)+Osn)
clc
global bDeleteIfExists
global bUseDiary
global xdata
global ydata
global order
global ggx

warning('off','all')
if isempty(sFxName)
  sFxName = getFuncName(fx);
end
xdata = xRange';
ydata = xdata;
for i=1:length(xdata)
  ydata(i)=fx(xdata(i));
end
ggx = gx;

fprintf('Fitting %s in range (%f, %f)\n', sFxName, min(xdata),max(xdata));

options = optimoptions('particleswarm', 'Display', 'iter');
[x,psAICc] = particleswarm(@optimFunc,length(Lb),Lb,Ub,options);
if isinf(psAICc), psAICc = -1e+99; end
S = zeros(order,1);
As = zeros(order,1);
Bs = zeros(order,1);
Os = zeros(order,1);
S = x(1:order);
As = x(order+1:2*order);
Bs = x(2*order+1:3*order);
Os = x(3*order+1:4*order);

if bUseDiary
  diaryFilename = strrep(diaryFilename, ".txt", strcat("_",
num2str(order),"_run", num2str(runNum),".txt"));
  if exist(diaryFilename, 'file')==2
    if bDeleteIfExists
      delete(diaryFilename);
    else
      return;
    end
  end
end
X = [];
flag = 1;
for i=1:order
  g = S(i)*gx(i,As(i),Bs(i));
  if flag > 0
    xs = sin(xdata.*g + Os(i));
  else
      xs = cos(xdata.*g + Os(i));
  end
  flag = 1 - flag;
  X = [X;xs'];
```

```
end
X = X';
lm = fitlm(X,ydata);
if bUseDiary
  diary(diaryFilename)
end
fprintf('Fitting %s in range (%f, %f)\n', sFxName, min(xdata),max(xdata));
sp = getFuncName(gx);
fprintf('Fourier Shammas Series factor is %s\n', sp);
format long
disp(lm);
anva = anova(lm,'summary');
disp(anva);
format short
fprintf('Model is ----------------------------------\n')
fprintf("y = %e +\n", lm.Coefficients{1,1});
k=2;
flag = 1;
for i=1:order
  g = S(i)*gx(i,As(i),Bs(i));
  if flag > 0
        if g >= 0 && Os(i) > 0
          fprintf("    %e * sin(%e * x + %f) + \n", lm.Coefficients{k,1}, g, Os(i));
        elseif g >= 0 && Os(i) < 0
          fprintf("    %e * sin(%e * x - %f) + \n", lm.Coefficients{k,1}, g, abs(Os(i)));
        elseif g >= 0 && Os(i) == 0
          fprintf("    %e * sin(%e * x) + \n", lm.Coefficients{k,1}, g);
        elseif g < 0 && Os(i) > 0
          fprintf("    %e * sin((%e) * x + %f) + \n", lm.Coefficients{k,1}, g, Os(i));
        elseif g < 0 && Os(i) == 0
          fprintf("    %e * sin((%e) * x) + \n", lm.Coefficients{k,1}, g);
        else % both g and Os(i) are negative
          fprintf("    %e * sin((%e) * x - %f) + \n", lm.Coefficients{k,1}, g, abs(Os(i)));
        end
  else
        if g >= 0 && Os(i) > 0
          fprintf("    %e * cos(%e * x + %f) + \n", lm.Coefficients{k,1}, g, Os(i));
        elseif g >= 0 && Os(i) < 0
          fprintf("    %e * cos(%e * x - %f) + \n", lm.Coefficients{k,1}, g, abs(Os(i)));
        elseif g >= 0 && Os(i) == 0
          fprintf("    %e * cos(%e * x) + \n", lm.Coefficients{k,1}, g);
        elseif g < 0 && Os(i) > 0
          fprintf("    %e * cos((%e) * x + %f) + \n", lm.Coefficients{k,1}, g, Os(i));
        elseif g < 0 && Os(i) == 0
          fprintf("    %e * cos((%e) * x) + \n", lm.Coefficients{k,1}, g);
        else % both g and Os(i) are negative
          fprintf("    %e * cos((%e) * x - %f) + \n", lm.Coefficients{k,1}, g, abs(Os(i)));
        end
  end
  flag = 1 - flag;
end
fprintf("\n");

lstFactor = [];
lstOffset = [];
for i=1:order
  g = S(i)*gx(i,As(i),Bs(i));
  lstFactor = [lstFactor,g];
  lstOffset = [lstOffset, Os(i)];
end
fprintf('List of factors: [');
for i=1:order-1
```



```
 fprintf('%f, ', lstFactor(i));
end
fprintf('%f]\n', lstFactor(order));
fprintf('List of offsets: [');
for i=1:order-1
 fprintf('%f, ', lstOffset(i));
end
fprintf('%f]\n', lstOffset(order));
fprintf('Fitting %s in range (%f, %f)\n', sFxName, min(xdata),max(xdata));
n = length(xdata);
sumsqr = 0;
for i=1:n
  flag = 1;
  yc = lm.Coefficients{1,1};
  for j=2:order+1 % length(lm.Coefficients{:,1})
      if flag > 0
      yc = yc + lm.Coefficients{j,1} * sin(xdata(i)*lstFactor(j-1)+Os(j-1));
    else
      yc = yc + lm.Coefficients{j,1} * cos(xdata(i)*lstFactor(j-1)+Os(j-1));
    end
    flag = 1 - flag;
  end
  sumsqr = sumsqr + (ydata(i) - yc)^2;
end
k = order + 1;
fprintf('MSS of errors squared = %e\n', sqrt(sumsqr)/n);
fprintf("R-Squared = %12.8f\n", lm.Rsquared.Ordinary);
fprintf("R-Squared Adjusted = %12.8f\n", lm.Rsquared.Adjusted);
AIC = lm.ModelCriterion.AIC;
AICc = AIC + 2*k*(1 + (k+1)/(n-k-1));
fprintf('Particle swarm AICc =  %e\n', psAICc);
fprintf('AIC =  %e\n', AIC);
fprintf('AICc =  %e\n', AICc);

if bUseDiary
  diary off
end
end

function AICc = optimFunc(x)
  global xdata
  global ydata
  global order
  global ggx

  S = zeros(order,1);
  As = zeros(order,1);
  Bs = zeros(order,1);
  Os = zeros(order,1);
  S = x(1:order);
  As = x(order+1:2*order);
  Bs = x(2*order+1:3*order);
  Os = x(3*order+1:4*order);
  X = [];
  flag = 1;
  for i=1:order
      g = S(i)*ggx(i,As(i),Bs(i));
```

```
    if flag > 0
      xs = sin(xdata*g + Os(i));
    else
      xs = cos(xdata*g + Os(i));
    end
    flag = 1 - flag;
    X = [X;xs'];
  end
  X = X';
  lm = fitlm(X,ydata);

  n = length(xdata);
  k = order + 1;
  AIC = lm.ModelCriterion.AIC;
  AICc = AIC + 2*k*(1 + (k+1)/(n-k-1));
  if isinf(AICc), AICc = -1e+99; end
end

function sFx = getFuncName(fx)
  sFx = func2str(fx);
  if sFx(1:2)=="@("
    i = strfind(sFx,")");
    sFx = sFx(i(1)+1:end);
  elseif sFx(1)=="@"
    sFx = strcat(sFx(2:end),".m");
  else
    % return sFx as is
  end
end
```

The above listing resembles the two listings before it. The code uses the variable flag to alternate between working with the sine and cosine functions.

The Driver Script

The following Matlab script goAll performs the various Fourier-Shammas series fittings for the various tested functions. All series use this script.

```
% Version 1.0.0 10/5/2020
global bUseDiary
global bDeleteIfExists
global order

bUseDiary = true;
bDeleteIfExists = false; % or false
selIdx = 0; % note a zero value will execute all the models
runNum = 1;
bShutdown = false;
order = 3;

tic;

% build lower limit values
lstC = zeros(order,1);
for i=1:order
  if i>1
```

```
    lstC(i) = (i-1)*pi;
  else
    lstC(1) = 0.1;
  end
end
lstAs = zeros(order,1);
lstBs = 0.1 + zeros(order,1);
lstOs = -2 + zeros(order,1);
% Lb = [0.1 pi  2*pi 0 0 0 0.1 0.1 0.1 -2 -2 -2];
Lb = [lstC; lstAs; lstBs; lstOs]';
% build upper limit values
lstC = zeros(order,1);
for i=1:order
  lstC(i) = i*pi;
end
lstAs = 1 + zeros(order,1);
lstBs = 3 + zeros(order,1);
lstOs = 2 + zeros(order,1);
% Ub = [pi 2*pi 3*pi 1 1 1 3 3 3 2 2 2];
Ub = [lstC; lstAs; lstBs; lstOs]';

if selIdx==0 || selIdx==1
  gx = @(i,A,B)A+B*i;

FourierShammasSeries2(@(x)acos(x),gx,[0:.01:1],Lb,Ub,runNum,"arccos(x)","arcc
os_1.txt")

FourierShammasSeries2(@(x)asin(x),gx,[0:.01:1],Lb,Ub,runNum,"arcsin(x)","arcs
in_1.txt")

FourierShammasSeries2(@(x)atan(x),gx,[0:.01:1],Lb,Ub,runNum,"arctan(x)","arct
an_1.txt")

FourierShammasSeries2(@(x)tan(x),gx,[0:.01:1],Lb,Ub,runNum,"tan(x)","tan_1.tx
t")

FourierShammasSeries2(@(x)sinh(x),gx,[0:.01:5],Lb,Ub,runNum,"sinh(x)","sinh_1
.txt")

FourierShammasSeries2(@(x)cosh(x),gx,[0:.01:5],Lb,Ub,runNum,"cosh(x)","cosh_1
.txt")

FourierShammasSeries2(@(x)tanh(x),gx,[0:.01:3],Lb,Ub,runNum,"tanh(x)","tanh_1
.txt")

FourierShammasSeries2(@(x)erf(x),gx,[0:.01:2.1],Lb,Ub,runNum,"erf(x)","erf_1.
txt")

FourierShammasSeries2(@(x)exp(x),gx,[0:.01:2],Lb,Ub,runNum,"exp(x)","exp_1.tx
t")

FourierShammasSeries2(@(x)log(x),gx,[1:.01:10],Lb,Ub,runNum,"ln(x)","ln_1.txt
")

FourierShammasSeries2(@(x)log10(x),gx,[1:.01:10],Lb,Ub,runNum,"log(x)","log_1
.txt")
```

```
FourierShammasSeries2(@(x)10.^x,gx,[0:.01:1],Lb,Ub,runNum,"10^x","pwr10_1.txt
")

FourierShammasSeries2(@(x)tinv(0.95,x),gx,[2:0.1:100],Lb,Ub,runNum,"tinv(0.95
,x)","tinv1_1.txt")

FourierShammasSeries2(@(x)tinv(0.975,x),gx,[2:0.1:100],Lb,Ub,runNum,"tinv(0.9
75,x)","tinv2_1.txt")

FourierShammasSeries2(@(x)log10(gamma(x)),gx,[2:0.1:100],Lb,Ub,runNum,"log10G
amma(x)","log10Gamma_1.txt")

FourierShammasSeries2(@(x)digamma(x),gx,[2:0.1:100],Lb,Ub,runNum,"digamma(x)"
,"digamma_1.txt")

FourierShammasSeries2(@(x)trigamma(x),gx,[1:0.1:100],Lb,Ub,runNum,"trigamma(x
)","trigamma_1.txt")

FourierShammasSeries2(@(x)besselj(0,x),gx,[2:0.1:30],Lb,Ub,runNum,"J0(x)","J0
x_1.txt")

FourierShammasSeries2(@(x)besselj(1,x),gx,[0:0.1:30],Lb,Ub,runNum,"J1(x)","J1
x_1.txt")

FourierShammasSeries2(@(x)besselj(2,x),gx,[0:0.1:30],Lb,Ub,runNum,"J2(x)","J2
x_1.txt")

FourierShammasSeries2(@(x)besselj(3,x),gx,[0:0.1:30],Lb,Ub,runNum,"J3(x)","J3
x_1.txt")

FourierShammasSeries2(@(x)besselj(4,x),gx,[0:0.1:30],Lb,Ub,runNum,"J4(x)","J4
x_1.txt")

FourierShammasSeries2(@(x)besselj(5,x),gx,[0:0.1:30],Lb,Ub,runNum,"J5(x)","J5
x_1.txt")

FourierShammasSeries2(@(x)integral(@(z)sin(z.^2),0,x),gx,[1:0.01:5],Lb,Ub,run
Num,"FresnelSine(x)","FresnelSine_1.txt")

FourierShammasSeries2(@(x)integral(@(z)cos(z.^2),0,x),gx,[0.5:0.01:5],Lb,Ub,r
unNum,"FresnelCosine(x)","FresnelCosine_1.txt")

FourierShammasSeries2(@(x)integral(@(z)sin(z)./z,0,x),gx,[1.3:0.1:20],Lb,Ub,r
unNum,"Si(x)","Si_1.txt")
  FourierShammasSeries2(@(x)0.57721566+log(x)-integral(@(z)(1-
cos(z))./z,0,x),gx,[0.5:0.1:20],Lb,Ub,runNum,"Ci(x)","CI_1.txt")

FourierShammasSeries2(@(x)asinh(x),gx,[0:0.1:100],Lb,Ub,runNum,"asinh(x)","as
inh_1.txt")

FourierShammasSeries2(@(x)acosh(x),gx,[1:0.1:100],Lb,Ub,runNum,"acosh(x)","ac
osh_1.txt")

FourierShammasSeries2(@(x)atanh(x),gx,[0:0.001:0.999],Lb,Ub,runNum,"atanh(x)"
,"atanh_1.txt")
end
```

```
if selIdx==0 || selIdx==2
  gx = @(i,A,B)A+B/i;

FourierShammasSeries2(@(x)acos(x),gx,[0:.01:1],Lb,Ub,runNum,"arccos(x)","arcc
os_2.txt")

FourierShammasSeries2(@(x)asin(x),gx,[0:.01:1],Lb,Ub,runNum,"arcsin(x)","arcs
in_2.txt")

FourierShammasSeries2(@(x)atan(x),gx,[0:.01:1],Lb,Ub,runNum,"arctan(x)","arct
an_2.txt")

FourierShammasSeries2(@(x)tan(x),gx,[0:.01:1],Lb,Ub,runNum,"tan(x)","tan_2.tx
t")

FourierShammasSeries2(@(x)sinh(x),gx,[0:.01:5],Lb,Ub,runNum,"sinh(x)","sinh_2
.txt")

FourierShammasSeries2(@(x)cosh(x),gx,[0:.01:5],Lb,Ub,runNum,"cosh(x)","cosh_2
.txt")

FourierShammasSeries2(@(x)tanh(x),gx,[0:.01:3],Lb,Ub,runNum,"tanh(x)","tanh_2
.txt")

FourierShammasSeries2(@(x)erf(x),gx,[0:.01:2.1],Lb,Ub,runNum,"erf(x)","erf_2.
txt")

FourierShammasSeries2(@(x)exp(x),gx,[0:.01:2],Lb,Ub,runNum,"exp(x)","exp_2.tx
t")

FourierShammasSeries2(@(x)log(x),gx,[1:.01:10],Lb,Ub,runNum,"ln(x)","ln_2.txt
")

FourierShammasSeries2(@(x)log10(x),gx,[1:.01:10],Lb,Ub,runNum,"log(x)","log_2
.txt")

FourierShammasSeries2(@(x)10.^x,gx,[0:.01:1],Lb,Ub,runNum,"10^x","pwr10_2.txt
")

FourierShammasSeries2(@(x)tinv(0.95,x),gx,[2:0.1:100],Lb,Ub,runNum,"tinv(0.95
,x)","tinv1_2.txt")

FourierShammasSeries2(@(x)tinv(0.975,x),gx,[2:0.1:100],Lb,Ub,runNum,"tinv(0.9
75,x)","tinv2_2.txt")

FourierShammasSeries2(@(x)log10(gamma(x)),gx,[2:0.1:100],Lb,Ub,runNum,"log10G
amma(x)","log10Gamma_2.txt")

FourierShammasSeries2(@(x)digamma(x),gx,[2:0.1:100],Lb,Ub,runNum,"digamma(x)"
,"diamma_2.txt")

FourierShammasSeries2(@(x)trigamma(x),gx,[1:0.1:100],Lb,Ub,runNum,"trigamma(x
)","trigamma_2.txt")

FourierShammasSeries2(@(x)trigamma(x),gx,[1:0.1:100],Lb,Ub,runNum,"trigamma(x
)","trigamma_2.txt")
```



```
FourierShammasSeries2(@(x)besselj(0,x),gx,[2:0.1:30],Lb,Ub,runNum,"J0(x)","J0
x_2.txt")

FourierShammasSeries2(@(x)besselj(1,x),gx,[0:0.1:30],Lb,Ub,runNum,"J1(x)","J1
x_2.txt")

FourierShammasSeries2(@(x)besselj(2,x),gx,[0:0.1:30],Lb,Ub,runNum,"J2(x)","J2
x_2.txt")

FourierShammasSeries2(@(x)besselj(3,x),gx,[0:0.1:30],Lb,Ub,runNum,"J3(x)","J3
x_2.txt")

FourierShammasSeries2(@(x)besselj(4,x),gx,[0:0.1:30],Lb,Ub,runNum,"J4(x)","J4
x_2.txt")

FourierShammasSeries2(@(x)besselj(5,x),gx,[0:0.1:30],Lb,Ub,runNum,"J5(x)","J5
x_2.txt")

FourierShammasSeries2(@(x)integral(@(z)sin(z.^2),0,x),gx,[1:0.01:5],Lb,Ub,run
Num,"FresnelSine(x)","FresnelSine_2.txt")

FourierShammasSeries2(@(x)integral(@(z)cos(z.^2),0,x),gx,[0.5:0.01:5],Lb,Ub,r
unNum,"FresnelCosine(x)","FresnelCosine_2.txt")

FourierShammasSeries2(@(x)integral(@(z)sin(z)./z,0,x),gx,[1.3:0.1:20],Lb,Ub,r
unNum,"Si(x)","Si_2.txt")
  FourierShammasSeries2(@(x)0.57721566+log(x)-integral(@(z)(1-
cos(z))./z,0,x),gx,[0.5:0.1:20],Lb,Ub,runNum,"Ci(x)","Ci_2.txt")

FourierShammasSeries2(@(x)asinh(x),gx,[0:0.1:100],Lb,Ub,runNum,"asinh(x)","as
inh_2.txt")

FourierShammasSeries2(@(x)acosh(x),gx,[1:0.1:100],Lb,Ub,runNum,"acosh(x)","ac
osh_2.txt")

FourierShammasSeries2(@(x)atanh(x),gx,[0:0.001:0.999],Lb,Ub,runNum,"atanh(x)"
,"atanh_2.txt")
end

if selIdx==0 || selIdx==3
  gx = @(i,A,B)A+B*sqrt(i);

FourierShammasSeries2(@(x)acos(x),gx,[0:.01:1],Lb,Ub,runNum,"arccos(x)","arcc
os_3.txt")

FourierShammasSeries2(@(x)asin(x),gx,[0:.01:1],Lb,Ub,runNum,"arcsin(x)","arcs
in_3.txt")

FourierShammasSeries2(@(x)atan(x),gx,[0:.01:1],Lb,Ub,runNum,"arctan(x)","arct
an_3.txt")

FourierShammasSeries2(@(x)tan(x),gx,[0:.01:1],Lb,Ub,runNum,"tan(x)","tan_3.tx
t")

FourierShammasSeries2(@(x)sinh(x),gx,[0:.01:5],Lb,Ub,runNum,"sinh(x)","sinh_3
.txt")
```

```
FourierShammasSeries2(@(x)cosh(x),gx,[0:.01:5],Lb,Ub,runNum,"cosh(x)","cosh_3
.txt")

FourierShammasSeries2(@(x)tanh(x),gx,[0:.01:3],Lb,Ub,runNum,"tanh(x)","tanh_3
.txt")

FourierShammasSeries2(@(x)erf(x),gx,[0:.01:2.1],Lb,Ub,runNum,"erf(x)","erf_3.
txt")

FourierShammasSeries2(@(x)exp(x),gx,[0:.01:2],Lb,Ub,runNum,"exp(x)","exp_3.tx
t")

FourierShammasSeries2(@(x)log(x),gx,[1:.01:10],Lb,Ub,runNum,"ln(x)","ln_3.txt
")

FourierShammasSeries2(@(x)log10(x),gx,[1:.01:10],Lb,Ub,runNum,"log(x)","log_3
.txt")

FourierShammasSeries2(@(x)10.^x,gx,[0:.01:1],Lb,Ub,runNum,"10^x","pwr10_3.txt
")

FourierShammasSeries2(@(x)tinv(0.95,x),gx,[2:0.1:100],Lb,Ub,runNum,"tinv(0.95
,x)","tinv1_3.txt")

FourierShammasSeries2(@(x)tinv(0.975,x),gx,[2:0.1:100],Lb,Ub,runNum,"tinv(0.9
75,x)","tinv2_3.txt")

FourierShammasSeries2(@(x)log10(gamma(x)),gx,[2:0.1:100],Lb,Ub,runNum,"log10G
amma(x)","log10Gamma_3.txt")

FourierShammasSeries2(@(x)digamma(x),gx,[2:0.1:100],Lb,Ub,runNum,"digamma(x)"
,"digamma_3.txt")

FourierShammasSeries2(@(x)trigamma(x),gx,[1:0.1:100],Lb,Ub,runNum,"trigamma(x
)","trigamma_3.txt")

FourierShammasSeries2(@(x)besselj(0,x),gx,[2:0.1:30],Lb,Ub,runNum,"J0(x)","J0
x_3.txt")

FourierShammasSeries2(@(x)besselj(1,x),gx,[0:0.1:30],Lb,Ub,runNum,"J1(x)","J1
x_3.txt")

FourierShammasSeries2(@(x)besselj(2,x),gx,[0:0.1:30],Lb,Ub,runNum,"J2(x)","J2
x_3.txt")

FourierShammasSeries2(@(x)besselj(3,x),gx,[0:0.1:30],Lb,Ub,runNum,"J3(x)","J3
x_3.txt")

FourierShammasSeries2(@(x)besselj(4,x),gx,[0:0.1:30],Lb,Ub,runNum,"J4(x)","J4
x_3.txt")

FourierShammasSeries2(@(x)besselj(5,x),gx,[0:0.1:30],Lb,Ub,runNum,"J5(x)","J5
x_3.txt")

FourierShammasSeries2(@(x)integral(@(z)sin(z.^2),0,x),gx,[1:0.01:5],Lb,Ub,run
Num,"FresnelSine(x)","FresnelSine_3.txt")
```



```
FourierShammasSeries2(@(x)integral(@(z)cos(z.^2),0,x),gx,[0.5:0.01:5],Lb,Ub,r
unNum,"FresnelCosine(x)","FresnelCosine_3.txt")

FourierShammasSeries2(@(x)integral(@(z)sin(z)./z,0,x),gx,[1.3:0.1:20],Lb,Ub,r
unNum,"Si(x)","Si_3.txt")
  FourierShammasSeries2(@(x)0.57721566+log(x)-integral(@(z)(1-
cos(z))./z,0,x),gx,[0.5:0.1:20],Lb,Ub,runNum,"Ci(x)","Ci_3.txt")

FourierShammasSeries2(@(x)asinh(x),gx,[0:0.1:100],Lb,Ub,runNum,"asinh(x)","as
inh_3.txt")

FourierShammasSeries2(@(x)acosh(x),gx,[1:0.1:100],Lb,Ub,runNum,"acosh(x)","ac
osh_3.txt")

FourierShammasSeries2(@(x)atanh(x),gx,[0:0.001:0.999],Lb,Ub,runNum,"atanh(x)"
,"atanh_3.txt")
end

if selIdx==0 || selIdx==4
  gx = @(i,A,B)A+B*log(i)^4;

FourierShammasSeries2(@(x)acos(x),gx,[0:.01:1],Lb,Ub,runNum,"arccos(x)","arcc
os_4.txt")

FourierShammasSeries2(@(x)asin(x),gx,[0:.01:1],Lb,Ub,runNum,"arcsin(x)","arcs
in_4.txt")

FourierShammasSeries2(@(x)atan(x),gx,[0:.01:1],Lb,Ub,runNum,"arctan(x)","arct
an_4.txt")

FourierShammasSeries2(@(x)tan(x),gx,[0:.01:1],Lb,Ub,runNum,"tan(x)","tan_4.tx
t")

FourierShammasSeries2(@(x)sinh(x),gx,[0:.01:5],Lb,Ub,runNum,"sinh(x)","sinh_4
.txt")

FourierShammasSeries2(@(x)cosh(x),gx,[0:.01:5],Lb,Ub,runNum,"cosh(x)","cosh_4
.txt")

FourierShammasSeries2(@(x)tanh(x),gx,[0:.01:3],Lb,Ub,runNum,"tanh(x)","tanh_4
.txt")

FourierShammasSeries2(@(x)erf(x),gx,[0:.01:2.1],Lb,Ub,runNum,"erf(x)","erf_4.
txt")

FourierShammasSeries2(@(x)exp(x),gx,[0:.01:2],Lb,Ub,runNum,"exp(x)","exp_4.tx
t")

FourierShammasSeries2(@(x)log(x),gx,[1:.01:10],Lb,Ub,runNum,"ln(x)","ln_4.txt
")

FourierShammasSeries2(@(x)log10(x),gx,[1:.01:10],Lb,Ub,runNum,"log(x)","log_4
.txt")

FourierShammasSeries2(@(x)10.^x,gx,[0:.01:1],Lb,Ub,runNum,"10^x","pwr10_4.txt
")
```

```
FourierShammasSeries2(@(x)tinv(0.95,x),gx,[2:0.1:100],Lb,Ub,runNum,"tinv(0.95
,x)","tinv1_4.txt")

FourierShammasSeries2(@(x)tinv(0.975,x),gx,[2:0.1:100],Lb,Ub,runNum,"tinv(0.9
75,x)","tinv2_4.txt")

FourierShammasSeries2(@(x)log10(gamma(x)),gx,[2:0.1:100],Lb,Ub,runNum,"log10G
amma(x)","log10Gamma_4.txt")

FourierShammasSeries2(@(x)digamma(x),gx,[2:0.1:100],Lb,Ub,runNum,"digamma(x)"
,"digamma_4.txt")

FourierShammasSeries2(@(x)trigamma(x),gx,[1:0.1:100],Lb,Ub,runNum,"tigamma(x)
","trigamma_4.txt")

FourierShammasSeries2(@(x)besselj(0,x),gx,[2:0.1:30],Lb,Ub,runNum,"J0(x)","J0
x_4.txt")

FourierShammasSeries2(@(x)besselj(1,x),gx,[0:0.1:30],Lb,Ub,runNum,"J1(x)","J1
x_4.txt")

FourierShammasSeries2(@(x)besselj(2,x),gx,[0:0.1:30],Lb,Ub,runNum,"J2(x)","J2
x_4.txt")

FourierShammasSeries2(@(x)besselj(3,x),gx,[0:0.1:30],Lb,Ub,runNum,"J3(x)","J3
x_4.txt")

FourierShammasSeries2(@(x)besselj(4,x),gx,[0:0.1:30],Lb,Ub,runNum,"J4(x)","J4
x_4.txt")

FourierShammasSeries2(@(x)besselj(5,x),gx,[0:0.1:30],Lb,Ub,runNum,"J5(x)","J5
x_4.txt")

FourierShammasSeries2(@(x)integral(@(z)sin(z.^2),0,x),gx,[1:0.01:5],Lb,Ub,run
Num,"FresnelSine(x)","FresnelSine_4.txt")

FourierShammasSeries2(@(x)integral(@(z)cos(z.^2),0,x),gx,[0.5:0.01:5],Lb,Ub,r
unNum,"FresnelCosine(x)","FresnelCosine_4.txt")

FourierShammasSeries2(@(x)integral(@(z)sin(z)./z,0,x),gx,[1.3:0.1:20],Lb,Ub,r
unNum,"Si(x)","Si_4.txt")
  FourierShammasSeries2(@(x)0.57721566+log(x)-integral(@(z)(1-
cos(z))./z,0,x),gx,[0.5:0.1:20],Lb,Ub,runNum,"Ci(x)","CI_4.txt")

FourierShammasSeries2(@(x)asinh(x),gx,[0:0.1:100],Lb,Ub,runNum,"asinh(x)","as
inh_4.txt")

FourierShammasSeries2(@(x)acosh(x),gx,[1:0.1:100],Lb,Ub,runNum,"acosh(x)","ac
osh_4.txt")

FourierShammasSeries2(@(x)atanh(x),gx,[0:0.001:0.999],Lb,Ub,runNum,"atanh(x)"
,"atanh_4.txt")
end


toc;
```

```
for i=1:7
  beep;
  pause(3)
end

if bShutdown
  system('shutdown -s');
else
  fprintf("\n\nDone!\n\n");
end
```

The above listing has the following global and operational variabls:

- The global variable order which is key in selecting the order of the Fourier-Shammas series being used. The version of FoutrierShammasSeries2() called by the above script, determines which Fourier-Shammas series is evaluated.
- The global variable bUseDiary is a Boolean flag used to tell the function FourierShammasSeries2() whether you want to copy the screen output to diary text files.
- The global variable bDeleteIfExists is a Boolean flag used to tell the function FourierShammasSeries2() whether you want to delete diary files if they exist.
- The variable selIdx allows you to select calculations for one of the seven groups (when set to the targeted group number) or all of the groups (when set to 0). I am using this scheme to reduce calculation time which can be done by working with a specific set of approximations.
- The Boolean variable bShutdown tells the Matlab script whether to shut down the computer when done.

General Comments on Results

The quality of the curve fitting varies between very good and (honestly) somewhat disappointing. Certain functions lend themselves for very good fitting. Others offer moderelty good fitting, and some seem to be hostile towards yielding good approximations.

To prevent the page count for this report from getting out of hand, I will start with presenting the main summaries that indicate the best type of Fourier-Shammas series. This will be followed by the summary for each of the three sets of series (each set has 5 versions). Finally, I will present sample output text to give you an idea of how to use the curve fitting models. The code ZIP file associated with this study has folders for the 16 Fourier-Shammas series along with the Matlab files used to generate these output text files. The ZIP file will have 16 folders, each with 120 output text files.

Summary Tables

The following table gives a summary for the performance of the various Fourier-Shammas series. The table shows the count of the number of models with the Adjusted R^2 of 1 and the count of the same statistic with values between 0.9999 and 1 (excluded). The last column calculates a weighted value using:

Wt = count_R2_Adj_1 + 0.8 * count_R2_Adj_0.9999

My goal is to pick the series and order with the maximum weighted sum. The results answer the following two questions:

- What is the best Fourier-Shammas series?
- What is the best order? Could lower orders excel in offering better model fittings?!!

Series	*Order*	*# Rsqr Adj =1*	*# Rsqr Adj > 0.9999 And < 1*	*Weighted Sum*
Sine	3	4	32	29.6
Sine	4	11	27	32.6
Sine	5	15	22	32.6
Sine	6	21	18	35.4
Sine	7	22	22	39.6
Cosine	3	6	33	32.4
Cosine	4	13	25	33.0
Cosine	5	14	26	34.8
Cosine	6	22	18	36.4
Cosine	7	24	20	40.0
Sine/Cosine	3	6	32	31.6
Sine/Cosine	4	13	24	32.2
Sine/Cosine	5	15	22	32.6
Sine/Cosine	6	22	17	35.6
Sine/Cosine	7	22	19	37.2
Cosine/Sine	7	22	19	.37.2

The above table shows that the best Fourier-Shammas series is the Cosine series with order 7. The second rank goes to the 7^{th} order Since/Cosine series and 7^{th} Cosine/Sine series.

The statistics in the above table (and other statistics, like the mean square root of errors squared and AICc) are tabulated in the next subsections, given for each series and order.

The tables in the next subsections provide you with summaries for the performances of the various Fourier-Shammas series and also offer you catalogs for the detailed results found in the output text files that you download in the accompanying ZIP file. Each subsection tells you what folder to look in and the general formats for the output text files. I am resorting to this approach to avoid ending up with a study that has a huge and intimidating page count. The ZIP file also contains the file *Fourier-Shammas Series Maps.pdf* which has tables that guide you to easily select output text files to view. The map file has tables for the various series with the following columns:

- Filename: indicates the output file name.
- Function: indicates the approximated function.
- The gx(i,A,B) expression.
- The Rsquare adjusted value.

Sine Series of Order 3

The next table shows a summary of results for the Sine series of the order 3:

$$Y = a_0 + a_1 * \sin(S_1 * gx(1,A_1,B_1) + Os_1) + a_2 * \sin(S_2 * gx(2,A_2,B_2) + Os_2) + a_3 * \sin(S_3 * gx(3,A_3,B_3) + Os_1) + a_4 * x + a_5 * x^2$$

The output text files for this series are located in the following folder:

Fourier-Shammas Series Approximations 3 Sine

The files are named using the following general format:

*fxName_n*_3_sin_run1.txt

Where fxName is the function name and n is the gx series number which varies between 1 and 4. Each output files specifies the function being approximated and the gx series being used. For example, the file for the inverse hyperboplic cosine (acosh) using the first gx series is named acosh_1_3_sin_run1.txt.

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
10^x	A+B*i	3.650636E-04	0.99999783	-8.297493E+02
10^x	A+B/i	1.964545E-04	0.99999937	-9.549166E+02
10^x	A+B*sqrt(i)	1.218638E-04	0.99999976	-1.051377E+03
10^x	A+B*log(i)^4	2.287565E-04	0.99999915	-9.241668E+02
acosh(x)	A+B*i	3.728649E-03	0.98296927	-1.417765E+03
acosh(x)	A+B/i	5.929624E-03	0.95692902	-4.982859E+02
acosh(x)	A+B*sqrt(i)	7.325893E-03	0.93425668	-7.918344E+01
acosh(x)	A+B*log(i)^4	4.343471E-03	0.97688977	-1.115257E+03
arccos(x)	A+B*i	1.251775E-03	0.99889253	-5.808355E+02
arccos(x)	A+B/i	1.180311E-03	0.99901538	-5.927100E+02
arccos(x)	A+B*sqrt(i)	1.241006E-03	0.99891151	-5.825808E+02
arccos(x)	A+B*log(i)^4	1.465164E-03	0.99848277	-5.490398E+02
arcsin(x)	A+B*i	1.249810E-03	0.99889601	-5.811528E+02
arcsin(x)	A+B/i	1.228734E-03	0.99893293	-5.845883E+02
arcsin(x)	A+B*sqrt(i)	1.189671E-03	0.9989997	-5.911144E+02
arcsin(x)	A+B*log(i)^4	1.128416E-03	0.99910005	-6.017925E+02
arctan(x)	A+B*i	4.289763E-07	1	-2.192527E+03
arctan(x)	A+B/i	6.125296E-07	1	-2.120576E+03
arctan(x)	A+B*sqrt(i)	2.791716E-07	1	-2.279301E+03
arctan(x)	A+B*log(i)^4	6.182260E-07	1	-2.118706E+03
asinh(x)	A+B*i	8.664614E-03	0.91831306	2.659058E+02
asinh(x)	A+B/i	7.695876E-03	0.93555781	2.854307E+01
asinh(x)	A+B*sqrt(i)	8.503923E-03	0.92131484	2.284287E+02
asinh(x)	A+B*log(i)^4	5.066971E-03	0.97206485	-8.081755E+02
atanh(x)	A+B*i	2.403210E-03	0.98265958	-2.300227E+03
atanh(x)	A+B/i	2.323776E-03	0.98378695	-2.367451E+03
atanh(x)	A+B*sqrt(i)	2.184100E-03	0.98567743	-2.491431E+03
atanh(x)	A+B*log(i)^4	2.241982E-03	0.98490823	-2.439118E+03
Ci(x)	A+B*i	2.519973E-03	0.93814279	-7.385910E+02
Ci(x)	A+B/i	2.420171E-03	0.94294538	-7.544317E+02
Ci(x)	A+B*sqrt(i)	2.796403E-03	0.92382752	-6.977894E+02
Ci(x)	A+B*log(i)^4	2.784964E-03	0.92444941	-6.993962E+02
cosh(x)	A+B*i	1.011948E-02	0.99984546	-5.010293E+01
cosh(x)	A+B/i	2.640371E-03	0.99998948	-1.396333E+03
cosh(x)	A+B*sqrt(i)	4.981615E-03	0.99996255	-7.602284E+02
cosh(x)	A+B*log(i)^4	4.903006E-03	0.99996372	-7.761660E+02
digamma(x)	A+B/i	5.045139E-03	0.96570853	-8.199775E+02
digamma(x)	A+B*i	2.402561E-03	0.99222342	-2.275566E+03
digamma(x)	A+B*sqrt(i)	6.051704E-03	0.95066045	-4.630611E+02
digamma(x)	A+B*log(i)^4	3.076170E-03	0.98725146	-1.790657E+03
erf(x)	A+B*i	3.872006E-06	0.99999996	-3.514625E+03
erf(x)	A+B/i	1.234115E-05	0.99999962	-3.025457E+03
erf(x)	A+B*sqrt(i)	6.394082E-06	0.9999999	-3.302950E+03
erf(x)	A+B*log(i)^4	6.065930E-06	0.99999991	-3.325183E+03
exp(x)	A+B*i	1.427644E-05	0.99999999	-2.832492E+03
exp(x)	A+B/i	5.823746E-05	0.99999979	-2.267313E+03
exp(x)	A+B*sqrt(i)	1.851662E-05	0.99999998	-2.727949E+03
exp(x)	A+B*log(i)^4	8.422550E-05	0.99999955	-2.118987E+03
FresnelCosine(x)	A+B*i	4.187338E-03	0.66832414	-8.868283E+02

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
FresnelCosine(x)	A+B/i	3.134863E-03	0.81410198	-1.147939E+03
FresnelCosine(x)	A+B*sqrt(i)	3.461316E-03	0.77336851	-1.058584E+03
FresnelCosine(x)	A+B*log(i)^4	3.433973E-03	0.77693497	-1.065738E+03
FresnelSine(x)	A+B*i	2.079201E-03	0.90489518	-1.395301E+03
FresnelSine(x)	A+B/i	2.489055E-03	0.86370537	-1.251005E+03
FresnelSine(x)	A+B*sqrt(i)	2.460819E-03	0.86678004	-1.260155E+03
FresnelSine(x)	A+B*log(i)^4	2.753628E-03	0.83319074	-1.169990E+03
J0(x)	A+B*i	1.467033E-03	0.97837334	-1.268810E+03
J0(x)	A+B/i	1.389346E-03	0.98060318	-1.299388E+03
J0(x)	A+B*sqrt(i)	7.730987E-04	0.99399408	-1.628822E+03
J0(x)	A+B*log(i)^4	1.426338E-03	0.97955654	-1.284620E+03
J1(x)	A+B*i	1.763597E-03	0.97720182	-1.228744E+03
J1(x)	A+B/i	2.962641E-03	0.93566317	-9.164715E+02
J1(x)	A+B*sqrt(i)	4.227374E-03	0.86900856	-7.024605E+02
J1(x)	A+B*log(i)^4	2.018297E-03	0.97014124	-1.147535E+03
J2(x)	A+B*i	2.603286E-03	0.94048907	-9.943141E+02
J2(x)	A+B/i	3.261118E-03	0.90661313	-8.586862E+02
J2(x)	A+B*sqrt(i)	4.236840E-03	0.84237068	-7.011141E+02
J2(x)	A+B*log(i)^4	3.254730E-03	0.90697861	-8.598665E+02
J3(x)	A+B*i	2.260215E-03	0.94910696	-1.079385E+03
J3(x)	A+B/i	3.345068E-03	0.8885273	-8.433852E+02
J3(x)	A+B*sqrt(i)	3.566303E-03	0.87329458	-8.048316E+02
J3(x)	A+B*log(i)^4	3.221795E-03	0.89659188	-8.659892E+02
J4(x)	A+B*i	2.725287E-03	0.91898376	-9.667429E+02
J4(x)	A+B/i	2.723055E-03	0.91911645	-9.672363E+02
J4(x)	A+B*sqrt(i)	3.983260E-03	0.82692878	-7.382678E+02
J4(x)	A+B*log(i)^4	2.833569E-03	0.91241795	-9.432870E+02
J5(x)	A+B*i	2.866672E-03	0.89996624	-9.362950E+02
J5(x)	A+B/i	3.066667E-03	0.88552155	-8.956964E+02
J5(x)	A+B*sqrt(i)	4.205416E-03	0.78471767	-7.055957E+02
J5(x)	A+B*log(i)^4	2.873269E-03	0.89950532	-9.349112E+02
ln(x)	A+B*i	5.215181E-04	0.99929085	-4.917967E+03
ln(x)	A+B/i	1.632302E-04	0.99993053	-7.011139E+03
ln(x)	A+B*sqrt(i)	1.328824E-04	0.99995396	-7.381805E+03
ln(x)	A+B*log(i)^4	2.300778E-04	0.99986198	-6.392592E+03
log(x)	A+B*i	1.978143E-04	0.99945907	-6.664853E+03
log(x)	A+B/i	7.430183E-05	0.99992368	-8.429360E+03
log(x)	A+B*sqrt(i)	1.039335E-04	0.99985067	-7.824581E+03
log(x)	A+B*log(i)^4	1.042741E-04	0.99984969	-7.818685E+03
log10Gamma(x)	A+B*i	5.408374E-03	0.99998687	-6.835730E+02
log10Gamma(x)	A+B/i	1.020537E-02	0.99995327	5.622292E+02
log10Gamma(x)	A+B*sqrt(i)	1.246785E-02	0.99993025	9.550985E+02
log10Gamma(x)	A+B*log(i)^4	1.246637E-02	0.99993026	9.548656E+02
Si(x)	A+B*i	2.533154E-03	0.89710847	-7.136466E+02
Si(x)	A+B/i	2.572534E-03	0.89388447	-7.078462E+02
Si(x)	A+B*sqrt(i)	1.841681E-03	0.9456142	-8.335102E+02
Si(x)	A+B*log(i)^4	1.186835E-03	0.9774141	-9.987203E+02
sinh(x)	A+B*i	9.912705E-03	0.99985385	-7.078916E+01
sinh(x)	A+B/i	1.069236E-02	0.99982995	5.073989E+00

Function	*gx(i,A,B)*	*MSSE*	*RsqrAdj*	*AICc*
sinh(x)	A+B*sqrt(i)	4.648007E-03	0.99996787	-8.296828E+02
sinh(x)	A+B*log(i)^4	5.187896E-03	0.99995997	-7.195733E+02
tan(x)	A+B*i	1.742191E-05	0.99999983	-1.444303E+03
tan(x)	A+B/i	7.733481E-05	0.9999966	-1.143239E+03
tan(x)	A+B*sqrt(i)	1.333828E-04	0.99998988	-1.033133E+03
tan(x)	A+B*log(i)^4	4.343075E-05	0.99999893	-1.259788E+03
tanh(x)	A+B*i	3.514412E-05	0.9999951	-3.585968E+03
tanh(x)	A+B/i	3.193752E-05	0.99999595	-3.643565E+03
tanh(x)	A+B*sqrt(i)	2.557276E-05	0.9999974	-3.777361E+03
tanh(x)	A+B*log(i)^4	7.082569E-05	0.99998008	-3.164107E+03
tinv(0.95,x)	A+B*i	1.980666E-03	0.76001523	-2.654432E+03
tinv(0.95,x)	A+B/i	2.386681E-03	0.65154237	-2.288577E+03
tinv(0.95,x)	A+B*sqrt(i)	2.727625E-03	0.54487513	-2.026596E+03
tinv(0.95,x)	A+B*log(i)^4	2.090300E-03	0.7327125	-2.548730E+03
tinv(0.975,x)	A+B*i	3.491477E-03	0.75435429	-1.542190E+03
tinv(0.975,x)	A+B/i	4.259093E-03	0.63446835	-1.152280E+03
tinv(0.975,x)	A+B*sqrt(i)	4.884982E-03	0.51914227	-8.832711E+02
tinv(0.975,x)	A+B*log(i)^4	3.831027E-03	0.70425253	-1.360101E+03
trigamma(x)	A+B*i	2.403639E-03	0.639675	-2.287986E+03
trigamma(x)	A+B/i	2.771742E-03	0.52086107	-2.005567E+03
trigamma(x)	A+B*sqrt(i)	3.083125E-03	0.40715937	-1.794548E+03
trigamma(x)	A+B*log(i)^4	2.814781E-03	0.50586566	-1.975027E+03

Sine Series of Order 4

The next table shows a summary of results for the Sine series of the order 4:

$$Y = a_0 + a_1 * \sin(S_1 * gx(1,A_1,B_1) + Os_1) + a_2 * \sin(S_2 * gx(2,A_2,B_2) + Os_2) + a_3 * \sin(S_3 * gx(3,A_3,B_3) + Os_3) + a_4 * \sin(S_4 * gx(4,A_4,B_4) + Os_4) + a_5*x + a_6*x^2$$

The output text files for this series are located in the following folder:

Fourier-Shammas Series Approximations 4 Sine

The files are named using the following general format:

*fxName_n_*4_sin_run1.txt

Where fxName is the function name and n is the gx series number which varies between 1 and 4. Each output files specifies the function being approximated and the gx series being used. For example, the file for the inverse hyperboplic cosine (acosh) using the first gx series is named acosh_1_4_sin_run1.txt.

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
10^x	A+B*i	9.043120E-06	1	-1.572544E+03
10^x	A+B/i	6.462105E-06	1	-1.640426E+03
10^x	A+B*sqrt(i)	4.526152E-06	1	-1.712355E+03
10^x	A+B*log(i)^4	8.588422E-06	1	-1.582965E+03
acosh(x)	A+B*i	3.497101E-03	0.9850036	-1.540814E+03
acosh(x)	A+B/i	5.983841E-03	0.9560933	-4.762255E+02
acosh(x)	A+B*sqrt(i)	4.169541E-03	0.97868197	-1.192237E+03
acosh(x)	A+B*log(i)^4	4.261202E-03	0.97773437	-1.149137E+03
arccos(x)	A+B*i	1.015259E-03	0.99926391	-6.189232E+02
arccos(x)	A+B/i	1.078793E-03	0.9991689	-6.066619E+02
arccos(x)	A+B*sqrt(i)	1.012764E-03	0.99926752	-6.194203E+02
arccos(x)	A+B*log(i)^4	9.784468E-04	0.99931632	-6.263836E+02
arcsin(x)	A+B*i	9.926567E-04	0.99929632	-6.234711E+02
arcsin(x)	A+B/i	1.079283E-03	0.99916814	-6.065702E+02
arcsin(x)	A+B*sqrt(i)	1.028584E-03	0.99924446	-6.162893E+02
arcsin(x)	A+B*log(i)^4	9.733682E-04	0.9993234	-6.274348E+02
arctan(x)	A+B*i	3.416003E-08	1	-2.699442E+03
arctan(x)	A+B/i	2.363417E-07	1	-2.308729E+03
arctan(x)	A+B*sqrt(i)	1.909205E-07	1	-2.351840E+03
arctan(x)	A+B*log(i)^4	4.547888E-08	1	-2.641631E+03
asinh(x)	A+B*i	3.140243E-03	0.9892597	-1.761998E+03
asinh(x)	A+B/i	7.727923E-03	0.93495475	4.088268E+01
asinh(x)	A+B*sqrt(i)	4.494867E-03	0.97799489	-1.044009E+03
asinh(x)	A+B*log(i)^4	2.744043E-03	0.99179891	-2.032004E+03
atanh(x)	A+B*i	1.911448E-03	0.98901912	-2.754096E+03
atanh(x)	A+B/i	1.966635E-03	0.98837589	-2.697170E+03
atanh(x)	A+B*sqrt(i)	1.866305E-03	0.98953167	-2.801897E+03
atanh(x)	A+B*log(i)^4	1.938602E-03	0.98870491	-2.725884E+03
Ci(x)	A+B*i	2.298660E-03	0.94826122	-7.705179E+02
Ci(x)	A+B/i	3.875977E-03	0.85289449	-5.657092E+02
Ci(x)	A+B*sqrt(i)	2.876832E-03	0.91896075	-6.825676E+02
Ci(x)	A+B*log(i)^4	2.643297E-03	0.93158388	-7.157554E+02
cosh(x)	A+B*i	4.925866E-03	0.99996331	-7.674645E+02
cosh(x)	A+B/i	1.210210E-03	0.99999779	-2.173978E+03
cosh(x)	A+B*sqrt(i)	1.191031E-03	0.99999785	-2.189985E+03
cosh(x)	A+B*log(i)^4	3.982644E-03	0.99997601	-9.804437E+02
digamma(x)	A+B/i	3.933029E-03	0.97913882	-1.304525E+03
digamma(x)	A+B*i	2.369396E-03	0.99242889	-2.298818E+03
digamma(x)	A+B*sqrt(i)	2.513286E-03	0.9914814	-2.183146E+03
digamma(x)	A+B*log(i)^4	3.613107E-03	0.98239459	-1.470985E+03
erf(x)	A+B*i	7.173444E-07	1	-4.222007E+03
erf(x)	A+B/i	2.533263E-06	0.99999998	-3.689566E+03
erf(x)	A+B*sqrt(i)	2.130573E-06	0.99999999	-3.762622E+03
erf(x)	A+B*log(i)^4	8.261862E-06	0.99999983	-3.190702E+03
exp(x)	A+B*i	1.224479E-05	0.99999999	-2.890100E+03
exp(x)	A+B/i	2.394690E-06	1	-3.546102E+03
exp(x)	A+B*sqrt(i)	8.600825E-06	1	-3.032103E+03
exp(x)	A+B*log(i)^4	1.191818E-05	0.99999999	-2.900968E+03
FresnelCosine(x)	A+B*i	3.119545E-03	0.81550145	-1.148312E+03

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
FresnelCosine(x)	A+B/i	2.600159E-03	0.87182293	-1.312579E+03
FresnelCosine(x)	A+B*sqrt(i)	2.103821E-03	0.91608728	-1.503639E+03
FresnelCosine(x)	A+B*log(i)^4	1.806604E-03	0.938122	-1.641019E+03
FresnelSine(x)	A+B*i	1.300943E-03	0.96267313	-1.767303E+03
FresnelSine(x)	A+B/i	2.130962E-03	0.89984869	-1.371529E+03
FresnelSine(x)	A+B*sqrt(i)	1.268664E-03	0.96450246	-1.787453E+03
FresnelSine(x)	A+B*log(i)^4	1.756451E-03	0.93195801	-1.526538E+03
J0(x)	A+B*i	1.258547E-03	0.98402579	-1.350883E+03
J0(x)	A+B/i	1.608089E-03	0.97392042	-1.213143E+03
J0(x)	A+B*sqrt(i)	3.353144E-03	0.88660742	-8.001563E+02
J0(x)	A+B*log(i)^4	5.851834E-04	0.99654646	-1.781256E+03
J1(x)	A+B*i	2.162480E-03	0.96560696	-1.101928E+03
J1(x)	A+B/i	1.001233E-03	0.99262714	-1.565483E+03
J1(x)	A+B*sqrt(i)	1.567273E-03	0.98193428	-1.295723E+03
J1(x)	A+B*log(i)^4	1.951108E-03	0.97200187	-1.163849E+03
J2(x)	A+B*i	2.582753E-03	0.94122625	-9.950129E+02
J2(x)	A+B/i	2.577683E-03	0.94145677	-9.961957E+02
J2(x)	A+B*sqrt(i)	2.436157E-03	0.94770884	-1.030190E+03
J2(x)	A+B*log(i)^4	1.655593E-03	0.97584959	-1.262720E+03
J3(x)	A+B*i	3.660991E-03	0.86602591	-7.849883E+02
J3(x)	A+B/i	1.454539E-03	0.97885177	-1.340661E+03
J3(x)	A+B*sqrt(i)	3.641938E-03	0.8674168	-7.881295E+02
J3(x)	A+B*log(i)^4	2.259820E-03	0.94895289	-1.075422E+03
J4(x)	A+B*i	2.851345E-03	0.91101606	-9.354541E+02
J4(x)	A+B/i	4.685248E-03	0.75974228	-6.364840E+02
J4(x)	A+B*sqrt(i)	4.368240E-03	0.79115443	-6.786592E+02
J4(x)	A+B*log(i)^4	2.694229E-03	0.92055232	-9.695746E+02
J5(x)	A+B*i	2.028666E-03	0.94973375	-1.140382E+03
J5(x)	A+B/i	2.536252E-03	0.92143302	-1.005950E+03
J5(x)	A+B*sqrt(i)	3.240810E-03	0.87171897	-8.583785E+02
J5(x)	A+B*log(i)^4	1.613877E-03	0.9681876	-1.278083E+03
ln(x)	A+B*i	5.153454E-04	0.99930677	-4.935401E+03
ln(x)	A+B/i	9.281012E-05	0.99997752	-8.024536E+03
ln(x)	A+B*sqrt(i)	7.261562E-05	0.99998624	-8.466703E+03
ln(x)	A+B*log(i)^4	2.361032E-04	0.99985449	-6.341985E+03
log(x)	A+B*i	2.063847E-04	0.99941052	-6.584403E+03
log(x)	A+B/i	3.278410E-05	0.99998513	-9.899719E+03
log(x)	A+B*sqrt(i)	1.748568E-04	0.99957687	-6.883128E+03
log(x)	A+B*log(i)^4	2.073443E-04	0.99940503	-6.576044E+03
log10Gamma(x)	A+B*i	5.426091E-03	0.99998678	-6.731358E+02
log10Gamma(x)	A+B/i	9.768494E-03	0.99995714	4.804092E+02
log10Gamma(x)	A+B*sqrt(i)	1.235367E-02	0.99993145	9.410689E+02
log10Gamma(x)	A+B*log(i)^4	1.829054E-03	0.9999985	-2.806654E+03
Si(x)	A+B*i	2.157699E-03	0.92494064	-7.698545E+02
Si(x)	A+B/i	1.288204E-03	0.97324575	-9.637926E+02
Si(x)	A+B*sqrt(i)	1.426087E-03	0.96721198	-9.255591E+02
Si(x)	A+B*log(i)^4	8.039579E-04	0.98957947	-1.141061E+03
sinh(x)	A+B*i	1.573593E-02	0.99963096	3.963048E+02
sinh(x)	A+B/i	1.501051E-03	0.99999664	-1.958176E+03

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
sinh(x)	A+B*sqrt(i)	5.169535E-03	0.99996017	-7.190852E+02
sinh(x)	A+B*log(i)^4	4.650189E-03	0.99996777	-8.251718E+02
tan(x)	A+B*i	1.789514E-05	0.99999982	-1.434674E+03
tan(x)	A+B/i	1.008257E-05	0.99999994	-1.550566E+03
tan(x)	A+B*sqrt(i)	2.062156E-05	0.99999976	-1.406029E+03
tan(x)	A+B*log(i)^4	1.460621E-05	0.99999988	-1.475697E+03
tanh(x)	A+B*i	3.883111E-05	0.99999399	-3.521841E+03
tanh(x)	A+B/i	5.990715E-06	0.99999986	-4.646986E+03
tanh(x)	A+B*sqrt(i)	3.540908E-06	0.99999995	-4.963534E+03
tanh(x)	A+B*log(i)^4	2.148949E-05	0.99999816	-3.878019E+03
tinv(0.95,x)	A+B*i	1.871619E-03	0.78549334	-2.761519E+03
tinv(0.95,x)	A+B/i	1.974633E-03	0.76123043	-2.656396E+03
tinv(0.95,x)	A+B*sqrt(i)	2.019112E-03	0.75035267	-2.612693E+03
tinv(0.95,x)	A+B*log(i)^4	1.459389E-03	0.86957873	-3.249636E+03
tinv(0.975,x)	A+B*i	3.538373E-03	0.74745276	-1.511993E+03
tinv(0.975,x)	A+B/i	3.960120E-03	0.6836614	-1.291057E+03
tinv(0.975,x)	A+B*sqrt(i)	4.931780E-03	0.5093828	-8.605442E+02
tinv(0.975,x)	A+B*log(i)^4	3.662313E-03	0.72945077	-1.444445E+03
trigamma(x)	A+B*i	2.449255E-03	0.62548944	-2.246704E+03
trigamma(x)	A+B/i	2.548156E-03	0.59463315	-2.168244E+03
trigamma(x)	A+B*sqrt(i)	2.423372E-03	0.63336312	-2.267761E+03
trigamma(x)	A+B*log(i)^4	2.132943E-03	0.71597636	-2.520778E+03

Sine Series of Order 5

The next table shows a summary of results for the Sine series of the order 5:

$$Y = a_0 + a_1 * \sin(S_1 * gx(1,A_1,B_1) + Os_1) + \ldots + a_5 * \sin(S_5 * gx(5,A_5,B_5) + Os_5)$$

The output text files for this series are located in the following folder:

Fourier-Shammas Series Approximations 5 Sine

The files are named using the following general format:

*fxName_n*_5_sin_run1.txt

Where fxName is the function name and n is the gx series number which varies between 1 and 4. Each output files specifies the function being approximated and the gx series being used. For example, the file for the inverse hyperboplic cosine (acosh) using the first gx series is named acosh_1_5_sin_run1.txt.

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
10^x	A+B*i	5.783943E-06	1	-1.658559E+03
10^x	A+B/i	1.789720E-06	1	-1.895510E+03
10^x	A+B*sqrt(i)	1.547546E-07	1	-2.390001E+03
10^x	A+B*log(i)^4	2.365784E-06	1	-1.839143E+03
acosh(x)	A+B*i	4.744579E-03	0.97236835	-9.321439E+02
acosh(x)	A+B/i	5.416084E-03	0.9639934	-6.697861E+02
acosh(x)	A+B*sqrt(i)	4.390504E-03	0.97633861	-1.085865E+03
acosh(x)	A+B*log(i)^4	2.789303E-03	0.99045001	-1.985004E+03
arccos(x)	A+B*i	8.340451E-04	0.999498	-6.543767E+02
arccos(x)	A+B/i	8.226809E-04	0.99951159	-6.571479E+02
arccos(x)	A+B*sqrt(i)	8.803522E-04	0.99944071	-6.434617E+02
arccos(x)	A+B*log(i)^4	8.198109E-04	0.99951499	-6.578539E+02
arcsin(x)	A+B*i	7.704855E-04	0.99957159	-6.703885E+02
arcsin(x)	A+B/i	8.501671E-04	0.9994784	-6.505093E+02
arcsin(x)	A+B*sqrt(i)	8.157772E-04	0.99951975	-6.588502E+02
arcsin(x)	A+B*log(i)^4	8.495843E-04	0.99947912	-6.506478E+02
arctan(x)	A+B*i	3.635707E-09	1	-3.147711E+03
arctan(x)	A+B/i	1.691177E-08	1	-2.837195E+03
arctan(x)	A+B*sqrt(i)	5.221378E-09	1	-3.074591E+03
arctan(x)	A+B*log(i)^4	5.634172E-09	1	-3.059225E+03
asinh(x)	A+B*i	4.056504E-03	0.98205968	-1.245419E+03
asinh(x)	A+B/i	6.457644E-03	0.95453529	-3.146036E+02
asinh(x)	A+B*sqrt(i)	4.688560E-03	0.97603347	-9.555218E+02
asinh(x)	A+B*log(i)^4	3.485087E-03	0.986758	-1.549380E+03
atanh(x)	A+B*i	1.461458E-03	0.99357428	-3.286924E+03
atanh(x)	A+B/i	1.677972E-03	0.99152932	-3.010622E+03
atanh(x)	A+B*sqrt(i)	1.556071E-03	0.99271536	-3.161465E+03
atanh(x)	A+B*log(i)^4	1.612431E-03	0.99217811	-3.090307E+03
Ci(x)	A+B*i	2.625641E-03	0.93213953	-7.142539E+02
Ci(x)	A+B/i	1.823994E-03	0.96725136	-8.570580E+02
Ci(x)	A+B*sqrt(i)	2.590006E-03	0.93396901	-7.196105E+02
Ci(x)	A+B*log(i)^4	1.701931E-03	0.97148782	-8.842098E+02
cosh(x)	A+B*i	1.606611E-02	0.99960888	4.211606E+02
cosh(x)	A+B/i	8.366444E-05	0.99999999	-4.847008E+03
cosh(x)	A+B*sqrt(i)	5.494985E-03	0.99995425	-6.538612E+02
cosh(x)	A+B*log(i)^4	4.391818E-03	0.99997077	-8.784019E+02
digamma(x)	A+B/i	4.156385E-03	0.97667824	-1.192128E+03
digamma(x)	A+B*i	1.571912E-03	0.99666431	-3.099885E+03
digamma(x)	A+B*sqrt(i)	2.558794E-03	0.99116106	-2.143913E+03
digamma(x)	A+B*log(i)^4	2.603299E-03	0.99085091	-2.110081E+03
erf(x)	A+B*i	3.295340E-06	0.99999997	-3.574461E+03
erf(x)	A+B/i	1.850501E-07	1	-4.789668E+03
erf(x)	A+B*sqrt(i)	1.652036E-07	1	-4.837543E+03
erf(x)	A+B*log(i)^4	2.667623E-06	0.99999998	-3.663638E+03
exp(x)	A+B*i	9.619641E-06	0.99999999	-2.982974E+03
exp(x)	A+B/i	2.977668E-07	1	-4.380025E+03
exp(x)	A+B*sqrt(i)	1.160719E-06	1	-3.833111E+03
exp(x)	A+B*log(i)^4	1.073514E-04	0.99999927	-2.013230E+03
FresnelCosine(x)	A+B*i	4.639895E-04	0.99590926	-2.863092E+03

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
FresnelCosine(x)	A+B/i	2.125452E-03	0.91416039	-1.490358E+03
FresnelCosine(x)	A+B*sqrt(i)	1.436428E-03	0.96079397	-1.843784E+03
FresnelCosine(x)	A+B*log(i)^4	1.825883E-03	0.93665224	-1.627390E+03
FresnelSine(x)	A+B*i	1.575175E-03	0.94513935	-1.609837E+03
FresnelSine(x)	A+B/i	1.385861E-03	0.95753387	-1.712529E+03
FresnelSine(x)	A+B*sqrt(i)	1.335027E-03	0.9605921	-1.742500E+03
FresnelSine(x)	A+B*log(i)^4	1.752730E-03	0.93207444	-1.524177E+03
J0(x)	A+B*i	7.710752E-04	0.99398203	-1.622134E+03
J0(x)	A+B/i	1.400096E-03	0.9801586	-1.286895E+03
J0(x)	A+B*sqrt(i)	1.102007E-03	0.98770791	-1.421442E+03
J0(x)	A+B*log(i)^4	3.487342E-04	0.99876903	-2.068067E+03
J1(x)	A+B*i	3.177566E-03	0.92548818	-8.661602E+02
J1(x)	A+B/i	1.526263E-03	0.98280926	-1.307603E+03
J1(x)	A+B*sqrt(i)	1.437996E-03	0.98474011	-1.343465E+03
J1(x)	A+B*log(i)^4	1.022790E-03	0.99228015	-1.548576E+03
J2(x)	A+B*i	1.523825E-03	0.97947148	-1.308565E+03
J2(x)	A+B/i	3.582373E-03	0.88654361	-7.939745E+02
J2(x)	A+B*sqrt(i)	1.806923E-03	0.97113534	-1.205983E+03
J2(x)	A+B*log(i)^4	4.262488E-03	0.83937476	-6.893302E+02
J3(x)	A+B*i	1.522750E-03	0.97674318	-1.308990E+03
J3(x)	A+B/i	1.999065E-03	0.95991824	-1.145149E+03
J3(x)	A+B*sqrt(i)	3.346782E-03	0.88765646	-8.349261E+02
J3(x)	A+B*log(i)^4	3.350019E-03	0.88743906	-8.343442E+02
J4(x)	A+B*i	2.962143E-03	0.90364065	-9.084222E+02
J4(x)	A+B/i	1.654166E-04	0.9996995	-2.645313E+03
J4(x)	A+B*sqrt(i)	5.118930E-03	0.71223341	-5.791086E+02
J4(x)	A+B*log(i)^4	3.649886E-03	0.85370125	-7.827348E+02
J5(x)	A+B*i	4.074160E-03	0.79657649	-7.165337E+02
J5(x)	A+B/i	3.303710E-03	0.86623923	-8.427241E+02
J5(x)	A+B*sqrt(i)	4.593709E-03	0.74138605	-6.442797E+02
J5(x)	A+B*log(i)^4	3.146122E-03	0.8786957	-8.721470E+02
ln(x)	A+B*i	5.004324E-04	0.99934558	-4.984289E+03
ln(x)	A+B/i	3.487776E-05	0.99999682	-9.784138E+03
ln(x)	A+B*sqrt(i)	3.772629E-04	0.99962808	-5.493409E+03
ln(x)	A+B*log(i)^4	2.210605E-04	0.9998723	-6.456588E+03
log(x)	A+B*i	1.933578E-04	0.99948201	-6.697865E+03
log(x)	A+B/i	1.554170E-05	0.99999665	-1.124074E+04
log(x)	A+B*sqrt(i)	5.146696E-05	0.9999633	-9.082997E+03
log(x)	A+B*log(i)^4	1.687110E-04	0.99960565	-6.943577E+03
log10Gamma(x)	A+B*i	3.290882E-03	0.99999513	-1.650235E+03
log10Gamma(x)	A+B/i	1.014841E-02	0.99995369	5.592930E+02
log10Gamma(x)	A+B*sqrt(i)	5.404767E-03	0.99998687	-6.768368E+02
log10Gamma(x)	A+B*log(i)^4	3.977628E-03	0.99999289	-1.278378E+03
Si(x)	A+B*i	1.974728E-03	0.93678537	-7.990380E+02
Si(x)	A+B/i	1.092101E-03	0.98066568	-1.021753E+03
Si(x)	A+B*sqrt(i)	1.012968E-03	0.98336608	-1.050035E+03
Si(x)	A+B*log(i)^4	1.429251E-03	0.96688534	-9.205912E+02
sinh(x)	A+B*i	1.695620E-02	0.99957064	4.751900E+02
sinh(x)	A+B/i	2.309543E-04	0.99999992	-3.829571E+03

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
sinh(x)	A+B*sqrt(i)	2.919080E-03	0.99998727	-1.287694E+03
sinh(x)	A+B*log(i)^4	4.531401E-03	0.99996934	-8.470515E+02
tan(x)	A+B*i	1.116410E-05	0.99999993	-1.525721E+03
tan(x)	A+B/i	2.121982E-06	1	-1.861112E+03
tan(x)	A+B*sqrt(i)	2.543316E-06	1	-1.824526E+03
tan(x)	A+B*log(i)^4	6.745507E-06	0.99999997	-1.627494E+03
tanh(x)	A+B*i	8.820843E-06	0.99999969	-4.409986E+03
tanh(x)	A+B/i	3.328149E-06	0.99999996	-4.996756E+03
tanh(x)	A+B*sqrt(i)	9.107990E-07	1	-5.776857E+03
tanh(x)	A+B*log(i)^4	1.480077E-05	0.99999912	-4.098414E+03
tinv(0.95,x)	A+B*i	2.049532E-03	0.74250977	-2.579329E+03
tinv(0.95,x)	A+B/i	2.312709E-03	0.67213639	-2.342303E+03
tinv(0.95,x)	A+B*sqrt(i)	1.841117E-03	0.79221518	-2.789732E+03
tinv(0.95,x)	A+B*log(i)^4	2.339574E-03	0.66447497	-2.319643E+03
tinv(0.975,x)	A+B*i	4.422419E-03	0.60508782	-1.070403E+03
tinv(0.975,x)	A+B/i	3.645720E-03	0.73162182	-1.449330E+03
tinv(0.975,x)	A+B*sqrt(i)	3.552706E-03	0.74514141	-1.500036E+03
tinv(0.975,x)	A+B*log(i)^4	2.741382E-03	0.84825311	-2.008680E+03
trigamma(x)	A+B*i	2.094883E-03	0.72574389	-2.552439E+03
trigamma(x)	A+B/i	2.472160E-03	0.61806473	-2.224231E+03
trigamma(x)	A+B*sqrt(i)	2.462036E-03	0.62118649	-2.232364E+03
trigamma(x)	A+B*log(i)^4	2.560338E-03	0.59033267	-2.154767E+03

Sine Series of Order 6

The next table shows a summary of results for the Sine series of the order 6:

$Y = a_0 + a_1 * \sin(S_1 * gx(1,A_1,B_1) + Os_1) + \ldots + a_6 * \sin(S_6 * gx(6,A_6,B_6) + Os_6)$

The output text files for this series are located in the following folder:

Fourier-Shammas Series Approximations 6 Sine

The files are named using the following general format:

*fxName_n*_6_sin_run1.txt

Where fxName is the function name and n is the gx series number which varies between 1 and 4. Each output files specifies the function being approximated and the gx series being used. For example, the file for the inverse hyperboplic cosine (acosh) using the first gx series is named acosh_1_6_sin_run1.txt.

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
10^x	A+B*i	2.262050E-06	1	-1.843889E+03
10^x	A+B/i	9.372038E-08	1	-2.486973E+03
10^x	A+B*sqrt(i)	1.903317E-07	1	-2.343891E+03
10^x	A+B*log(i)^4	2.694578E-06	1	-1.808545E+03
acosh(x)	A+B*i	4.363852E-03	0.97660126	-1.093905E+03
acosh(x)	A+B/i	4.336076E-03	0.97689818	-1.106561E+03
acosh(x)	A+B*sqrt(i)	3.745602E-03	0.98276164	-1.396700E+03
acosh(x)	A+B*log(i)^4	4.002662E-03	0.98031432	-1.265141E+03
arccos(x)	A+B*i	7.290328E-04	0.99961237	-6.772489E+02
arccos(x)	A+B/i	7.443073E-04	0.99959596	-6.730604E+02
arccos(x)	A+B*sqrt(i)	6.788238E-04	0.99966393	-6.916630E+02
arccos(x)	A+B*log(i)^4	8.184868E-04	0.99951141	-6.538697E+02
arcsin(x)	A+B*i	7.286418E-04	0.99961279	-6.773573E+02
arcsin(x)	A+B/i	7.069758E-04	0.99963547	-6.834548E+02
arcsin(x)	A+B*sqrt(i)	7.174422E-04	0.9996246	-6.804862E+02
arcsin(x)	A+B*log(i)^4	7.737502E-04	0.99956336	-6.652238E+02
arctan(x)	A+B*i	3.139691E-09	1	-3.173024E+03
arctan(x)	A+B/i	1.857685E-09	1	-3.277299E+03
arctan(x)	A+B*sqrt(i)	1.766992E-09	1	-3.289148E+03
arctan(x)	A+B*log(i)^4	7.251844E-09	1	-3.003929E+03
asinh(x)	A+B*i	4.048855E-03	0.98210929	-1.245169E+03
asinh(x)	A+B/i	4.841715E-03	0.97441641	-8.871423E+02
asinh(x)	A+B*sqrt(i)	5.708424E-03	0.96443724	-5.574650E+02
asinh(x)	A+B*log(i)^4	2.126515E-03	0.99506485	-2.534356E+03
atanh(x)	A+B*i	1.391486E-03	0.99416899	-3.381021E+03
atanh(x)	A+B/i	1.407783E-03	0.9940316	-3.357733E+03
atanh(x)	A+B*sqrt(i)	1.360796E-03	0.99442336	-3.425626E+03
atanh(x)	A+B*log(i)^4	1.650622E-03	0.99179495	-3.039461E+03
Ci(x)	A+B*i	4.292631E-03	0.81765858	-5.174052E+02
Ci(x)	A+B/i	1.394178E-03	0.98076579	-9.582465E+02
Ci(x)	A+B*sqrt(i)	1.446653E-03	0.97929062	-9.437628E+02
Ci(x)	A+B*log(i)^4	1.586928E-03	0.97507973	-9.074843E+02
cosh(x)	A+B*i	9.102869E-03	0.99987419	-1.440408E+02
cosh(x)	A+B/i	4.000438E-05	1	-5.582251E+03
cosh(x)	A+B*sqrt(i)	5.429580E-03	0.99995524	-6.618021E+02
cosh(x)	A+B*log(i)^4	5.576522E-03	0.99995278	-6.350451E+02
digamma(x)	A+B/i	4.030868E-03	0.97804303	-1.248262E+03
digamma(x)	A+B*i	3.771107E-03	0.98078178	-1.378956E+03
digamma(x)	A+B*sqrt(i)	3.382585E-03	0.98453775	-1.592282E+03
digamma(x)	A+B*log(i)^4	1.246448E-03	0.99790046	-3.551029E+03
erf(x)	A+B*i	6.616165E-07	1	-4.247875E+03
erf(x)	A+B/i	3.483092E-08	1	-5.490319E+03
erf(x)	A+B*sqrt(i)	2.582684E-08	1	-5.616536E+03
erf(x)	A+B*log(i)^4	4.443779E-06	0.99999995	-3.444144E+03
exp(x)	A+B*i	5.054793E-06	1	-3.237502E+03
exp(x)	A+B/i	1.596976E-08	1	-5.548668E+03
exp(x)	A+B*sqrt(i)	2.876857E-07	1	-4.389724E+03
exp(x)	A+B*log(i)^4	8.820092E-06	1	-3.013710E+03
FresnelCosine(x)	A+B*i	1.851197E-03	0.93473695	-1.610907E+03

Function	*gx(i,A,B)*	*MSSE*	*RsqrAdj*	*AICc*
FresnelCosine(x)	A+B/i	2.093223E-03	0.91655637	-1.500076E+03
FresnelCosine(x)	A+B*sqrt(i)	1.488393E-03	0.95781116	-1.807665E+03
FresnelCosine(x)	A+B*log(i)^4	1.707956E-03	0.94444598	-1.683550E+03
FresnelSine(x)	A+B*i	1.160217E-03	0.97016112	-1.850985E+03
FresnelSine(x)	A+B/i	1.635195E-03	0.94072891	-1.575775E+03
FresnelSine(x)	A+B*sqrt(i)	4.164975E-04	0.99615471	-2.672619E+03
FresnelSine(x)	A+B*log(i)^4	2.210086E-03	0.89172649	-1.334156E+03
J0(x)	A+B*i	1.416091E-03	0.97962859	-1.276407E+03
J0(x)	A+B/i	5.296751E-04	0.99714992	-1.829073E+03
J0(x)	A+B*sqrt(i)	8.812965E-04	0.99210989	-1.542942E+03
J0(x)	A+B*log(i)^4	1.347992E-03	0.98154079	-1.304105E+03
J1(x)	A+B*i	2.078306E-03	0.96801624	-1.117650E+03
J1(x)	A+B/i	3.408194E-03	0.91398794	-8.198833E+02
J1(x)	A+B*sqrt(i)	3.005646E-03	0.93310613	-8.955487E+02
J1(x)	A+B*log(i)^4	1.823778E-03	0.97537057	-1.196297E+03
J2(x)	A+B*i	9.049245E-04	0.99273582	-1.618187E+03
J2(x)	A+B/i	2.697165E-05	0.99999355	-3.733053E+03
J2(x)	A+B*sqrt(i)	1.806239E-03	0.97105907	-1.202115E+03
J2(x)	A+B*log(i)^4	1.005383E-03	0.99103346	-1.554813E+03
J3(x)	A+B*i	4.193865E-03	0.82299044	-6.950044E+02
J3(x)	A+B/i	1.479348E-03	0.97797536	-1.322301E+03
J3(x)	A+B*sqrt(i)	2.260173E-03	0.94858959	-1.067150E+03
J3(x)	A+B*log(i)^4	3.489707E-03	0.87744082	-8.056549E+02
J4(x)	A+B*i	1.129004E-04	0.99985954	-2.871157E+03
J4(x)	A+B/i	1.184459E-04	0.9998454	-2.842291E+03
J4(x)	A+B*sqrt(i)	4.940458E-03	0.73103797	-5.963756E+02
J4(x)	A+B*log(i)^4	1.446666E-03	0.97693821	-1.335750E+03
J5(x)	A+B*i	7.845946E-05	0.9999243	-3.090239E+03
J5(x)	A+B/i	6.024932E-04	0.99553621	-1.863066E+03
J5(x)	A+B*sqrt(i)	3.202834E-03	0.87385546	-8.572956E+02
J5(x)	A+B*log(i)^4	4.659618E-04	0.99733007	-2.017764E+03
ln(x)	A+B*i	4.677838E-04	0.99942754	-5.101832E+03
ln(x)	A+B/i	1.641913E-05	0.99999929	-1.113774E+04
ln(x)	A+B*sqrt(i)	7.130334E-05	0.9999867	-8.491507E+03
ln(x)	A+B*log(i)^4	3.206252E-04	0.99973107	-5.782508E+03
log(x)	A+B*i	1.941710E-04	0.99947706	-6.686271E+03
log(x)	A+B/i	1.760416E-05	0.9999957	-1.101216E+04
log(x)	A+B*sqrt(i)	1.520240E-04	0.99967944	-7.127222E+03
log(x)	A+B*log(i)^4	1.228635E-04	0.99979062	-7.510983E+03
log10Gamma(x)	A+B*i	5.253899E-03	0.99998758	-7.283538E+02
log10Gamma(x)	A+B/i	9.352386E-03	0.99996063	4.030553E+02
log10Gamma(x)	A+B*sqrt(i)	5.441770E-03	0.99998667	-6.594209E+02
log10Gamma(x)	A+B*log(i)^4	3.422757E-03	0.99999473	-1.569118E+03
Si(x)	A+B*i	3.262733E-03	0.82647623	-6.060772E+02
Si(x)	A+B/i	1.471268E-03	0.96471587	-9.055389E+02
Si(x)	A+B*sqrt(i)	1.536148E-03	0.96153533	-8.893132E+02
Si(x)	A+B*log(i)^4	2.442233E-03	0.9027768	-7.149866E+02
sinh(x)	A+B*i	1.083740E-02	0.99982425	3.072154E+01
sinh(x)	A+B/i	2.415847E-05	1	-6.087614E+03

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
sinh(x)	A+B*sqrt(i)	2.728185E-03	0.99998886	-1.351404E+03
sinh(x)	A+B*log(i)^4	3.880010E-03	0.99997747	-9.984980E+02
tan(x)	A+B*i	1.330751E-06	1	-1.951056E+03
tan(x)	A+B/i	1.582327E-06	1	-1.916079E+03
tan(x)	A+B*sqrt(i)	9.215551E-07	1	-2.025278E+03
tan(x)	A+B*log(i)^4	2.098076E-06	1	-1.859090E+03
tanh(x)	A+B*i	7.480860E-06	0.99999978	-4.505081E+03
tanh(x)	A+B/i	1.362164E-06	0.99999999	-5.530451E+03
tanh(x)	A+B*sqrt(i)	1.156604E-06	0.99999999	-5.628930E+03
tanh(x)	A+B*log(i)^4	1.212550E-05	0.99999941	-4.214337E+03
tinv(0.95,x)	A+B*i	1.480078E-03	0.86557927	-3.213964E+03
tinv(0.95,x)	A+B/i	2.148007E-03	0.71688138	-2.483225E+03
tinv(0.95,x)	A+B*sqrt(i)	1.929103E-03	0.77164638	-2.694111E+03
tinv(0.95,x)	A+B*log(i)^4	1.486631E-03	0.86438644	-3.205297E+03
tinv(0.975,x)	A+B*i	4.214632E-03	0.64095763	-1.160794E+03
tinv(0.975,x)	A+B/i	3.884758E-03	0.69496183	-1.320701E+03
tinv(0.975,x)	A+B*sqrt(i)	3.459580E-03	0.75807932	-1.548123E+03
tinv(0.975,x)	A+B*log(i)^4	3.560839E-03	0.74371036	-1.491521E+03
trigamma(x)	A+B*i	2.258680E-03	0.68085565	-2.399200E+03
trigamma(x)	A+B/i	2.269460E-03	0.67780199	-2.389763E+03
trigamma(x)	A+B*sqrt(i)	2.172784E-03	0.70466779	-2.476045E+03
trigamma(x)	A+B*log(i)^4	2.042475E-03	0.73902949	-2.598625E+03

Sine Series of Order 7

The next table shows a summary of results for the Sine series of the order 7:

$$Y = a_0 + a_1 * \sin(S_1 * gx(1,A_1,B_1) + Os_1) + \ldots + a_7 * \sin(S_7 * gx(7,A_7,B_7) + Os_7)$$

The output text files for this series are located in the following folder:

Fourier-Shammas Series Approximations 7 Sine

The files are named using the following general format:

*fxName_n_*7_sin_run1.txt

Where fxName is the function name and n is the gx series number which varies between 1 and 4. Each output files specifies the function being approximated and the gx series being used. For example, the file for the inverse hyperboplic cosine (acosh) using the first gx series is named acosh_1_7_sin_run1.txt.

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
10^x	A+B*i	1.216499E-06	1	-1.964828E+03
10^x	A+B/i	8.793805E-09	1	-Inf
10^x	A+B*sqrt(i)	1.479552E-08	1	-2.855512E+03
10^x	A+B*log(i)^4	4.591448E-07	1	-2.161650E+03
acosh(x)	A+B*i	6.275283E-03	0.95156484	-3.698840E+02
acosh(x)	A+B/i	5.716508E-03	0.95980651	-5.547263E+02
acosh(x)	A+B*sqrt(i)	4.824767E-03	0.97136832	-8.908647E+02
acosh(x)	A+B*log(i)^4	2.163933E-03	0.99424054	-2.480102E+03
arccos(x)	A+B*i	5.590944E-04	0.99976957	-7.264989E+02
arccos(x)	A+B/i	6.268564E-04	0.99971033	-7.033902E+02
arccos(x)	A+B*sqrt(i)	5.602309E-04	0.99976863	-7.260887E+02
arccos(x)	A+B*log(i)^4	6.633059E-04	0.99967566	-6.919734E+02
arcsin(x)	A+B*i	6.234619E-04	0.99971346	-7.044871E+02
arcsin(x)	A+B/i	5.692746E-04	0.9997611	-7.228538E+02
arcsin(x)	A+B*sqrt(i)	5.667076E-04	0.99976325	-7.237668E+02
arcsin(x)	A+B*log(i)^4	6.564407E-04	0.99968234	-6.940750E+02
arctan(x)	A+B*i	2.445453E-09	1	-3.219142E+03
arctan(x)	A+B/i	6.326969E-10	1	-3.491224E+03
arctan(x)	A+B*sqrt(i)	6.893689E-10	1	-3.468397E+03
arctan(x)	A+B*log(i)^4	3.645360E-08	1	-2.673382E+03
asinh(x)	A+B*i	4.501287E-03	0.9778653	-1.029067E+03
asinh(x)	A+B/i	6.661765E-03	0.9515182	-2.442407E+02
asinh(x)	A+B*sqrt(i)	2.690206E-03	0.99209374	-2.059588E+03
asinh(x)	A+B*log(i)^4	5.424549E-03	0.96785397	-6.555510E+02
atanh(x)	A+B*i	1.189659E-03	0.99573353	-3.690399E+03
atanh(x)	A+B/i	1.168656E-03	0.99588285	-3.726024E+03
atanh(x)	A+B*sqrt(i)	1.202359E-03	0.99564195	-3.669162E+03
atanh(x)	A+B*log(i)^4	1.306991E-03	0.99485045	-3.502277E+03
Ci(x)	A+B*i	3.287487E-03	0.89248469	-6.178072E+02
Ci(x)	A+B/i	1.867862E-03	0.96529182	-8.394162E+02
Ci(x)	A+B*sqrt(i)	2.946022E-03	0.91365959	-6.607969E+02
Ci(x)	A+B*log(i)^4	1.772565E-03	0.96874306	-8.599440E+02
cosh(x)	A+B*i	5.481244E-03	0.99995429	-6.482472E+02
cosh(x)	A+B/i	1.202895E-05	1	-6.782262E+03
cosh(x)	A+B*sqrt(i)	3.832235E-03	0.99997766	-1.006847E+03
cosh(x)	A+B*log(i)^4	3.917438E-03	0.99997665	-9.848134E+02
digamma(x)	A+B/i	3.016668E-03	0.98768948	-1.812873E+03
digamma(x)	A+B*i	1.932255E-03	0.99494931	-2.686875E+03
digamma(x)	A+B*sqrt(i)	2.920795E-03	0.98845953	-1.876240E+03
digamma(x)	A+B*log(i)^4	3.036597E-03	0.98752629	-1.799954E+03
erf(x)	A+B*i	5.862359E-06	0.99999991	-3.323069E+03
erf(x)	A+B/i	4.966149E-09	1	-6.308146E+03
erf(x)	A+B*sqrt(i)	1.149754E-08	1	-5.953891E+03
erf(x)	A+B*log(i)^4	1.223867E-06	1	-3.984148E+03
exp(x)	A+B*i	2.157387E-05	0.99999997	-2.649972E+03
exp(x)	A+B/i	6.199150E-09	1	-Inf
exp(x)	A+B*sqrt(i)	3.816891E-07	1	-4.271894E+03
exp(x)	A+B*log(i)^4	1.013994E-06	1	-3.879121E+03
FresnelCosine(x)	A+B*i	4.482202E-04	0.99616536	-2.886144E+03

Function	*gx(i,A,B)*	*MSSE*	*RsqrAdj*	*AICc*
FresnelCosine(x)	A+B/i	1.330178E-03	0.96622767	-1.904963E+03
FresnelCosine(x)	A+B*sqrt(i)	1.006809E-03	0.98065202	-2.156194E+03
FresnelCosine(x)	A+B*log(i)^4	8.461549E-04	0.986334	-2.312997E+03
FresnelSine(x)	A+B*i	2.080813E-03	0.90377809	-1.378413E+03
FresnelSine(x)	A+B/i	9.392112E-04	0.98039647	-2.016383E+03
FresnelSine(x)	A+B*sqrt(i)	8.642568E-04	0.98340056	-2.083085E+03
FresnelSine(x)	A+B*log(i)^4	4.009519E-04	0.99642734	-2.699044E+03
J0(x)	A+B*i	1.042043E-03	0.98892872	-1.444663E+03
J0(x)	A+B/i	8.510170E-04	0.9926158	-1.558472E+03
J0(x)	A+B*sqrt(i)	1.530525E-03	0.97611596	-1.228615E+03
J0(x)	A+B*log(i)^4	2.773203E-03	0.92158668	-8.945664E+02
J1(x)	A+B*i	1.738668E-03	0.97753928	-1.220957E+03
J1(x)	A+B/i	5.327282E-05	0.99997891	-3.319196E+03
J1(x)	A+B*sqrt(i)	2.569403E-03	0.95094821	-9.858428E+02
J1(x)	A+B*log(i)^4	1.270807E-03	0.98800088	-1.409664E+03
J2(x)	A+B*i	1.747147E-03	0.97282932	-1.218028E+03
J2(x)	A+B/i	1.162467E-04	0.99987972	-2.849463E+03
J2(x)	A+B*sqrt(i)	1.967632E-03	0.96553889	-1.146482E+03
J2(x)	A+B*log(i)^4	1.406755E-03	0.98238515	-1.348480E+03
J3(x)	A+B*i	1.388631E-03	0.98052751	-1.356287E+03
J3(x)	A+B/i	1.471671E-03	0.97812899	-1.321323E+03
J3(x)	A+B*sqrt(i)	3.598896E-03	0.86920651	-7.829968E+02
J3(x)	A+B*log(i)^4	4.666254E-04	0.99780121	-2.012796E+03
J4(x)	A+B*i	2.901443E-03	0.90691829	-9.126790E+02
J4(x)	A+B/i	4.708946E-05	0.99997548	-3.393469E+03
J4(x)	A+B*sqrt(i)	4.567209E-03	0.76935813	-6.395551E+02
J4(x)	A+B*log(i)^4	7.432161E-05	0.99993892	-3.118745E+03
J5(x)	A+B*i	4.689544E-03	0.7286433	-6.236425E+02
J5(x)	A+B/i	2.992414E-04	0.9988951	-2.280250E+03
J5(x)	A+B*sqrt(i)	3.986795E-03	0.80387759	-7.213757E+02
J5(x)	A+B*log(i)^4	2.668560E-03	0.9121315	-9.630479E+02
ln(x)	A+B*i	4.445642E-04	0.99948238	-5.189539E+03
ln(x)	A+B/i	8.755107E-06	0.9999998	-1.226682E+04
ln(x)	A+B*sqrt(i)	1.267966E-04	0.99995789	-7.450166E+03
ln(x)	A+B*log(i)^4	1.725267E-04	0.99992204	-6.895208E+03
log(x)	A+B*i	2.009335E-04	0.99943937	-6.620544E+03
log(x)	A+B/i	8.693821E-06	0.99999895	-1.227948E+04
log(x)	A+B*sqrt(i)	5.837543E-05	0.99995268	-8.847958E+03
log(x)	A+B*log(i)^4	1.074467E-04	0.99983969	-7.748558E+03
log10Gamma(x)	A+B*i	3.640413E-03	0.99999403	-1.444126E+03
log10Gamma(x)	A+B/i	7.246925E-03	0.99997634	-9.332816E+01
log10Gamma(x)	A+B*sqrt(i)	5.456676E-03	0.99998658	-6.500210E+02
log10Gamma(x)	A+B*log(i)^4	1.155112E-03	0.9999994	-3.696305E+03
Si(x)	A+B*i	1.224904E-03	0.9754073	-9.702631E+02
Si(x)	A+B/i	1.717247E-03	0.95166436	-8.432279E+02
Si(x)	A+B*sqrt(i)	1.152320E-03	0.97823552	-9.932312E+02
Si(x)	A+B*log(i)^4	9.635411E-04	0.98478253	-1.060504E+03
sinh(x)	A+B*i	1.534610E-03	0.99999647	-1.923849E+03
sinh(x)	A+B/i	3.020784E-06	1	-8.166826E+03

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
sinh(x)	A+B*sqrt(i)	1.755806E-03	0.99999538	-1.788928E+03
sinh(x)	A+B*log(i)^4	7.343496E-04	0.99999919	-2.662370E+03
tan(x)	A+B*i	4.745792E-07	1	-2.154971E+03
tan(x)	A+B/i	2.612968E-07	1	-2.275519E+03
tan(x)	A+B*sqrt(i)	2.847881E-07	1	-2.258129E+03
tan(x)	A+B*log(i)^4	1.043325E-06	1	-1.995848E+03
tanh(x)	A+B*i	3.979417E-05	0.99999363	-3.494803E+03
tanh(x)	A+B/i	3.071668E-07	1	-6.422982E+03
tanh(x)	A+B*sqrt(i)	3.392316E-07	1	-6.363208E+03
tanh(x)	A+B*log(i)^4	7.977759E-06	0.99999974	-4.462255E+03
tinv(0.95,x)	A+B*i	1.665824E-03	0.82954842	-2.977974E+03
tinv(0.95,x)	A+B/i	1.734241E-03	0.81525968	-2.899003E+03
tinv(0.95,x)	A+B*sqrt(i)	2.297372E-03	0.67580553	-2.347296E+03
tinv(0.95,x)	A+B*log(i)^4	1.393626E-03	0.88070138	-3.328016E+03
tinv(0.975,x)	A+B*i	4.312191E-03	0.62375699	-1.111863E+03
tinv(0.975,x)	A+B/i	3.762186E-03	0.71361328	-1.379571E+03
tinv(0.975,x)	A+B*sqrt(i)	3.868100E-03	0.69726133	-1.325099E+03
tinv(0.975,x)	A+B*log(i)^4	3.066566E-03	0.80972705	-1.780685E+03
trigamma(x)	A+B*i	1.920911E-03	0.76893516	-2.716213E+03
trigamma(x)	A+B/i	2.317971E-03	0.66353844	-2.343810E+03
trigamma(x)	A+B*sqrt(i)	2.363286E-03	0.65025454	-2.305437E+03
trigamma(x)	A+B*log(i)^4	2.351365E-03	0.65377415	-2.315460E+03

Cosine Series of Order 3

The next table shows a summary of results for the Sine series of the order 3:

$$Y = a_0 + a_1 * \cos(C_1 * gx(1,A_1,B_1) + Oc_1) + \ldots + a_3 * \cos(C_3 * gx(3,A_3,B_3) + Oc_3)$$

The output text files for this series are located in the following folder:

Fourier-Shammas Series Approximations 3 Cosine

The files are named using the following general format:

*fxName_n*_3_cos_run1.txt

Where fxName is the function name and n is the gx series number which varies between 1 and 4. Each output files specifies the function being approximated and the gx series being used. For example, the file for the inverse hyperboplic cosine (acosh) using the first gx series is named acosh_1_3_cos_run1.txt.

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
10^x	A+B*i	6.780165E-05	0.99999993	-1.169813E+03
10^x	A+B/i	4.254118E-05	0.99999997	-1.263968E+03
10^x	A+B*sqrt(i)	6.239488E-05	0.99999994	-1.186600E+03
10^x	A+B*log(i)^4	1.022035E-04	0.99999983	-1.086917E+03
acosh(x)	A+B*i	3.525328E-03	0.98477597	-1.528901E+03
acosh(x)	A+B/i	6.118412E-03	0.95414275	-4.361663E+02
acosh(x)	A+B*sqrt(i)	7.326548E-03	0.93424492	-7.900621E+01
acosh(x)	A+B*log(i)^4	4.479382E-03	0.97542086	-1.054189E+03
arccos(x)	A+B*i	1.327939E-03	0.99875367	-5.689042E+02
arccos(x)	A+B/i	1.351446E-03	0.99870915	-5.653597E+02
arccos(x)	A+B*sqrt(i)	1.231822E-03	0.99892756	-5.840813E+02
arccos(x)	A+B*log(i)^4	1.258897E-03	0.9988799	-5.796895E+02
arcsin(x)	A+B*i	1.372551E-03	0.99866852	-5.622296E+02
arcsin(x)	A+B/i	1.370329E-03	0.99867283	-5.625568E+02
arcsin(x)	A+B*sqrt(i)	1.396766E-03	0.99862113	-5.586970E+02
arcsin(x)	A+B*log(i)^4	1.290254E-03	0.9988234	-5.747197E+02
arctan(x)	A+B*i	2.344106E-07	1	-2.314601E+03
arctan(x)	A+B/i	2.388803E-07	1	-2.310786E+03
arctan(x)	A+B*sqrt(i)	2.010726E-07	1	-2.345590E+03
arctan(x)	A+B*log(i)^4	1.001725E-07	1	-2.486338E+03
asinh(x)	A+B*i	5.377597E-03	0.96853478	-6.890596E+02
asinh(x)	A+B/i	7.369963E-03	0.94090036	-5.808719E+01
asinh(x)	A+B*sqrt(i)	8.458405E-03	0.92215492	2.176841E+02
asinh(x)	A+B*log(i)^4	5.175483E-03	0.97085555	-7.657544E+02
atanh(x)	A+B*i	2.360175E-03	0.98327506	-2.336367E+03
atanh(x)	A+B/i	2.557929E-03	0.98035496	-2.175442E+03
atanh(x)	A+B*sqrt(i)	2.326764E-03	0.98374523	-2.364881E+03
atanh(x)	A+B*log(i)^4	2.347353E-03	0.9834563	-2.347262E+03
Ci(x)	A+B*i	3.998405E-03	0.84427002	-5.576252E+02
Ci(x)	A+B/i	2.721882E-03	0.92783322	-7.083774E+02
Ci(x)	A+B*sqrt(i)	2.569709E-03	0.93567696	-7.309296E+02
Ci(x)	A+B*log(i)^4	2.394468E-03	0.94415081	-7.586171E+02
cosh(x)	A+B*i	1.937423E-02	0.99943352	6.006776E+02
cosh(x)	A+B/i	3.110246E-03	0.9999854	-1.232223E+03
cosh(x)	A+B*sqrt(i)	7.394416E-03	0.99991748	-3.644675E+02
cosh(x)	A+B*log(i)^4	4.576626E-03	0.99996839	-8.451901E+02
digamma(x)	A+B/i	5.111704E-03	0.96479769	-7.942605E+02
digamma(x)	A+B*i	2.487089E-03	0.99166659	-2.207724E+03
digamma(x)	A+B*sqrt(i)	6.194052E-03	0.94831203	-4.174454E+02
digamma(x)	A+B*log(i)^4	5.886115E-03	0.9533236	-5.174941E+02
erf(x)	A+B*i	1.055258E-06	1	-4.063220E+03
erf(x)	A+B/i	3.625629E-06	0.99999997	-3.542369E+03
erf(x)	A+B*sqrt(i)	2.188686E-06	0.99999999	-3.755364E+03
erf(x)	A+B*log(i)^4	1.004832E-06	1	-4.083883E+03
exp(x)	A+B*i	3.406509E-05	0.99999993	-2.482888E+03
exp(x)	A+B/i	4.275797E-05	0.99999988	-2.391520E+03
exp(x)	A+B*sqrt(i)	1.546564E-05	0.99999998	-2.800328E+03
exp(x)	A+B*log(i)^4	5.367560E-05	0.99999982	-2.300104E+03
FresnelCosine(x)	A+B*i	3.463424E-03	0.77309246	-1.058035E+03

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
FresnelCosine(x)	A+B/i	2.888796E-03	0.84214025	-1.221674E+03
FresnelCosine(x)	A+B*sqrt(i)	2.712659E-03	0.86080357	-1.278419E+03
FresnelCosine(x)	A+B*log(i)^4	3.436205E-03	0.77664493	-1.065152E+03
FresnelSine(x)	A+B*i	2.086882E-03	0.90419112	-1.392343E+03
FresnelSine(x)	A+B/i	2.094470E-03	0.90349318	-1.389433E+03
FresnelSine(x)	A+B*sqrt(i)	2.382359E-03	0.87513977	-1.286142E+03
FresnelSine(x)	A+B*log(i)^4	2.886250E-03	0.81673585	-1.132265E+03
J0(x)	A+B*i	1.337680E-03	0.982019	-1.320686E+03
J0(x)	A+B/i	1.375109E-03	0.98099869	-1.305177E+03
J0(x)	A+B*sqrt(i)	1.450670E-03	0.97885309	-1.275114E+03
J0(x)	A+B*log(i)^4	6.363635E-04	0.9959307	-1.738208E+03
J1(x)	A+B*i	1.488979E-03	0.98374906	-1.330642E+03
J1(x)	A+B/i	3.019727E-03	0.93315994	-9.049822E+02
J1(x)	A+B*sqrt(i)	5.826470E-03	0.75116441	-5.093208E+02
J1(x)	A+B*log(i)^4	2.574193E-03	0.95142825	-1.001080E+03
J2(x)	A+B*i	2.661783E-03	0.93778454	-9.809366E+02
J2(x)	A+B/i	2.778454E-03	0.93221096	-9.551117E+02
J2(x)	A+B*sqrt(i)	4.171571E-03	0.84718986	-7.104602E+02
J2(x)	A+B*log(i)^4	2.821303E-03	0.93010397	-9.458986E+02
J3(x)	A+B*i	3.673081E-03	0.86559368	-7.870718E+02
J3(x)	A+B/i	2.491294E-03	0.93816863	-1.020785E+03
J3(x)	A+B*sqrt(i)	3.657145E-03	0.86675742	-7.896893E+02
J3(x)	A+B*log(i)^4	2.324655E-03	0.94616365	-1.062462E+03
J4(x)	A+B*i	2.814185E-03	0.91361214	-9.474194E+02
J4(x)	A+B/i	2.723447E-03	0.91909313	-9.671495E+02
J4(x)	A+B*sqrt(i)	4.460856E-03	0.78293791	-6.700972E+02
J4(x)	A+B*log(i)^4	2.723160E-03	0.91911017	-9.672129E+02
J5(x)	A+B*i	2.520771E-03	0.92265053	-1.013704E+03
J5(x)	A+B/i	3.217476E-03	0.87398534	-8.667969E+02
J5(x)	A+B*sqrt(i)	4.415705E-03	0.76264927	-6.762216E+02
J5(x)	A+B*log(i)^4	2.519481E-03	0.92272969	-1.014013E+03
ln(x)	A+B*i	4.854013E-04	0.99938567	-5.047293E+03
ln(x)	A+B/i	1.564359E-04	0.99993619	-7.087751E+03
ln(x)	A+B*sqrt(i)	1.525729E-04	0.99993931	-7.132808E+03
ln(x)	A+B*log(i)^4	2.492751E-04	0.99983799	-6.248180E+03
log(x)	A+B*i	2.377574E-04	0.99921856	-6.333426E+03
log(x)	A+B/i	8.272676E-05	0.99990539	-8.235811E+03
log(x)	A+B*sqrt(i)	7.917719E-05	0.99991334	-8.314838E+03
log(x)	A+B*log(i)^4	1.089619E-04	0.99983587	-7.739441E+03
log10Gamma(x)	A+B*i	6.117569E-03	0.99998321	-4.418226E+02
log10Gamma(x)	A+B/i	8.466877E-03	0.99996783	1.958214E+02
log10Gamma(x)	A+B*sqrt(i)	1.090393E-02	0.99994665	6.921322E+02
log10Gamma(x)	A+B*log(i)^4	3.582813E-03	0.99999424	-1.491525E+03
Si(x)	A+B*i	1.741313E-03	0.9513805	-8.545809E+02
Si(x)	A+B/i	2.470871E-03	0.90210588	-7.230069E+02
Si(x)	A+B*sqrt(i)	1.307314E-03	0.97259586	-9.623671E+02
Si(x)	A+B*log(i)^4	2.467339E-03	0.90238554	-7.235447E+02
sinh(x)	A+B*i	4.699444E-03	0.99996715	-8.186551E+02
sinh(x)	A+B/i	3.995686E-03	0.99997625	-9.812084E+02

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
sinh(x)	A+B*sqrt(i)	5.839497E-03	0.99994928	-6.010201E+02
sinh(x)	A+B*log(i)^4	4.991204E-03	0.99996295	-7.583015E+02
tan(x)	A+B*i	8.016220E-05	0.99999634	-1.135985E+03
tan(x)	A+B/i	3.941291E-05	0.99999912	-1.279397E+03
tan(x)	A+B*sqrt(i)	5.169569E-05	0.99999848	-1.224598E+03
tan(x)	A+B*log(i)^4	6.670961E-05	0.99999747	-1.173093E+03
tanh(x)	A+B*i	2.991432E-05	0.99999645	-3.682962E+03
tanh(x)	A+B/i	3.685239E-05	0.99999461	-3.557395E+03
tanh(x)	A+B*sqrt(i)	2.553797E-05	0.99999741	-3.778181E+03
tanh(x)	A+B*log(i)^4	2.999587E-05	0.99999643	-3.681323E+03
tinv(0.95,x)	A+B*i	2.738333E-03	0.54129465	-2.018909E+03
tinv(0.95,x)	A+B/i	2.686739E-03	0.55841701	-2.056228E+03
tinv(0.95,x)	A+B*sqrt(i)	2.708239E-03	0.5513214	-2.040590E+03
tinv(0.95,x)	A+B*log(i)^4	2.621027E-03	0.5797534	-2.104811E+03
tinv(0.975,x)	A+B*i	4.936193E-03	0.50900736	-8.628097E+02
tinv(0.975,x)	A+B/i	4.458122E-03	0.59950729	-1.062672E+03
tinv(0.975,x)	A+B*sqrt(i)	4.946681E-03	0.50691874	-8.586455E+02
tinv(0.975,x)	A+B*log(i)^4	3.887011E-03	0.69554553	-1.331637E+03
trigamma(x)	A+B*i	2.419457E-03	0.63491706	-2.274986E+03
trigamma(x)	A+B/i	2.733029E-03	0.53415188	-2.033444E+03
trigamma(x)	A+B*sqrt(i)	3.085088E-03	0.40640401	-1.793286E+03
trigamma(x)	A+B*log(i)^4	2.570560E-03	0.5878919	-2.154915E+03

Cosine Series of Order 4

The next table shows a summary of results for the Sine series of the order 4:

$$Y = a_0 + a_1 * \cos(C_1 * gx(1,A_1,B_1) + Oc_1) + \ldots + a_4 * \cos(C_4 * gx(4,A_4,B_4) + Oc_4)$$

The output text files for this series are located in the following folder:

Fourier-Shammas Series Approximations 4 Cosine

The files are named using the following general format:

*fxName_n*_4_cos_run1.txt

Where fxName is the function name and n is the gx series number which varies between 1 and 4. Each output files specifies the function being approximated and the gx series being used. For example, the file for the inverse hyperboplic cosine (acosh) using the first gx series is named acosh_1_4_cos_run1.txt.

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
10^x	A+B*i	1.179050E-05	1	-1.518955E+03
10^x	A+B/i	2.263460E-05	0.99999999	-1.387214E+03
10^x	A+B*sqrt(i)	1.493355E-05	1	-1.471219E+03
10^x	A+B*log(i)^4	2.102403E-05	0.99999999	-1.402124E+03
acosh(x)	A+B*i	4.777794E-03	0.97200855	-9.223417E+02
acosh(x)	A+B/i	5.330030E-03	0.96516386	-7.055544E+02
acosh(x)	A+B*sqrt(i)	3.598932E-03	0.98411753	-1.483925E+03
acosh(x)	A+B*log(i)^4	3.961006E-03	0.98076104	-1.293929E+03
arccos(x)	A+B*i	9.909110E-04	0.99929879	-6.238266E+02
arccos(x)	A+B/i	9.668870E-04	0.99933238	-6.287843E+02
arccos(x)	A+B*sqrt(i)	1.049138E-03	0.99921396	-6.122925E+02
arccos(x)	A+B*log(i)^4	1.021637E-03	0.99925463	-6.176581E+02
arcsin(x)	A+B*i	9.858753E-04	0.9993059	-6.248558E+02
arcsin(x)	A+B/i	9.783403E-04	0.99931647	-6.264056E+02
arcsin(x)	A+B*sqrt(i)	1.042986E-03	0.99922315	-6.134804E+02
arcsin(x)	A+B*log(i)^4	9.857209E-04	0.99930612	-6.248874E+02
arctan(x)	A+B*i	2.533961E-07	1	-2.294655E+03
arctan(x)	A+B/i	1.936419E-07	1	-2.348981E+03
arctan(x)	A+B*sqrt(i)	9.521347E-08	1	-2.492379E+03
arctan(x)	A+B*log(i)^4	8.737572E-08	1	-2.509731E+03
asinh(x)	A+B*i	5.285009E-03	0.96957844	-7.198091E+02
asinh(x)	A+B/i	7.038893E-03	0.94603666	-1.460825E+02
asinh(x)	A+B*sqrt(i)	4.174077E-03	0.98102372	-1.192243E+03
asinh(x)	A+B*log(i)^4	3.588733E-03	0.98597275	-1.494733E+03
atanh(x)	A+B*i	2.087163E-03	0.98690742	-2.578206E+03
atanh(x)	A+B/i	1.932316E-03	0.98877804	-2.732379E+03
atanh(x)	A+B*sqrt(i)	2.036314E-03	0.98753759	-2.627535E+03
atanh(x)	A+B*log(i)^4	2.046372E-03	0.98741417	-2.617681E+03
Ci(x)	A+B*i	4.137773E-03	0.83235137	-5.400880E+02
Ci(x)	A+B/i	2.511337E-03	0.93824438	-7.358303E+02
Ci(x)	A+B*sqrt(i)	2.474095E-03	0.94006242	-7.416871E+02
Ci(x)	A+B*log(i)^4	2.053445E-03	0.95871115	-8.147385E+02
cosh(x)	A+B*i	6.576753E-03	0.99993459	-4.778452E+02
cosh(x)	A+B/i	7.598903E-04	0.99999913	-2.640284E+03
cosh(x)	A+B*sqrt(i)	6.342031E-03	0.99993918	-5.142601E+02
cosh(x)	A+B*log(i)^4	4.545532E-03	0.99996876	-8.479804E+02
digamma(x)	A+B/i	4.934167E-03	0.96716687	-8.595945E+02
digamma(x)	A+B*i	1.517517E-03	0.99689436	-3.173006E+03
digamma(x)	A+B*sqrt(i)	2.376347E-03	0.9923844	-2.293070E+03
digamma(x)	A+B*log(i)^4	2.321137E-03	0.99273416	-2.339191E+03
erf(x)	A+B*i	4.430584E-07	1	-4.425349E+03
erf(x)	A+B/i	9.122354E-07	1	-4.120582E+03
erf(x)	A+B*sqrt(i)	6.694030E-07	1	-4.251196E+03
erf(x)	A+B*log(i)^4	8.716407E-07	1	-4.139792E+03
exp(x)	A+B*i	6.852782E-06	1	-3.123439E+03
exp(x)	A+B/i	5.234927E-06	1	-3.231698E+03
exp(x)	A+B*sqrt(i)	5.234708E-06	1	-3.231715E+03
exp(x)	A+B*log(i)^4	1.026146E-05	0.99999999	-2.961135E+03
FresnelCosine(x)	A+B*i	1.806528E-03	0.93812718	-1.641057E+03

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
FresnelCosine(x)	A+B/i	2.323489E-03	0.89764915	-1.414057E+03
FresnelCosine(x)	A+B*sqrt(i)	1.827380E-03	0.93669058	-1.630705E+03
FresnelCosine(x)	A+B*log(i)^4	2.397392E-03	0.89103463	-1.385814E+03
FresnelSine(x)	A+B*i	1.694644E-03	0.93666236	-1.555268E+03
FresnelSine(x)	A+B/i	1.460752E-03	0.95293932	-1.674382E+03
FresnelSine(x)	A+B*sqrt(i)	2.133790E-03	0.89958275	-1.370465E+03
FresnelSine(x)	A+B*log(i)^4	1.784400E-03	0.92977539	-1.513877E+03
J0(x)	A+B*i	2.127561E-03	0.95434963	-1.055823E+03
J0(x)	A+B/i	1.498514E-03	0.97735345	-1.252805E+03
J0(x)	A+B*sqrt(i)	1.707925E-03	0.97058167	-1.179292E+03
J0(x)	A+B*log(i)^4	1.976791E-03	0.96059042	-1.097130E+03
J1(x)	A+B*i	7.967386E-04	0.99533128	-1.703016E+03
J1(x)	A+B/i	2.991066E-03	0.93420116	-9.066549E+02
J1(x)	A+B*sqrt(i)	2.917351E-03	0.93740444	-9.216772E+02
J1(x)	A+B*log(i)^4	7.600521E-04	0.99575133	-1.731394E+03
J2(x)	A+B*i	4.171513E-03	0.84667783	-7.064002E+02
J2(x)	A+B/i	3.461413E-03	0.8944339	-8.187346E+02
J2(x)	A+B*sqrt(i)	2.735010E-03	0.93409241	-9.605308E+02
J2(x)	A+B*log(i)^4	1.670610E-03	0.97540949	-1.257285E+03
J3(x)	A+B*i	2.263717E-03	0.94877667	-1.074385E+03
J3(x)	A+B/i	1.558111E-03	0.97573277	-1.299253E+03
J3(x)	A+B*sqrt(i)	5.781323E-03	0.66589889	-5.099353E+02
J3(x)	A+B*log(i)^4	3.799440E-03	0.85570122	-7.626422E+02
J4(x)	A+B*i	2.770395E-03	0.91599686	-9.527923E+02
J4(x)	A+B/i	1.298847E-03	0.98153589	-1.408815E+03
J4(x)	A+B*sqrt(i)	1.742076E-03	0.96678405	-1.232067E+03
J4(x)	A+B*log(i)^4	2.723759E-03	0.9188012	-9.630123E+02
J5(x)	A+B*i	1.880032E-03	0.95682964	-1.186188E+03
J5(x)	A+B/i	8.287698E-04	0.99161075	-1.679287E+03
J5(x)	A+B*sqrt(i)	3.049958E-03	0.88638308	-8.949171E+02
J5(x)	A+B*log(i)^4	4.723996E-03	0.72743238	-6.315258E+02
ln(x)	A+B*i	5.201567E-04	0.99929376	-4.918655E+03
ln(x)	A+B/i	7.241345E-05	0.99998631	-8.471727E+03
ln(x)	A+B*sqrt(i)	1.252893E-04	0.99995903	-7.483810E+03
ln(x)	A+B*log(i)^4	3.137635E-04	0.99974303	-5.829549E+03
log(x)	A+B*i	2.027032E-04	0.99943136	-6.616837E+03
log(x)	A+B/i	3.308050E-05	0.99998486	-9.883500E+03
log(x)	A+B*sqrt(i)	1.824826E-04	0.99953915	-6.806206E+03
log(x)	A+B*log(i)^4	1.118853E-04	0.99982676	-7.687709E+03
log10Gamma(x)	A+B*i	5.825969E-03	0.99998475	-5.336251E+02
log10Gamma(x)	A+B/i	1.021728E-02	0.99995311	5.685387E+02
log10Gamma(x)	A+B*sqrt(i)	1.090399E-02	0.99994659	6.961632E+02
log10Gamma(x)	A+B*log(i)^4	2.435512E-03	0.99999734	-2.244819E+03
Si(x)	A+B*i	2.299120E-03	0.91477903	-7.459844E+02
Si(x)	A+B/i	2.449557E-03	0.90326169	-7.221532E+02
Si(x)	A+B*sqrt(i)	2.948277E-03	0.85986076	-6.524753E+02
Si(x)	A+B*log(i)^4	1.348022E-03	0.97070341	-9.467264E+02
sinh(x)	A+B*i	1.841787E-02	0.99949444	5.539946E+02
sinh(x)	A+B/i	9.057064E-04	0.99999878	-2.464392E+03

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
sinh(x)	A+B*sqrt(i)	6.343980E-03	0.99994002	-5.139521E+02
sinh(x)	A+B*log(i)^4	4.190512E-03	0.99997383	-9.294649E+02
tan(x)	A+B*i	1.217088E-05	0.99999991	-1.512541E+03
tan(x)	A+B/i	2.438552E-05	0.99999966	-1.372163E+03
tan(x)	A+B*sqrt(i)	3.946175E-06	0.99999999	-1.740054E+03
tan(x)	A+B*log(i)^4	2.218884E-05	0.99999972	-1.391232E+03
tanh(x)	A+B*i	6.343755E-06	0.99999984	-4.612515E+03
tanh(x)	A+B/i	8.973283E-06	0.99999968	-4.403753E+03
tanh(x)	A+B*sqrt(i)	1.613681E-05	0.99999896	-4.050469E+03
tanh(x)	A+B*log(i)^4	3.071150E-05	0.99999624	-3.663061E+03
tinv(0.95,x)	A+B*i	1.456138E-03	0.87015927	-3.254012E+03
tinv(0.95,x)	A+B/i	2.689934E-03	0.55691273	-2.049876E+03
tinv(0.95,x)	A+B*sqrt(i)	1.860823E-03	0.78796069	-2.772868E+03
tinv(0.95,x)	A+B*log(i)^4	1.902921E-03	0.77825826	-2.728976E+03
tinv(0.975,x)	A+B*i	3.516175E-03	0.75061148	-1.524340E+03
tinv(0.975,x)	A+B/i	3.723075E-03	0.72039887	-1.412161E+03
tinv(0.975,x)	A+B*sqrt(i)	3.646222E-03	0.73182294	-1.453085E+03
tinv(0.975,x)	A+B*log(i)^4	2.640213E-03	0.8593909	-2.086481E+03
trigamma(x)	A+B*i	2.472601E-03	0.6183158	-2.227901E+03
trigamma(x)	A+B/i	2.452023E-03	0.62464234	-2.244465E+03
trigamma(x)	A+B*sqrt(i)	2.418891E-03	0.63471772	-2.271429E+03
trigamma(x)	A+B*log(i)^4	2.580027E-03	0.5844296	-2.143608E+03

Cosine Series of Order 5

The next table shows a summary of results for the Sine series of the order 5:

$$Y = a_0 + a_1 * \cos(C_1 * gx(1,A_1,B_1) + Oc_1) + \ldots + a_5 * \cos(C_5 * gx(5,A_5,B_5) + Oc_5)$$

The output text files for this series are located in the following folder:

Fourier-Shammas Series Approximations 5 Cosine

The files are named using the following general format:

*fxName_n*_5_cos_run1.txt

Where fxName is the function name and n is the gx series number which varies between 1 and 4. Each output files specifies the function being approximated and the gx series being used. For example, the file for the inverse hyperboplic cosine (acosh) using the first gx series is named acosh_1_5_cos_run1.txt.

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
10^x	A+B*i	1.235405E-06	1	-1.970384E+03
10^x	A+B/i	1.022668E-06	1	-2.008559E+03
10^x	A+B*sqrt(i)	5.914507E-07	1	-2.119173E+03
10^x	A+B*log(i)^4	7.299811E-06	1	-1.611541E+03
acosh(x)	A+B*i	2.108761E-03	0.99454159	-2.539352E+03
acosh(x)	A+B/i	6.784050E-03	0.94350766	-2.234370E+02
acosh(x)	A+B*sqrt(i)	4.205296E-03	0.97829276	-1.171288E+03
acosh(x)	A+B*log(i)^4	4.204430E-03	0.9783017	-1.171696E+03
arccos(x)	A+B*i	8.019826E-04	0.99953585	-6.622952E+02
arccos(x)	A+B/i	8.413702E-04	0.99948914	-6.526104E+02
arccos(x)	A+B*sqrt(i)	8.443978E-04	0.99948546	-6.518848E+02
arccos(x)	A+B*log(i)^4	8.556072E-04	0.99947171	-6.492209E+02
arcsin(x)	A+B*i	8.392406E-04	0.99949173	-6.531223E+02
arcsin(x)	A+B/i	8.059431E-04	0.99953126	-6.613001E+02
arcsin(x)	A+B*sqrt(i)	8.185037E-04	0.99951653	-6.581762E+02
arcsin(x)	A+B*log(i)^4	7.861717E-04	0.99955397	-6.663174E+02
arctan(x)	A+B*i	3.848901E-08	1	-2.671078E+03
arctan(x)	A+B/i	1.448856E-08	1	-2.868435E+03
arctan(x)	A+B*sqrt(i)	1.022431E-08	1	-2.938850E+03
arctan(x)	A+B*log(i)^4	1.993571E-08	1	-2.803966E+03
asinh(x)	A+B*i	3.739776E-03	0.98475184	-1.408173E+03
asinh(x)	A+B/i	6.512735E-03	0.95375624	-2.975965E+02
asinh(x)	A+B*sqrt(i)	4.240464E-03	0.98039563	-1.156629E+03
asinh(x)	A+B*log(i)^4	4.270111E-03	0.98012054	-1.142680E+03
atanh(x)	A+B*i	1.536321E-03	0.99289911	-3.187012E+03
atanh(x)	A+B/i	1.612373E-03	0.99217868	-3.090380E+03
atanh(x)	A+B*sqrt(i)	1.578604E-03	0.99250286	-3.132712E+03
atanh(x)	A+B*log(i)^4	1.618621E-03	0.99211795	-3.082645E+03
Ci(x)	A+B*i	2.693012E-03	0.9286124	-7.043225E+02
Ci(x)	A+B/i	2.213734E-03	0.95176111	-7.811464E+02
Ci(x)	A+B*sqrt(i)	2.886945E-03	0.91796041	-6.770633E+02
Ci(x)	A+B*log(i)^4	3.022324E-03	0.91008578	-6.590990E+02
cosh(x)	A+B*i	4.815259E-03	0.99996487	-7.861713E+02
cosh(x)	A+B/i	2.945540E-04	0.99999987	-3.585842E+03
cosh(x)	A+B*sqrt(i)	1.438064E-03	0.99999687	-1.997080E+03
cosh(x)	A+B*log(i)^4	4.501460E-03	0.9999693	-8.536941E+02
digamma(x)	A+B/i	5.762265E-03	0.95517541	-5.511718E+02
digamma(x)	A+B*i	3.312998E-03	0.9851826	-1.637094E+03
digamma(x)	A+B*sqrt(i)	3.533995E-03	0.98313984	-1.510397E+03
digamma(x)	A+B*log(i)^4	2.052556E-03	0.99431252	-2.576436E+03
erf(x)	A+B*i	1.583484E-06	0.99999999	-3.883737E+03
erf(x)	A+B/i	2.794279E-07	1	-4.615754E+03
erf(x)	A+B*sqrt(i)	3.781136E-07	1	-4.488120E+03
erf(x)	A+B*log(i)^4	6.971867E-08	1	-5.201607E+03
exp(x)	A+B*i	3.273122E-05	0.99999993	-2.490716E+03
exp(x)	A+B/i	2.775627E-07	1	-4.408271E+03
exp(x)	A+B*sqrt(i)	7.071978E-07	1	-4.032296E+03
exp(x)	A+B*log(i)^4	1.547873E-05	0.99999998	-2.791759E+03
FresnelCosine(x)	A+B*i	4.487680E-04	0.99617326	-2.893178E+03

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
FresnelCosine(x)	A+B/i	2.116216E-03	0.91490474	-1.494286E+03
FresnelCosine(x)	A+B*sqrt(i)	5.262550E-04	0.99473767	-2.749508E+03
FresnelCosine(x)	A+B*log(i)^4	1.842320E-03	0.93550656	-1.619306E+03
FresnelSine(x)	A+B*i	1.149752E-03	0.97077118	-1.862323E+03
FresnelSine(x)	A+B/i	1.286461E-03	0.96340714	-1.772220E+03
FresnelSine(x)	A+B*sqrt(i)	2.047180E-03	0.90733507	-1.399636E+03
FresnelSine(x)	A+B*log(i)^4	2.058364E-03	0.90631983	-1.395266E+03
J0(x)	A+B*i	1.112538E-03	0.98747186	-1.416097E+03
J0(x)	A+B/i	9.962118E-04	0.98995477	-1.478164E+03
J0(x)	A+B*sqrt(i)	6.369618E-04	0.99589339	-1.729518E+03
J0(x)	A+B*log(i)^4	7.204562E-04	0.99474622	-1.660294E+03
J1(x)	A+B*i	9.157231E-04	0.9938118	-1.615143E+03
J1(x)	A+B/i	9.035537E-04	0.99397518	-1.623196E+03
J1(x)	A+B*sqrt(i)	2.908984E-03	0.937552	-9.193239E+02
J1(x)	A+B*log(i)^4	1.277999E-03	0.98794695	-1.414474E+03
J2(x)	A+B*i	3.930780E-03	0.86340183	-7.381013E+02
J2(x)	A+B/i	1.811102E-03	0.97100167	-1.204593E+03
J2(x)	A+B*sqrt(i)	5.188662E-03	0.76198836	-5.709633E+02
J2(x)	A+B*log(i)^4	4.278574E-03	0.83816016	-6.870627E+02
J3(x)	A+B*i	2.525740E-03	0.93601614	-1.004368E+03
J3(x)	A+B/i	2.588901E-04	0.99932776	-2.375656E+03
J3(x)	A+B*sqrt(i)	4.918389E-03	0.75737315	-6.031672E+02
J3(x)	A+B*log(i)^4	1.526307E-03	0.97663441	-1.307586E+03
J4(x)	A+B*i	1.345641E-03	0.98011434	-1.383426E+03
J4(x)	A+B/i	1.076843E-04	0.99987265	-2.903730E+03
J4(x)	A+B*sqrt(i)	2.811506E-03	0.91319197	-9.398422E+02
J4(x)	A+B*log(i)^4	7.572084E-04	0.9937033	-1.729568E+03
J5(x)	A+B*i	2.641541E-03	0.9144855	-9.773815E+02
J5(x)	A+B/i	7.857900E-05	0.99992433	-3.093420E+03
J5(x)	A+B*sqrt(i)	1.511657E-03	0.97199525	-1.313391E+03
J5(x)	A+B*log(i)^4	2.545682E-03	0.9205794	-9.996339E+02
ln(x)	A+B*i	4.483424E-04	0.99947473	-5.182356E+03
ln(x)	A+B/i	3.438990E-05	0.99999691	-9.809522E+03
ln(x)	A+B*sqrt(i)	1.821398E-04	0.99991331	-6.805567E+03
ln(x)	A+B*log(i)^4	3.796908E-04	0.99962327	-5.481849E+03
log(x)	A+B*i	2.121886E-04	0.9993762	-6.530399E+03
log(x)	A+B/i	1.613884E-05	0.99999639	-1.117280E+04
log(x)	A+B*sqrt(i)	2.200406E-05	0.99999329	-1.061418E+04
log(x)	A+B*log(i)^4	1.115317E-04	0.99982766	-7.689385E+03
log10Gamma(x)	A+B*i	1.287559E-03	0.99999925	-3.491391E+03
log10Gamma(x)	A+B/i	6.572071E-03	0.99998058	-2.931720E+02
log10Gamma(x)	A+B*sqrt(i)	4.229002E-03	0.99999196	-1.158145E+03
log10Gamma(x)	A+B*log(i)^4	6.456210E-03	0.99998126	-3.280692E+02
Si(x)	A+B*i	1.839997E-03	0.94511705	-8.256086E+02
Si(x)	A+B/i	1.299744E-03	0.97261463	-9.563050E+02
Si(x)	A+B*sqrt(i)	1.414258E-03	0.96757647	-9.245564E+02
Si(x)	A+B*log(i)^4	1.272816E-03	0.97373759	-9.641767E+02
sinh(x)	A+B*i	5.763877E-03	0.99995039	-6.059912E+02
sinh(x)	A+B/i	1.989014E-04	0.99999994	-3.979280E+03

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
sinh(x)	A+B*sqrt(i)	1.664919E-03	0.99999586	-1.850309E+03
sinh(x)	A+B*log(i)^4	1.296365E-02	0.99974904	2.060701E+02
tan(x)	A+B*i	4.104082E-06	0.99999999	-1.727866E+03
tan(x)	A+B/i	5.248551E-06	0.99999998	-1.678180E+03
tan(x)	A+B*sqrt(i)	5.527030E-06	0.99999998	-1.667737E+03
tan(x)	A+B*log(i)^4	4.634873E-06	0.99999999	-1.703298E+03
tanh(x)	A+B*i	3.083530E-06	0.99999996	-5.042713E+03
tanh(x)	A+B/i	2.317933E-06	0.99999998	-5.214524E+03
tanh(x)	A+B*sqrt(i)	1.006120E-06	1	-5.716937E+03
tanh(x)	A+B*log(i)^4	2.375620E-05	0.99999774	-3.813569E+03
tinv(0.95,x)	A+B*i	1.999856E-03	0.75484043	-2.627469E+03
tinv(0.95,x)	A+B/i	1.861547E-03	0.78757801	-2.768080E+03
tinv(0.95,x)	A+B*sqrt(i)	1.906723E-03	0.77714281	-2.721035E+03
tinv(0.95,x)	A+B*log(i)^4	2.018904E-03	0.75014807	-2.608870E+03
tinv(0.975,x)	A+B*i	2.744472E-03	0.84791083	-2.006470E+03
tinv(0.975,x)	A+B/i	3.891627E-03	0.6941961	-1.321264E+03
tinv(0.975,x)	A+B*sqrt(i)	3.725395E-03	0.71976313	-1.406914E+03
tinv(0.975,x)	A+B*log(i)^4	2.670307E-03	0.85601967	-2.060219E+03
trigamma(x)	A+B*i	2.409666E-03	0.63713048	-2.274978E+03
trigamma(x)	A+B/i	2.590731E-03	0.58054888	-2.131378E+03
trigamma(x)	A+B*sqrt(i)	2.732557E-03	0.53336721	-2.025742E+03
trigamma(x)	A+B*log(i)^4	2.076503E-03	0.73053543	-2.569906E+03

Cosine Series of Order 6

The next table shows a summary of results for the Sine series of the order 6:

$$Y = a_0 + a_1 * \cos(C_1 * gx(1,A_1,B_1) + Oc_1) + \ldots + a_6 * \cos(C_6 * gx(6,A_6,B_6) + Oc_6)$$

The output text files for this series are located in the following folder:

Fourier-Shammas Series Approximations 6 Cosine

The files are named using the following general format:

*fxName_n*_6_cos_run1.txt

Where fxName is the function name and n is the gx series number which varies between 1 and 4. Each output files specifies the function being approximated and the gx series being used. For example, the file for the inverse hyperboplic cosine (acosh) using the first gx series is named acosh_1_6_cos_run1.txt.

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
10^x	A+B*i	4.497158E-07	1	-2.170202E+03
10^x	A+B/i	1.375388E-07	1	-2.409515E+03
10^x	A+B*sqrt(i)	4.823951E-07	1	-2.156032E+03
10^x	A+B*log(i)^4	7.334725E-06	1	-1.606267E+03
acosh(x)	A+B*i	3.830994E-03	0.98196669	-1.352022E+03
acosh(x)	A+B/i	6.537885E-03	0.94747971	-2.926643E+02
acosh(x)	A+B*sqrt(i)	2.488389E-03	0.99239167	-2.207233E+03
acosh(x)	A+B*log(i)^4	2.204638E-03	0.9940279	-2.447199E+03
arccos(x)	A+B*i	7.428383E-04	0.99959755	-6.734595E+02
arccos(x)	A+B/i	6.835200E-04	0.99965926	-6.902704E+02
arccos(x)	A+B*sqrt(i)	7.172873E-04	0.99962476	-6.805298E+02
arccos(x)	A+B*log(i)^4	7.216041E-04	0.99962023	-6.793178E+02
arcsin(x)	A+B*i	7.151439E-04	0.999627	-6.811343E+02
arcsin(x)	A+B/i	7.704259E-04	0.9995671	-6.660935E+02
arcsin(x)	A+B*sqrt(i)	7.271618E-04	0.99961436	-6.777680E+02
arcsin(x)	A+B*log(i)^4	7.322152E-04	0.99960898	-6.763690E+02
arctan(x)	A+B*i	5.242430E-08	1	-2.604350E+03
arctan(x)	A+B/i	3.600432E-09	1	-3.145348E+03
arctan(x)	A+B*sqrt(i)	7.577148E-10	1	-3.458417E+03
arctan(x)	A+B*log(i)^4	1.138694E-08	1	-2.912784E+03
asinh(x)	A+B*i	2.657590E-03	0.99229205	-2.088041E+03
asinh(x)	A+B/i	6.810401E-03	0.94938161	-2.040959E+02
asinh(x)	A+B*sqrt(i)	5.571382E-03	0.96612425	-6.061131E+02
asinh(x)	A+B*log(i)^4	2.589112E-03	0.99268415	-2.140302E+03
atanh(x)	A+B*i	1.394530E-03	0.99414345	-3.376651E+03
atanh(x)	A+B/i	1.379832E-03	0.99426625	-3.397841E+03
atanh(x)	A+B*sqrt(i)	1.364734E-03	0.99439104	-3.419846E+03
atanh(x)	A+B*log(i)^4	1.288413E-03	0.99500084	-3.534942E+03
Ci(x)	A+B*i	3.464627E-03	0.8812179	-6.014088E+02
Ci(x)	A+B/i	1.955742E-03	0.96215042	-8.255683E+02
Ci(x)	A+B*sqrt(i)	3.679557E-03	0.86602332	-5.778153E+02
Ci(x)	A+B*log(i)^4	2.117162E-03	0.95564463	-7.944800E+02
cosh(x)	A+B*i	2.075628E-02	0.99934587	6.818668E+02
cosh(x)	A+B/i	1.773657E-05	1	-6.397239E+03
cosh(x)	A+B*sqrt(i)	1.679635E-03	0.99999572	-1.837434E+03
cosh(x)	A+B*log(i)^4	4.807491E-03	0.99996491	-7.837320E+02
digamma(x)	A+B/i	5.507157E-03	0.95901449	-6.359866E+02
digamma(x)	A+B*i	2.910514E-03	0.98855239	-1.887191E+03
digamma(x)	A+B*sqrt(i)	1.850624E-03	0.99537181	-2.775598E+03
digamma(x)	A+B*log(i)^4	2.282615E-03	0.9929589	-2.363973E+03
erf(x)	A+B*i	4.692905E-07	1	-4.392817E+03
erf(x)	A+B/i	1.283509E-08	1	-5.911610E+03
erf(x)	A+B*sqrt(i)	8.517839E-09	1	-6.084639E+03
erf(x)	A+B*log(i)^4	4.550292E-07	1	-4.405840E+03
exp(x)	A+B*i	1.537292E-05	0.99999998	-2.790369E+03
exp(x)	A+B/i	3.565131E-08	1	-5.229006E+03
exp(x)	A+B*sqrt(i)	1.167248E-07	1	-4.752346E+03
exp(x)	A+B*log(i)^4	2.818170E-06	1	-3.472370E+03
FresnelCosine(x)	A+B*i	1.267224E-03	0.96941776	-1.952768E+03

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
FresnelCosine(x)	A+B/i	6.344077E-04	0.99233523	-2.576855E+03
FresnelCosine(x)	A+B*sqrt(i)	1.458919E-03	0.95946549	-1.825706E+03
FresnelCosine(x)	A+B*log(i)^4	2.590449E-03	0.87220541	-1.307836E+03
FresnelSine(x)	A+B*i	1.223021E-03	0.96684325	-1.808706E+03
FresnelSine(x)	A+B/i	1.441006E-03	0.95397054	-1.677163E+03
FresnelSine(x)	A+B*sqrt(i)	7.958852E-04	0.98595878	-2.153264E+03
FresnelSine(x)	A+B*log(i)^4	1.759959E-03	0.93133915	-1.516805E+03
J0(x)	A+B*i	7.227734E-04	0.99469307	-1.654386E+03
J0(x)	A+B/i	1.585804E-03	0.97445312	-1.212794E+03
J0(x)	A+B*sqrt(i)	1.952735E-03	0.96126306	-1.095819E+03
J0(x)	A+B*log(i)^4	7.850119E-04	0.99373976	-1.607963E+03
J1(x)	A+B*i	6.340232E-05	0.99997023	-3.218515E+03
J1(x)	A+B/i	2.430171E-03	0.95626952	-1.023492E+03
J1(x)	A+B*sqrt(i)	9.376972E-04	0.99348918	-1.596771E+03
J1(x)	A+B*log(i)^4	1.784758E-03	0.97641317	-1.209317E+03
J2(x)	A+B*i	1.679977E-03	0.97496379	-1.245740E+03
J2(x)	A+B/i	1.088896E-03	0.98948197	-1.506776E+03
J2(x)	A+B*sqrt(i)	3.137827E-03	0.91265863	-8.696399E+02
J2(x)	A+B*log(i)^4	3.283563E-03	0.90435712	-8.423100E+02
J3(x)	A+B*i	5.369078E-03	0.70988693	-5.462902E+02
J3(x)	A+B/i	5.838835E-05	0.99996569	-3.268110E+03
J3(x)	A+B*sqrt(i)	7.263389E-04	0.99469059	-1.750528E+03
J3(x)	A+B*log(i)^4	8.602452E-05	0.99992552	-3.034825E+03
J4(x)	A+B*i	3.025672E-03	0.89912107	-8.915510E+02
J4(x)	A+B/i	6.561328E-04	0.99525605	-1.811723E+03
J4(x)	A+B*sqrt(i)	2.996649E-04	0.99901047	-2.283510E+03
J4(x)	A+B*log(i)^4	1.342343E-03	0.98014438	-1.380807E+03
J5(x)	A+B*i	4.721924E-03	0.72581885	-6.236110E+02
J5(x)	A+B/i	4.658310E-04	0.99733157	-2.017933E+03
J5(x)	A+B*sqrt(i)	1.441040E-03	0.97446406	-1.338095E+03
J5(x)	A+B*log(i)^4	2.021656E-04	0.99949741	-2.520443E+03
ln(x)	A+B*i	5.457280E-04	0.99922088	-4.824118E+03
ln(x)	A+B/i	1.732813E-05	0.99999921	-1.104064E+04
ln(x)	A+B*sqrt(i)	4.844457E-04	0.99938604	-5.038763E+03
ln(x)	A+B*log(i)^4	3.548589E-04	0.99967057	-5.599699E+03
log(x)	A+B*i	2.229312E-04	0.99931067	-6.437372E+03
log(x)	A+B/i	7.443218E-06	0.99999923	-1.256338E+04
log(x)	A+B*sqrt(i)	2.128854E-04	0.9993714	-6.520460E+03
log(x)	A+B*log(i)^4	1.079900E-04	0.99983825	-7.743505E+03
log10Gamma(x)	A+B*i	5.677188E-03	0.99998549	-5.763270E+02
log10Gamma(x)	A+B/i	6.432528E-03	0.99998138	-3.312507E+02
log10Gamma(x)	A+B*sqrt(i)	4.710966E-03	0.99999001	-9.423638E+02
log10Gamma(x)	A+B*log(i)^4	1.052652E-03	0.9999995	-3.882577E+03
Si(x)	A+B*i	1.929430E-03	0.93931887	-8.036054E+02
Si(x)	A+B/i	7.854921E-04	0.98994274	-1.141505E+03
Si(x)	A+B*sqrt(i)	1.588046E-03	0.95889243	-8.768202E+02
Si(x)	A+B*log(i)^4	6.718985E-04	0.99264127	-1.200237E+03
sinh(x)	A+B*i	5.383626E-03	0.99995663	-6.703186E+02
sinh(x)	A+B/i	3.750108E-05	1	-5.647000E+03

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
sinh(x)	A+B*sqrt(i)	1.420978E-03	0.99999698	-2.005000E+03
sinh(x)	A+B*log(i)^4	4.380209E-03	0.99997129	-8.769969E+02
tan(x)	A+B*i	1.489224E-06	1	-1.928329E+03
tan(x)	A+B/i	1.063654E-06	1	-1.996311E+03
tan(x)	A+B*sqrt(i)	1.723952E-06	1	-1.898763E+03
tan(x)	A+B*log(i)^4	3.385755E-06	0.99999999	-1.762422E+03
tanh(x)	A+B*i	2.580307E-05	0.99999733	-3.759717E+03
tanh(x)	A+B/i	9.484891E-07	1	-5.748351E+03
tanh(x)	A+B*sqrt(i)	3.751777E-07	1	-6.306688E+03
tanh(x)	A+B*log(i)^4	8.883256E-06	0.99999968	-4.401645E+03
tinv(0.95,x)	A+B*i	1.808544E-03	0.79929646	-2.820725E+03
tinv(0.95,x)	A+B/i	2.114020E-03	0.72576999	-2.514518E+03
tinv(0.95,x)	A+B*sqrt(i)	2.047079E-03	0.74286213	-2.577650E+03
tinv(0.95,x)	A+B*log(i)^4	2.083370E-03	0.73366408	-2.543172E+03
tinv(0.975,x)	A+B*i	3.045756E-03	0.81249344	-1.798078E+03
tinv(0.975,x)	A+B/i	3.244148E-03	0.78727049	-1.674268E+03
tinv(0.975,x)	A+B*sqrt(i)	3.478875E-03	0.75537321	-1.537211E+03
tinv(0.975,x)	A+B*log(i)^4	3.586934E-03	0.73994027	-1.477195E+03
trigamma(x)	A+B*i	2.982377E-03	0.44357956	-1.848322E+03
trigamma(x)	A+B/i	2.604487E-03	0.57565204	-2.116854E+03
trigamma(x)	A+B*sqrt(i)	2.481511E-03	0.6147786	-2.212719E+03
trigamma(x)	A+B*log(i)^4	2.037530E-03	0.74029162	-2.603429E+03

Cosine Series of Order 7

The next table shows a summary of results for the Sine series of the order 7:

$$Y = a_0 + a_1 * \cos(C_1 * gx(1,A_1,B_1) + Oc_1) + \ldots + a_7 * \cos(C_7 * gx(7,A_7,B_7) + Oc_7)$$

The output text files for this series are located in the following folder:

Fourier-Shammas Series Approximations 7 Cosine

The files are named using the following general format:

*fxName_n_*7_cos_run1.txt

Where fxName is the function name and n is the gx series number which varies between 1 and 4. Each output files specifies the function being approximated and the gx series being used. For example, the file for the inverse hyperboplic cosine (acosh) using the first gx series is named acosh_1_7_cos_run1.txt.

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
10^x	A+B*i	6.743020E-07	1	-2.084019E+03
10^x	A+B/i	1.021083E-08	1	-Inf
10^x	A+B*sqrt(i)	1.475173E-08	1	-2.856098E+03
10^x	A+B*log(i)^4	2.042822E-06	1	-1.860120E+03
acosh(x)	A+B*i	3.829398E-03	0.98196338	-1.348816E+03
acosh(x)	A+B/i	4.546892E-03	0.97457135	-1.008434E+03
acosh(x)	A+B*sqrt(i)	2.990002E-03	0.98900394	-1.839229E+03
acosh(x)	A+B*log(i)^4	3.193615E-03	0.98745533	-1.708656E+03
arccos(x)	A+B*i	6.005479E-04	0.99973413	-7.120510E+02
arccos(x)	A+B/i	5.859935E-04	0.99974687	-7.170068E+02
arccos(x)	A+B*sqrt(i)	5.895259E-04	0.9997438	-7.157928E+02
arccos(x)	A+B*log(i)^4	7.189340E-04	0.99961898	-6.757057E+02
arcsin(x)	A+B*i	5.926509E-04	0.99974108	-7.147248E+02
arcsin(x)	A+B/i	5.890742E-04	0.9997442	-7.159476E+02
arcsin(x)	A+B*sqrt(i)	6.214318E-04	0.99971532	-7.051459E+02
arcsin(x)	A+B*log(i)^4	6.968992E-04	0.99964198	-6.819937E+02
arctan(x)	A+B*i	2.938997E-09	1	-3.182009E+03
arctan(x)	A+B/i	3.252243E-10	1	-Inf
arctan(x)	A+B*sqrt(i)	1.252821E-09	1	-3.353725E+03
arctan(x)	A+B*log(i)^4	1.092659E-08	1	-2.916759E+03
asinh(x)	A+B*i	3.514545E-03	0.98650608	-1.524469E+03
asinh(x)	A+B/i	6.809740E-03	0.94934046	-2.002577E+02
asinh(x)	A+B*sqrt(i)	2.477076E-03	0.99329685	-2.224830E+03
asinh(x)	A+B*log(i)^4	2.478475E-03	0.99328928	-2.223700E+03
atanh(x)	A+B*i	1.372639E-03	0.99432016	-3.404263E+03
atanh(x)	A+B/i	1.192588E-03	0.99571249	-3.685481E+03
atanh(x)	A+B*sqrt(i)	1.163786E-03	0.99591709	-3.734376E+03
atanh(x)	A+B*log(i)^4	1.382444E-03	0.99423873	-3.390028E+03
Ci(x)	A+B*i	2.534972E-03	0.9360724	-7.197041E+02
Ci(x)	A+B/i	2.267336E-03	0.94885846	-7.634423E+02
Ci(x)	A+B*sqrt(i)	3.012362E-03	0.90972732	-6.520677E+02
Ci(x)	A+B*log(i)^4	1.429993E-03	0.97965724	-9.441292E+02
cosh(x)	A+B*i	2.284842E-03	0.99999206	-1.525032E+03
cosh(x)	A+B/i	7.140264E-06	1	-7.304872E+03
cosh(x)	A+B*sqrt(i)	1.867724E-03	0.99999469	-1.727012E+03
cosh(x)	A+B*log(i)^4	8.300212E-04	0.99999895	-2.539659E+03
digamma(x)	A+B/i	5.056248E-03	0.96541576	-7.995551E+02
digamma(x)	A+B*i	2.711378E-03	0.99005507	-2.022210E+03
digamma(x)	A+B*sqrt(i)	2.244244E-03	0.99318663	-2.393201E+03
digamma(x)	A+B*log(i)^4	6.253559E-04	0.99947098	-4.900251E+03
erf(x)	A+B*i	1.167316E-06	1	-4.004112E+03
erf(x)	A+B/i	2.982498E-09	1	-6.523324E+03
erf(x)	A+B*sqrt(i)	9.365843E-10	1	-7.000241E+03
erf(x)	A+B*log(i)^4	2.727623E-07	1	-4.617642E+03
exp(x)	A+B*i	6.988275E-06	1	-3.103126E+03
exp(x)	A+B/i	8.297127E-09	1	-Inf
exp(x)	A+B*sqrt(i)	2.787044E-07	1	-4.398304E+03
exp(x)	A+B*log(i)^4	2.252917E-06	1	-3.558193E+03
FresnelCosine(x)	A+B*i	1.813705E-03	0.93721223	-1.625290E+03

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
FresnelCosine(x)	A+B/i	1.153232E-03	0.97461515	-2.033719E+03
FresnelCosine(x)	A+B*sqrt(i)	1.156554E-03	0.97446869	-2.031124E+03
FresnelCosine(x)	A+B*log(i)^4	3.567168E-04	0.99757122	-3.092109E+03
FresnelSine(x)	A+B*i	1.110977E-03	0.97257049	-1.881683E+03
FresnelSine(x)	A+B/i	7.808332E-04	0.98645047	-2.164495E+03
FresnelSine(x)	A+B*sqrt(i)	9.248102E-04	0.98099303	-2.028775E+03
FresnelSine(x)	A+B*log(i)^4	1.766703E-03	0.93063588	-1.509655E+03
J0(x)	A+B*i	1.134614E-03	0.98687429	-1.396832E+03
J0(x)	A+B/i	8.206475E-04	0.99313343	-1.578894E+03
J0(x)	A+B*sqrt(i)	4.552350E-04	0.99788701	-1.910069E+03
J0(x)	A+B*log(i)^4	9.675270E-04	0.99045551	-1.486361E+03
J1(x)	A+B*i	6.604109E-04	0.99675945	-1.803700E+03
J1(x)	A+B/i	1.092265E-04	0.99991136	-2.886962E+03
J1(x)	A+B*sqrt(i)	2.415621E-03	0.95664414	-1.022997E+03
J1(x)	A+B*log(i)^4	8.272438E-04	0.99491539	-1.668107E+03
J2(x)	A+B*i	2.305473E-03	0.95268902	-1.051092E+03
J2(x)	A+B/i	4.366440E-05	0.99998303	-3.438930E+03
J2(x)	A+B*sqrt(i)	2.208295E-05	0.99999566	-3.849330E+03
J2(x)	A+B*log(i)^4	1.957893E-03	0.96587918	-1.149469E+03
J3(x)	A+B*i	1.295527E-03	0.98305113	-1.398066E+03
J3(x)	A+B/i	3.691302E-04	0.99862404	-2.153891E+03
J3(x)	A+B*sqrt(i)	3.728185E-03	0.85964029	-7.617496E+02
J3(x)	A+B*log(i)^4	2.533847E-03	0.9351652	-9.942317E+02
J4(x)	A+B*i	1.903989E-03	0.95991654	-1.166276E+03
J4(x)	A+B/i	2.616125E-04	0.99924325	-2.361151E+03
J4(x)	A+B*sqrt(i)	9.789663E-05	0.99989403	-2.952888E+03
J4(x)	A+B*log(i)^4	1.317293E-03	0.98081325	-1.388036E+03
J5(x)	A+B*i	5.248234E-03	0.66013548	-5.558835E+02
J5(x)	A+B/i	3.435563E-04	0.99854362	-2.197114E+03
J5(x)	A+B*sqrt(i)	2.228504E-03	0.93872178	-1.071533E+03
J5(x)	A+B*log(i)^4	6.551881E-05	0.99994703	-3.194636E+03
ln(x)	A+B*i	4.936666E-04	0.99936173	-5.000751E+03
ln(x)	A+B/i	9.874688E-06	0.99999974	-1.204997E+04
ln(x)	A+B*sqrt(i)	1.589417E-04	0.99993384	-7.042999E+03
ln(x)	A+B*log(i)^4	2.381617E-04	0.99985145	-6.314247E+03
log(x)	A+B*i	2.085417E-04	0.99939611	-6.553573E+03
log(x)	A+B/i	6.647896E-06	0.99999939	-1.276297E+04
log(x)	A+B*sqrt(i)	1.817805E-04	0.99954116	-6.801058E+03
log(x)	A+B*log(i)^4	6.380000E-05	0.99994348	-8.687836E+03
log10Gamma(x)	A+B*i	7.243508E-03	0.99997636	-9.425349E+01
log10Gamma(x)	A+B/i	8.992663E-03	0.99996356	3.301339E+02
log10Gamma(x)	A+B*sqrt(i)	3.150823E-03	0.99999553	-1.727505E+03
log10Gamma(x)	A+B*log(i)^4	7.767485E-03	0.99997282	4.277429E+01
Si(x)	A+B*i	1.898271E-03	0.94093664	-8.055448E+02
Si(x)	A+B/i	1.797161E-03	0.947061	-8.261253E+02
Si(x)	A+B*sqrt(i)	1.958922E-03	0.93710211	-7.937193E+02
Si(x)	A+B*log(i)^4	5.085593E-04	0.99576079	-1.300781E+03
sinh(x)	A+B*i	5.299457E-03	0.99995789	-6.820424E+02
sinh(x)	A+B/i	6.380504E-06	1	-7.417599E+03

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
sinh(x)	A+B*sqrt(i)	1.906399E-03	0.99999455	-1.706475E+03
sinh(x)	A+B*log(i)^4	6.899811E-04	0.99999929	-2.724815E+03
tan(x)	A+B*i	2.393438E-06	1	-1.828123E+03
tan(x)	A+B/i	2.016224E-07	1	-2.327890E+03
tan(x)	A+B*sqrt(i)	3.190561E-07	1	-2.235178E+03
tan(x)	A+B*log(i)^4	1.352552E-06	1	-1.943413E+03
tanh(x)	A+B*i	3.223701E-05	0.99999582	-3.621588E+03
tanh(x)	A+B/i	4.607066E-07	1	-6.178949E+03
tanh(x)	A+B*sqrt(i)	7.339715E-07	1	-5.898593E+03
tanh(x)	A+B*log(i)^4	2.906121E-06	0.99999997	-5.070178E+03
tinv(0.95,x)	A+B*i	1.886963E-03	0.7812895	-2.733413E+03
tinv(0.95,x)	A+B/i	1.511267E-03	0.85971044	-3.169016E+03
tinv(0.95,x)	A+B*sqrt(i)	2.275021E-03	0.68208298	-2.366478E+03
tinv(0.95,x)	A+B*log(i)^4	1.689776E-03	0.82461132	-2.949963E+03
tinv(0.975,x)	A+B*i	2.096589E-03	0.91105961	-2.526729E+03
tinv(0.975,x)	A+B/i	3.303687E-03	0.77916382	-1.634554E+03
tinv(0.975,x)	A+B*sqrt(i)	4.460572E-03	0.59741878	-1.045487E+03
tinv(0.975,x)	A+B*log(i)^4	2.212478E-03	0.90095551	-2.421171E+03
trigamma(x)	A+B*i	2.072806E-03	0.73094762	-2.565376E+03
trigamma(x)	A+B/i	2.573622E-03	0.5852286	-2.136450E+03
trigamma(x)	A+B*sqrt(i)	2.423551E-03	0.63218979	-2.255529E+03
trigamma(x)	A+B*log(i)^4	2.665587E-03	0.55505632	-2.066861E+03

Alternating Sine/Cosine Series of Order 3

The next table shows a summary of results for the Sine series of the order 3:

$$Y = a_0 + a_1 * \sin(S_1 * gx(1,A_1,B_1) + Os_1) + a_2 * \cos(S_2 * gx(2,A_2,B_2) + Os_2)$$
$$+ a_3 * \sin(S_3 * gx(3,A_3,B_3) + Os_3)$$

The output text files for this series are located in the following folder:

Fourier-Shammas Series Approximations 3 Sine Cosine

The files are named using the following general format:

*fxName_n*_3_run1.txt

Where fxName is the function name and n is the gx series number which varies between 1 and 4. Each output files specifies the function being approximated and the gx series being used. For example, the file for the inverse hyperboplic cosine (acosh) using the first gx series is named acosh_1_3_run1.txt.

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
10^x	A+B*i	7.296351E-05	0.99999991	-1.154992E+03
10^x	A+B/i	7.964428E-05	0.9999999	-1.137295E+03
10^x	A+B*sqrt(i)	7.873855E-05	0.9999999	-1.139605E+03
10^x	A+B*log(i)^4	7.742904E-05	0.9999999	-1.142993E+03
acosh(x)	A+B*i	3.712953E-03	0.98311234	-1.426126E+03
acosh(x)	A+B/i	7.103324E-03	0.93819071	-1.403326E+02
acosh(x)	A+B*sqrt(i)	7.335333E-03	0.93408714	-7.663108E+01
acosh(x)	A+B*log(i)^4	4.338807E-03	0.97693937	-1.117386E+03
arccos(x)	A+B*i	1.232522E-03	0.99892634	-5.839664E+02
arccos(x)	A+B/i	1.065355E-03	0.99919783	-6.134089E+02
arccos(x)	A+B*sqrt(i)	1.285387E-03	0.99883226	-5.754831E+02
arccos(x)	A+B*log(i)^4	1.091163E-03	0.99915849	-6.085738E+02
arcsin(x)	A+B*i	1.213275E-03	0.99895961	-5.871459E+02
arcsin(x)	A+B/i	1.317075E-03	0.99877398	-5.705637E+02
arcsin(x)	A+B*sqrt(i)	1.249874E-03	0.9988959	-5.811425E+02
arcsin(x)	A+B*log(i)^4	1.292469E-03	0.99881936	-5.743732E+02
arctan(x)	A+B*i	2.580034E-07	1	-2.295230E+03
arctan(x)	A+B/i	3.793481E-07	1	-2.217363E+03
arctan(x)	A+B*sqrt(i)	2.071295E-07	1	-2.339595E+03
arctan(x)	A+B*log(i)^4	9.030133E-07	1	-2.042171E+03
asinh(x)	A+B*i	4.585934E-03	0.97711717	-1.007874E+03
asinh(x)	A+B/i	7.664873E-03	0.93607597	2.046179E+01
asinh(x)	A+B*sqrt(i)	8.472098E-03	0.92190269	2.209224E+02
asinh(x)	A+B*log(i)^4	5.012883E-03	0.97265806	-8.296610E+02
atanh(x)	A+B*i	2.079288E-03	0.98701908	-2.589787E+03
atanh(x)	A+B/i	2.192682E-03	0.98556465	-2.483588E+03
atanh(x)	A+B*sqrt(i)	2.459003E-03	0.98184508	-2.254326E+03
atanh(x)	A+B*log(i)^4	2.300410E-03	0.98411136	-2.387663E+03
Ci(x)	A+B*i	2.241111E-03	0.95107562	-7.845634E+02
Ci(x)	A+B/i	2.830169E-03	0.92197683	-6.930843E+02
Ci(x)	A+B*sqrt(i)	3.888672E-03	0.85270051	-5.685338E+02
Ci(x)	A+B*log(i)^4	2.635477E-03	0.9323423	-7.210231E+02
cosh(x)	A+B*i	1.626274E-02	0.99960086	4.252604E+02
cosh(x)	A+B/i	3.008263E-03	0.99998634	-1.265629E+03
cosh(x)	A+B*sqrt(i)	5.754950E-03	0.99995002	-6.156336E+02
cosh(x)	A+B*log(i)^4	7.552659E-03	0.99991391	-3.432505E+02
digamma(x)	A+B/i	5.419160E-03	0.96043567	-6.796639E+02
digamma(x)	A+B*i	6.037199E-03	0.95089668	-4.677692E+02
digamma(x)	A+B*sqrt(i)	5.985851E-03	0.95172841	-4.845281E+02
digamma(x)	A+B*log(i)^4	3.091465E-03	0.98712437	-1.780926E+03
erf(x)	A+B*i	1.212533E-06	1	-4.004593E+03
erf(x)	A+B/i	1.851609E-05	0.99999915	-2.854251E+03
erf(x)	A+B*sqrt(i)	1.130740E-06	1	-4.034065E+03
erf(x)	A+B*log(i)^4	8.138008E-06	0.99999984	-3.201175E+03
exp(x)	A+B*i	3.186792E-05	0.99999994	-2.509690E+03
exp(x)	A+B/i	1.773561E-05	0.99999998	-2.745273E+03
exp(x)	A+B*sqrt(i)	1.541414E-05	0.99999999	-2.801669E+03
exp(x)	A+B*log(i)^4	9.770117E-05	0.9999994	-2.059324E+03
FresnelCosine(x)	A+B*i	2.739524E-03	0.85803284	-1.269530E+03

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
FresnelCosine(x)	A+B/i	3.140335E-03	0.81345244	-1.146366E+03
FresnelCosine(x)	A+B*sqrt(i)	3.565792E-03	0.75948077	-1.031761E+03
FresnelCosine(x)	A+B*log(i)^4	3.468591E-03	0.77241491	-1.056690E+03
FresnelSine(x)	A+B*i	2.244440E-03	0.88917802	-1.333970E+03
FresnelSine(x)	A+B/i	2.152428E-03	0.89807817	-1.367541E+03
FresnelSine(x)	A+B*sqrt(i)	2.419942E-03	0.87116918	-1.273589E+03
FresnelSine(x)	A+B*log(i)^4	2.753627E-03	0.8331908	-1.169990E+03
J0(x)	A+B*i	1.075476E-03	0.98837719	-1.443299E+03
J0(x)	A+B/i	1.373707E-03	0.9810374	-1.305750E+03
J0(x)	A+B*sqrt(i)	1.388393E-03	0.98062978	-1.299774E+03
J0(x)	A+B*log(i)^4	1.349092E-03	0.9817109	-1.315912E+03
J1(x)	A+B*i	1.472910E-03	0.98409793	-1.337174E+03
J1(x)	A+B/i	2.907378E-03	0.93804097	-9.278068E+02
J1(x)	A+B*sqrt(i)	5.209551E-03	0.80106914	-5.766951E+02
J1(x)	A+B*log(i)^4	1.724342E-03	0.97820542	-1.242295E+03
J2(x)	A+B*i	2.575894E-03	0.94173485	-1.000682E+03
J2(x)	A+B/i	4.234657E-03	0.84253304	-7.014243E+02
J2(x)	A+B*sqrt(i)	5.437017E-03	0.7404182	-5.509676E+02
J2(x)	A+B*log(i)^4	4.206877E-03	0.84459233	-7.053866E+02
J3(x)	A+B*i	2.259108E-03	0.94915681	-1.079680E+03
J3(x)	A+B/i	3.712451E-03	0.86269697	-7.806536E+02
J3(x)	A+B*sqrt(i)	4.734604E-03	0.77668077	-6.342437E+02
J3(x)	A+B*log(i)^4	2.609637E-03	0.93215483	-9.928473E+02
J4(x)	A+B*i	4.573380E-03	0.77184918	-6.551004E+02
J4(x)	A+B/i	4.394021E-03	0.78939354	-6.791851E+02
J4(x)	A+B*sqrt(i)	3.982963E-03	0.82695461	-7.383127E+02
J4(x)	A+B*log(i)^4	4.052868E-03	0.82082704	-7.278386E+02
J5(x)	A+B*i	4.723709E-03	0.72838303	-6.356305E+02
J5(x)	A+B/i	2.518847E-03	0.92276856	-1.014164E+03
J5(x)	A+B*sqrt(i)	4.724008E-03	0.72834874	-6.355925E+02
J5(x)	A+B*log(i)^4	2.523876E-03	0.92245984	-1.012963E+03
ln(x)	A+B*i	5.167866E-04	0.99930366	-4.934391E+03
ln(x)	A+B/i	1.529002E-04	0.99993904	-7.128947E+03
ln(x)	A+B*sqrt(i)	1.348643E-04	0.99995258	-7.355127E+03
ln(x)	A+B*log(i)^4	1.027942E-04	0.99997245	-7.844442E+03
log(x)	A+B*i	2.251596E-04	0.99929918	-6.431529E+03
log(x)	A+B/i	6.990312E-05	0.99993245	-8.539327E+03
log(x)	A+B*sqrt(i)	9.088722E-05	0.99988581	-8.066286E+03
log(x)	A+B*log(i)^4	1.486237E-04	0.99969465	-7.180066E+03
log10Gamma(x)	A+B*i	5.486246E-03	0.99998649	-6.555247E+02
log10Gamma(x)	A+B/i	1.020560E-02	0.99995326	5.622738E+02
log10Gamma(x)	A+B*sqrt(i)	1.226409E-02	0.99993251	9.227697E+02
log10Gamma(x)	A+B*log(i)^4	4.684158E-03	0.99999015	-9.656345E+02
Si(x)	A+B*i	2.411985E-03	0.90671631	-7.320762E+02
Si(x)	A+B/i	1.615554E-03	0.95814958	-8.827664E+02
Si(x)	A+B*sqrt(i)	1.537965E-03	0.96207292	-9.012725E+02
Si(x)	A+B*log(i)^4	1.406478E-03	0.9682808	-9.348762E+02
sinh(x)	A+B*i	6.387009E-03	0.99993932	-5.112194E+02
sinh(x)	A+B/i	1.085851E-02	0.99982463	2.052450E+01

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
sinh(x)	A+B*sqrt(i)	7.400179E-03	0.99991855	-3.636869E+02
sinh(x)	A+B*log(i)^4	4.286739E-03	0.99997267	-9.107568E+02
tan(x)	A+B*i	6.834038E-05	0.99999734	-1.168215E+03
tan(x)	A+B/i	9.769609E-05	0.99999457	-1.096028E+03
tan(x)	A+B*sqrt(i)	1.092704E-04	0.99999321	-1.073411E+03
tan(x)	A+B*log(i)^4	4.347330E-05	0.99999892	-1.259590E+03
tanh(x)	A+B*i	3.272364E-05	0.99999575	-3.628926E+03
tanh(x)	A+B/i	4.218377E-05	0.99999294	-3.476056E+03
tanh(x)	A+B*sqrt(i)	2.286247E-05	0.99999792	-3.844804E+03
tanh(x)	A+B*log(i)^4	6.781579E-05	0.99998174	-3.190250E+03
tinv(0.95,x)	A+B*i	2.742548E-03	0.53988131	-2.015891E+03
tinv(0.95,x)	A+B/i	2.663896E-03	0.56589401	-2.072981E+03
tinv(0.95,x)	A+B*sqrt(i)	2.727070E-03	0.54506024	-2.026995E+03
tinv(0.95,x)	A+B*log(i)^4	2.073617E-03	0.73696218	-2.564453E+03
tinv(0.975,x)	A+B*i	3.631051E-03	0.73432212	-1.465285E+03
tinv(0.975,x)	A+B/i	4.259093E-03	0.63446841	-1.152280E+03
tinv(0.975,x)	A+B*sqrt(i)	4.918656E-03	0.51248983	-8.697924E+02
tinv(0.975,x)	A+B*log(i)^4	4.951158E-03	0.50602568	-8.568703E+02
trigamma(x)	A+B*i	3.083435E-03	0.40704011	-1.794349E+03
trigamma(x)	A+B/i	2.771742E-03	0.52086108	-2.005567E+03
trigamma(x)	A+B*sqrt(i)	3.061838E-03	0.41531714	-1.808279E+03
trigamma(x)	A+B*log(i)^4	2.581795E-03	0.58428168	-2.146271E+03

Alternating Sine/Cosine Series of Order 4

The next table shows a summary of results for the Sine series of the order 4:

$$Y = a_0 + a_1 * \sin(S_1 * gx(1,A_1,B_1) + Os_1) + a_2 * \cos(S_2 * gx(2,A_2,B_2) + Os_2)$$

$$+ \ldots + a_4 * \sin(S_4 * gx(4,A_4,B_4) + Os_4)$$

The output text files for this series are located in the following folder:

Fourier-Shammas Series Approximations 4 Sine Cosine

The files are named using the following general format:

*fxName_n_*4_run1.txt

Where fxName is the function name and n is the gx series number which varies between 1 and 4. Each output files specifies the function being approximated and the gx series being used. For example, the file for the inverse hyperboplic cosine (acosh) using the first gx series is named acosh_1_4_run1.txt.

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
10^x	A+B*i	2.517341E-05	0.99999999	-1.365740E+03
10^x	A+B/i	1.804190E-05	0.99999999	-1.433024E+03
10^x	A+B*sqrt(i)	1.543554E-05	1	-1.464541E+03
10^x	A+B*log(i)^4	7.788594E-06	1	-1.602711E+03
acosh(x)	A+B*i	3.620105E-03	0.9839301	-1.472299E+03
acosh(x)	A+B/i	5.076859E-03	0.96839463	-8.020067E+02
acosh(x)	A+B*sqrt(i)	4.120459E-03	0.9791809	-1.215706E+03
acosh(x)	A+B*log(i)^4	3.775015E-03	0.98252537	-1.389250E+03
arccos(x)	A+B*i	1.022039E-03	0.99925404	-6.175786E+02
arccos(x)	A+B/i	1.081216E-03	0.99916516	-6.062088E+02
arccos(x)	A+B*sqrt(i)	9.936338E-04	0.99929493	-6.232723E+02
arccos(x)	A+B*log(i)^4	9.810336E-04	0.9993127	-6.258502E+02
arcsin(x)	A+B*i	9.836621E-04	0.99930901	-6.253097E+02
arcsin(x)	A+B/i	9.896042E-04	0.99930064	-6.240932E+02
arcsin(x)	A+B*sqrt(i)	1.026025E-03	0.99924822	-6.167925E+02
arcsin(x)	A+B*log(i)^4	1.001411E-03	0.99928385	-6.216974E+02
arctan(x)	A+B*i	4.084297E-08	1	-2.663349E+03
arctan(x)	A+B/i	1.209335E-07	1	-2.444077E+03
arctan(x)	A+B*sqrt(i)	5.153205E-08	1	-2.616390E+03
arctan(x)	A+B*log(i)^4	3.929647E-08	1	-2.671146E+03
asinh(x)	A+B*i	3.631711E-03	0.98563477	-1.470900E+03
asinh(x)	A+B/i	5.802506E-03	0.96332913	-5.327908E+02
asinh(x)	A+B*sqrt(i)	4.296115E-03	0.97989788	-1.134550E+03
asinh(x)	A+B*log(i)^4	3.507005E-03	0.98660438	-1.540853E+03
atanh(x)	A+B*i	1.973813E-03	0.98829088	-2.689884E+03
atanh(x)	A+B/i	1.968129E-03	0.98835822	-2.695651E+03
atanh(x)	A+B*sqrt(i)	1.779015E-03	0.99048801	-2.897698E+03
atanh(x)	A+B*log(i)^4	1.924780E-03	0.9888654	-2.740194E+03
Ci(x)	A+B*i	4.357489E-03	0.81407437	-5.198066E+02
Ci(x)	A+B/i	1.683775E-03	0.97223903	-8.925429E+02
Ci(x)	A+B*sqrt(i)	2.325385E-03	0.94705117	-7.659867E+02
Ci(x)	A+B*log(i)^4	1.625082E-03	0.97414067	-9.064510E+02
cosh(x)	A+B*i	9.371429E-03	0.99986719	-1.230126E+02
cosh(x)	A+B/i	8.326214E-04	0.99999895	-2.548696E+03
cosh(x)	A+B*sqrt(i)	9.810338E-04	0.99999854	-2.384340E+03
cosh(x)	A+B*log(i)^4	4.127582E-03	0.99997424	-9.446263E+02
digamma(x)	A+B/i	5.505193E-03	0.95912764	-6.447399E+02
digamma(x)	A+B*i	2.997356E-03	0.98788395	-1.837560E+03
digamma(x)	A+B*sqrt(i)	3.383006E-03	0.98456559	-1.600091E+03
digamma(x)	A+B*log(i)^4	1.291643E-03	0.99775007	-3.489202E+03
erf(x)	A+B*i	5.414670E-07	1	-4.340704E+03
erf(x)	A+B/i	1.129884E-06	1	-4.030286E+03
erf(x)	A+B*sqrt(i)	6.217740E-07	1	-4.282344E+03
erf(x)	A+B*log(i)^4	4.494413E-07	1	-4.419313E+03
exp(x)	A+B*i	3.488967E-06	1	-3.394808E+03
exp(x)	A+B/i	2.886296E-06	1	-3.471040E+03
exp(x)	A+B*sqrt(i)	5.217764E-06	1	-3.233018E+03
exp(x)	A+B*log(i)^4	1.342649E-05	0.99999999	-2.853064E+03
FresnelCosine(x)	A+B*i	3.436906E-03	0.77605274	-1.060922E+03

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
FresnelCosine(x)	A+B/i	1.843107E-03	0.93559619	-1.622976E+03
FresnelCosine(x)	A+B*sqrt(i)	1.826124E-03	0.93677759	-1.631325E+03
FresnelCosine(x)	A+B*log(i)^4	1.873299E-03	0.93346892	-1.608320E+03
FresnelSine(x)	A+B*i	1.435385E-03	0.95455963	-1.688431E+03
FresnelSine(x)	A+B/i	1.573158E-03	0.94541795	-1.614926E+03
FresnelSine(x)	A+B*sqrt(i)	1.808087E-03	0.92789859	-1.503301E+03
FresnelSine(x)	A+B*log(i)^4	2.753471E-03	0.83278851	-1.165985E+03
J0(x)	A+B*i	1.754146E-03	0.96896787	-1.164285E+03
J0(x)	A+B/i	6.907596E-04	0.99518791	-1.688039E+03
J0(x)	A+B*sqrt(i)	7.477651E-04	0.99436089	-1.643474E+03
J0(x)	A+B*log(i)^4	7.017971E-04	0.99503289	-1.679130E+03
J1(x)	A+B*i	2.081158E-03	0.96814508	-1.125003E+03
J1(x)	A+B/i	1.472185E-03	0.98405991	-1.333402E+03
J1(x)	A+B*sqrt(i)	2.907306E-03	0.93783472	-9.237534E+02
J1(x)	A+B*log(i)^4	2.158542E-03	0.96573213	-1.103026E+03
J2(x)	A+B*i	2.788295E-03	0.9314993	-9.489151E+02
J2(x)	A+B/i	1.036262E-03	0.99053858	-1.544781E+03
J2(x)	A+B*sqrt(i)	4.131728E-03	0.84958846	-7.121692E+02
J2(x)	A+B*log(i)^4	1.785808E-03	0.97190124	-1.217142E+03
J3(x)	A+B*i	2.606546E-03	0.93208678	-9.894924E+02
J3(x)	A+B/i	2.733897E-03	0.92528847	-9.607759E+02
J3(x)	A+B*sqrt(i)	2.584673E-03	0.93322181	-9.945655E+02
J3(x)	A+B*log(i)^4	2.254482E-03	0.94919375	-1.076846E+03
J4(x)	A+B*i	2.405789E-03	0.93665283	-1.037742E+03
J4(x)	A+B/i	2.720528E-03	0.91899372	-9.637268E+02
J4(x)	A+B*sqrt(i)	2.660093E-03	0.92255274	-9.772506E+02
J4(x)	A+B*log(i)^4	1.794943E-03	0.96473747	-1.214070E+03
J5(x)	A+B*i	4.758869E-03	0.72339321	-6.270980E+02
J5(x)	A+B/i	4.724050E-03	0.72742608	-6.315188E+02
J5(x)	A+B*sqrt(i)	4.420878E-03	0.76128901	-6.714484E+02
J5(x)	A+B*log(i)^4	8.075585E-04	0.99203468	-1.694895E+03
ln(x)	A+B*i	4.905667E-04	0.99937183	-5.024196E+03
ln(x)	A+B/i	7.271550E-05	0.9999862	-8.464226E+03
ln(x)	A+B*sqrt(i)	8.532246E-05	0.999981	-8.176117E+03
ln(x)	A+B*log(i)^4	3.459222E-04	0.99968765	-5.653720E+03
log(x)	A+B*i	2.153913E-04	0.99935795	-6.507431E+03
log(x)	A+B/i	3.245168E-05	0.99998543	-9.918084E+03
log(x)	A+B*sqrt(i)	2.401379E-04	0.99920194	-6.311451E+03
log(x)	A+B*log(i)^4	1.528883E-04	0.99967651	-7.125065E+03
log10Gamma(x)	A+B*i	4.289229E-03	0.99999174	-1.134426E+03
log10Gamma(x)	A+B/i	1.178521E-02	0.99993761	8.486432E+02
log10Gamma(x)	A+B*sqrt(i)	5.381271E-03	0.99998699	-6.894093E+02
log10Gamma(x)	A+B*log(i)^4	2.334348E-03	0.99999755	-2.328056E+03
Si(x)	A+B*i	2.510105E-03	0.8984203	-7.129724E+02
Si(x)	A+B/i	1.095712E-03	0.98064399	-1.024646E+03
Si(x)	A+B*sqrt(i)	2.117108E-03	0.92773814	-7.769952E+02
Si(x)	A+B*log(i)^4	1.804349E-03	0.94751149	-8.370993E+02
sinh(x)	A+B*i	1.834750E-02	0.9994983	5.501587E+02
sinh(x)	A+B/i	6.548525E-04	0.99999936	-2.789345E+03

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
sinh(x)	A+B*sqrt(i)	1.263055E-03	0.99999762	-2.131153E+03
sinh(x)	A+B*log(i)^4	4.401153E-03	0.99997113	-8.803231E+02
tan(x)	A+B*i	1.266110E-05	0.99999991	-1.504565E+03
tan(x)	A+B/i	1.429130E-05	0.99999988	-1.480099E+03
tan(x)	A+B*sqrt(i)	1.288521E-05	0.9999999	-1.501021E+03
tan(x)	A+B*log(i)^4	1.367533E-05	0.99999989	-1.488999E+03
tanh(x)	A+B*i	1.089901E-05	0.99999953	-4.286713E+03
tanh(x)	A+B/i	8.933477E-06	0.99999968	-4.406430E+03
tanh(x)	A+B*sqrt(i)	3.853287E-06	0.99999994	-4.912639E+03
tanh(x)	A+B*log(i)^4	4.694121E-05	0.99999122	-3.407657E+03
tinv(0.95,x)	A+B*i	1.733095E-03	0.81607068	-2.912386E+03
tinv(0.95,x)	A+B/i	2.372057E-03	0.65544681	-2.296615E+03
tinv(0.95,x)	A+B*sqrt(i)	1.933764E-03	0.77101178	-2.697430E+03
tinv(0.95,x)	A+B*log(i)^4	1.482117E-03	0.86548488	-3.219316E+03
tinv(0.975,x)	A+B*i	3.909510E-03	0.69169528	-1.316293E+03
tinv(0.975,x)	A+B/i	4.399746E-03	0.60952732	-1.084513E+03
tinv(0.975,x)	A+B*sqrt(i)	3.654325E-03	0.73062962	-1.448729E+03
tinv(0.975,x)	A+B*log(i)^4	3.671651E-03	0.72806929	-1.439449E+03
trigamma(x)	A+B*i	2.195910E-03	0.69895935	-2.463113E+03
trigamma(x)	A+B/i	2.808290E-03	0.50764309	-1.975582E+03
trigamma(x)	A+B*sqrt(i)	2.548161E-03	0.59463183	-2.168241E+03
trigamma(x)	A+B*log(i)^4	2.350371E-03	0.6551193	-2.328384E+03

Alternating Sine/Cosine Series of Order 5

The next table shows a summary of results for the Sine series of the order 5:

$$Y = a_0 + a_1 * \sin(S_1 * gx(1,A_1,B_1) + Os_1) + a_2 * \cos(S_2 * gx(2,A_2,B_2) + Os_2)$$

$$+ \ldots + a_5 * \sin(S_5 * gx(5,A_5,B_5) + Os_5)$$

The output text files for this series are located in the following folder:

Fourier-Shammas Series Approximations 5 Sine Cosine

The files are named using the following general format:

*fxName_n*_5_run1.txt

Where fxName is the function name and n is the gx series number which varies between 1 and 4. Each output files specifies the function being approximated and the gx series being used. For example, the file for the inverse hyperboplic cosine (acosh) using the first gx series is named acosh_1_5_run1.txt.

Function	*gx(i,A,B)*	*MSSE*	*RsqrAdj*	*AICc*
10^x	A+B*i	7.571722E-06	1	-1.604154E+03
10^x	A+B/i	1.877541E-06	1	-1.885834E+03
10^x	A+B*sqrt(i)	1.173387E-06	1	-1.980788E+03
10^x	A+B*log(i)^4	3.504767E-06	1	-1.759754E+03
acosh(x)	A+B*i	4.750069E-03	0.97230437	-9.298517E+02
acosh(x)	A+B/i	7.212741E-03	0.93614248	-1.019905E+02
acosh(x)	A+B*sqrt(i)	4.166976E-03	0.97868656	-1.189431E+03
acosh(x)	A+B*log(i)^4	3.579313E-03	0.98427427	-1.490735E+03
arccos(x)	A+B*i	8.266745E-04	0.99950683	-6.561697E+02
arccos(x)	A+B/i	8.359595E-04	0.99949569	-6.539136E+02
arccos(x)	A+B*sqrt(i)	8.173640E-04	0.99951788	-6.584577E+02
arccos(x)	A+B*log(i)^4	7.129549E-04	0.99963318	-6.860643E+02
arcsin(x)	A+B*i	8.497423E-04	0.99947893	-6.506103E+02
arcsin(x)	A+B/i	8.313886E-04	0.99950119	-6.550211E+02
arcsin(x)	A+B*sqrt(i)	7.971182E-04	0.99954147	-6.635242E+02
arcsin(x)	A+B*log(i)^4	7.893380E-04	0.99955037	-6.655055E+02
arctan(x)	A+B*i	9.460549E-09	1	-2.954532E+03
arctan(x)	A+B/i	1.674177E-08	1	-2.839236E+03
arctan(x)	A+B*sqrt(i)	1.674168E-09	1	-3.303751E+03
arctan(x)	A+B*log(i)^4	6.088385E-09	1	-3.043563E+03
asinh(x)	A+B*i	2.734985E-03	0.99184477	-2.034599E+03
asinh(x)	A+B/i	7.128568E-03	0.9445973	-1.167141E+02
asinh(x)	A+B*sqrt(i)	4.811582E-03	0.97475926	-9.036692E+02
asinh(x)	A+B*log(i)^4	4.619000E-03	0.97673933	-9.854462E+02
atanh(x)	A+B*i	1.742613E-03	0.99086411	-2.935022E+03
atanh(x)	A+B/i	1.612746E-03	0.99217506	-3.089918E+03
atanh(x)	A+B*sqrt(i)	1.607178E-03	0.992229	-3.096834E+03
atanh(x)	A+B*log(i)^4	1.617549E-03	0.99212838	-3.083970E+03
Ci(x)	A+B*i	1.691566E-03	0.97183407	-8.866047E+02
Ci(x)	A+B/i	1.980701E-03	0.96138249	-8.247485E+02
Ci(x)	A+B*sqrt(i)	3.330625E-03	0.89080621	-6.210225E+02
Ci(x)	A+B*log(i)^4	2.739398E-03	0.92613193	-6.976278E+02
cosh(x)	A+B*i	1.872930E-02	0.99946847	5.748447E+02
cosh(x)	A+B/i	3.512029E-04	0.99999981	-3.409589E+03
cosh(x)	A+B*sqrt(i)	2.027664E-03	0.99999377	-1.652807E+03
cosh(x)	A+B*log(i)^4	4.685069E-03	0.99996674	-8.136352E+02
digamma(x)	A+B/i	4.242175E-03	0.97570556	-1.152043E+03
digamma(x)	A+B*i	1.669649E-03	0.9962366	-2.981535E+03
digamma(x)	A+B*sqrt(i)	2.984561E-03	0.98797485	-1.841929E+03
digamma(x)	A+B*log(i)^4	3.084639E-03	0.98715487	-1.777218E+03
erf(x)	A+B*i	3.258864E-06	0.99999997	-3.579158E+03
erf(x)	A+B/i	4.705293E-07	1	-4.395844E+03
erf(x)	A+B*sqrt(i)	2.700056E-06	0.99999998	-3.658539E+03
erf(x)	A+B*log(i)^4	3.277784E-06	0.99999997	-3.576715E+03
exp(x)	A+B*i	1.439639E-06	1	-3.746539E+03
exp(x)	A+B/i	2.355721E-07	1	-4.474211E+03
exp(x)	A+B*sqrt(i)	6.938817E-07	1	-4.039937E+03
exp(x)	A+B*log(i)^4	7.521501E-06	1	-3.081883E+03
FresnelCosine(x)	A+B*i	9.306436E-04	0.98354295	-2.235287E+03

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
FresnelCosine(x)	A+B/i	1.530143E-03	0.95551132	-1.786776E+03
FresnelCosine(x)	A+B*sqrt(i)	1.221758E-03	0.97163677	-1.989789E+03
FresnelCosine(x)	A+B*log(i)^4	2.401487E-03	0.8904164	-1.380220E+03
FresnelSine(x)	A+B*i	1.053913E-03	0.97544087	-1.932126E+03
FresnelSine(x)	A+B/i	4.674186E-04	0.99516924	-2.584184E+03
FresnelSine(x)	A+B*sqrt(i)	8.213475E-04	0.98508384	-2.132080E+03
FresnelSine(x)	A+B*log(i)^4	3.933589E-04	0.99657878	-2.722532E+03
J0(x)	A+B*i	3.931187E-04	0.99843576	-2.000739E+03
J0(x)	A+B/i	5.392281E-04	0.99705692	-1.823131E+03
J0(x)	A+B*sqrt(i)	7.940165E-04	0.99361861	-1.605657E+03
J0(x)	A+B*log(i)^4	1.517901E-03	0.9766792	-1.241492E+03
J1(x)	A+B*i	1.892259E-03	0.97357608	-1.178203E+03
J1(x)	A+B/i	1.521699E-03	0.98291192	-1.309406E+03
J1(x)	A+B*sqrt(i)	1.309239E-03	0.98735048	-1.399935E+03
J1(x)	A+B*log(i)^4	3.349371E-03	0.91721293	-8.344607E+02
J2(x)	A+B*i	4.267860E-03	0.83896967	-6.885720E+02
J2(x)	A+B/i	1.598965E-03	0.97739704	-1.279589E+03
J2(x)	A+B*sqrt(i)	4.290904E-03	0.83722601	-6.853303E+02
J2(x)	A+B*log(i)^4	1.557885E-04	0.99978544	-2.681414E+03
J3(x)	A+B*i	2.007436E-03	0.95958184	-1.142633E+03
J3(x)	A+B/i	1.696756E-03	0.97112434	-1.243853E+03
J3(x)	A+B*sqrt(i)	2.944935E-03	0.91301497	-9.119296E+02
J3(x)	A+B*log(i)^4	1.472130E-03	0.9782637	-1.329342E+03
J4(x)	A+B*i	2.418383E-04	0.99935771	-2.416672E+03
J4(x)	A+B/i	2.718325E-03	0.91885073	-9.601323E+02
J4(x)	A+B*sqrt(i)	6.297886E-04	0.99564416	-1.840489E+03
J4(x)	A+B*log(i)^4	2.751682E-03	0.9168469	-9.527899E+02
J5(x)	A+B*i	1.133870E-03	0.9842438	-1.486509E+03
J5(x)	A+B/i	1.728175E-03	0.96339837	-1.232808E+03
J5(x)	A+B*sqrt(i)	1.965738E-03	0.95264387	-1.155269E+03
J5(x)	A+B*log(i)^4	1.663896E-03	0.9660705	-1.255626E+03
ln(x)	A+B*i	4.882431E-04	0.99937707	-5.028725E+03
ln(x)	A+B/i	5.455986E-05	0.99999222	-8.977835E+03
ln(x)	A+B*sqrt(i)	4.086031E-04	0.99956372	-5.349606E+03
ln(x)	A+B*log(i)^4	2.354677E-04	0.99985511	-6.342815E+03
log(x)	A+B*i	2.124458E-04	0.99937469	-6.528217E+03
log(x)	A+B/i	3.417131E-05	0.99998382	-9.821012E+03
log(x)	A+B*sqrt(i)	2.134719E-04	0.99936864	-6.519534E+03
log(x)	A+B*log(i)^4	1.803032E-04	0.99954959	-6.823830E+03
log10Gamma(x)	A+B*i	5.407963E-03	0.99998685	-6.756768E+02
log10Gamma(x)	A+B/i	1.020883E-02	0.99995314	5.709406E+02
log10Gamma(x)	A+B*sqrt(i)	8.924285E-03	0.99996419	3.070963E+02
log10Gamma(x)	A+B*log(i)^4	5.402125E-03	0.99998688	-6.777959E+02
Si(x)	A+B*i	9.948158E-04	0.9839569	-1.056834E+03
Si(x)	A+B/i	2.514321E-03	0.89751871	-7.082068E+02
Si(x)	A+B*sqrt(i)	1.140673E-03	0.97890762	-1.005391E+03
Si(x)	A+B*log(i)^4	1.205919E-03	0.97642566	-9.844768E+02
sinh(x)	A+B*i	4.988603E-03	0.99996284	-7.507345E+02
sinh(x)	A+B/i	1.776948E-04	0.99999995	-4.092248E+03

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
sinh(x)	A+B*sqrt(i)	5.827923E-04	0.99999949	-2.902109E+03
sinh(x)	A+B*log(i)^4	3.917799E-03	0.99997708	-9.928437E+02
tan(x)	A+B*i	2.701458E-06	1	-1.812341E+03
tan(x)	A+B/i	5.977027E-06	0.99999998	-1.651926E+03
tan(x)	A+B*sqrt(i)	2.186517E-06	1	-1.855060E+03
tan(x)	A+B*log(i)^4	1.040949E-05	0.99999994	-1.539858E+03
tanh(x)	A+B*i	8.600299E-05	0.99997044	-3.039072E+03
tanh(x)	A+B/i	2.820846E-06	0.99999997	-5.096314E+03
tanh(x)	A+B*sqrt(i)	1.515700E-06	0.99999999	-5.470252E+03
tanh(x)	A+B*log(i)^4	9.576609E-06	0.99999963	-4.360498E+03
tinv(0.95,x)	A+B*i	1.343123E-03	0.88941843	-3.408498E+03
tinv(0.95,x)	A+B/i	2.423467E-03	0.6399809	-2.250522E+03
tinv(0.95,x)	A+B*sqrt(i)	2.150348E-03	0.71655515	-2.485118E+03
tinv(0.95,x)	A+B*log(i)^4	1.462203E-03	0.86894109	-3.241832E+03
tinv(0.975,x)	A+B*i	3.485037E-03	0.75475769	-1.537768E+03
tinv(0.975,x)	A+B/i	4.407596E-03	0.60773076	-1.076990E+03
tinv(0.975,x)	A+B*sqrt(i)	3.452725E-03	0.75928413	-1.556043E+03
tinv(0.975,x)	A+B*log(i)^4	2.422334E-03	0.88151905	-2.251440E+03
trigamma(x)	A+B*i	2.286058E-03	0.67340374	-2.379348E+03
trigamma(x)	A+B/i	2.624549E-03	0.56952684	-2.105674E+03
trigamma(x)	A+B*sqrt(i)	2.261053E-03	0.68050937	-2.401147E+03
trigamma(x)	A+B*log(i)^4	1.949172E-03	0.76256911	-2.695327E+03

Alternating Sine/Cosine Series of Order 6

The next table shows a summary of results for the Sine series of the order 6:

$$Y = a_0 + a_1 * \sin(S_1 * gx(1,A_1,B_1) + Os_1) + a_2 * \cos(S_2 * gx(2,A_2,B_2) + Os_2)$$

$$+ \ldots + a_6 * \sin(S_6 * gx(6,A_6,B_6) + Os_6)$$

The output text files for this series are located in the following folder:

Fourier-Shammas Series Approximations 6 Sine Cosine

The files are named using the following general format:

*fxName_n*_6_run1.txt

Where fxName is the function name and n is the gx series number which varies between 1 and 4. Each output files specifies the function being approximated and the gx series being used. For example, the file for the inverse hyperboplic cosine (acosh) using the first gx series is named acosh_1_6_run1.txt.

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
10^x	A+B*i	6.548687E-06	1	-1.629165E+03
10^x	A+B/i	1.048563E-07	1	-2.464319E+03
10^x	A+B*sqrt(i)	7.902675E-07	1	-2.056324E+03
10^x	A+B*log(i)^4	4.374398E-06	1	-1.710671E+03
acosh(x)	A+B*i	2.373090E-03	0.9930804	-2.301264E+03
acosh(x)	A+B/i	4.106250E-03	0.97928221	-1.214499E+03
acosh(x)	A+B*sqrt(i)	3.785397E-03	0.9823934	-1.375754E+03
acosh(x)	A+B*log(i)^4	2.323393E-03	0.99336719	-2.343212E+03
arccos(x)	A+B*i	6.881320E-04	0.99965465	-6.889120E+02
arccos(x)	A+B/i	7.301488E-04	0.99961118	-6.769399E+02
arccos(x)	A+B*sqrt(i)	7.520836E-04	0.99958747	-6.709609E+02
arccos(x)	A+B*log(i)^4	8.337119E-04	0.99949306	-6.501467E+02
arcsin(x)	A+B*i	7.462893E-04	0.9995938	-6.725232E+02
arcsin(x)	A+B/i	7.122126E-04	0.99963005	-6.819640E+02
arcsin(x)	A+B*sqrt(i)	6.612404E-04	0.99968111	-6.969643E+02
arcsin(x)	A+B*log(i)^4	6.788224E-04	0.99966393	-6.916635E+02
arctan(x)	A+B*i	1.701554E-08	1	-2.831649E+03
arctan(x)	A+B/i	1.553343E-09	1	-3.315175E+03
arctan(x)	A+B*sqrt(i)	1.527675E-09	1	-3.318451E+03
arctan(x)	A+B*log(i)^4	5.413779E-09	1	-3.062952E+03
asinh(x)	A+B*i	4.395475E-03	0.97891495	-1.080722E+03
asinh(x)	A+B/i	5.143000E-03	0.97113337	-7.662863E+02
asinh(x)	A+B*sqrt(i)	6.002877E-03	0.96067381	-4.567727E+02
asinh(x)	A+B*log(i)^4	2.739841E-03	0.99180755	-2.027019E+03
atanh(x)	A+B*i	1.411711E-03	0.99399825	-3.352160E+03
atanh(x)	A+B/i	1.354979E-03	0.99447094	-3.434193E+03
atanh(x)	A+B*sqrt(i)	1.342948E-03	0.99456869	-3.452031E+03
atanh(x)	A+B*log(i)^4	1.310424E-03	0.99482858	-3.501063E+03
Ci(x)	A+B*i	3.267192E-03	0.89436997	-6.244091E+02
Ci(x)	A+B/i	3.865134E-03	0.85216843	-5.585274E+02
Ci(x)	A+B*sqrt(i)	1.877843E-03	0.96510551	-8.415013E+02
Ci(x)	A+B*log(i)^4	1.100531E-03	0.98801488	-1.050959E+03
cosh(x)	A+B*i	7.468544E-03	0.99991531	-3.423260E+02
cosh(x)	A+B/i	2.433471E-05	1	-6.080331E+03
cosh(x)	A+B*sqrt(i)	6.362802E-04	0.99999939	-2.810068E+03
cosh(x)	A+B*log(i)^4	2.763490E-03	0.9999884	-1.338520E+03
digamma(x)	A+B/i	3.833171E-03	0.980144	-1.346929E+03
digamma(x)	A+B*i	2.677531E-03	0.99031177	-2.050890E+03
digamma(x)	A+B*sqrt(i)	1.971178E-03	0.99474918	-2.651779E+03
digamma(x)	A+B*log(i)^4	3.860821E-03	0.97985651	-1.332828E+03
erf(x)	A+B*i	2.784052E-06	0.99999998	-3.641471E+03
erf(x)	A+B/i	3.236674E-08	1	-5.521283E+03
erf(x)	A+B*sqrt(i)	5.461676E-09	1	-6.272176E+03
erf(x)	A+B*log(i)^4	6.509762E-07	1	-4.254717E+03
exp(x)	A+B*i	3.924191E-06	1	-3.339279E+03
exp(x)	A+B/i	1.564045E-08	1	-5.556547E+03
exp(x)	A+B*sqrt(i)	4.063670E-07	1	-4.250878E+03
exp(x)	A+B*log(i)^4	6.118165E-06	1	-3.160750E+03
FresnelCosine(x)	A+B*i	1.979856E-03	0.92535009	-1.550301E+03

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
FresnelCosine(x)	A+B/i	1.849657E-03	0.93484543	-1.611658E+03
FresnelCosine(x)	A+B*sqrt(i)	1.197177E-03	0.97270527	-2.004059E+03
FresnelCosine(x)	A+B*log(i)^4	1.798579E-03	0.93839421	-1.636917E+03
FresnelSine(x)	A+B*i	2.348012E-03	0.87779054	-1.285605E+03
FresnelSine(x)	A+B/i	3.808265E-04	0.99678517	-2.744427E+03
FresnelSine(x)	A+B*sqrt(i)	1.022760E-03	0.97681263	-1.952119E+03
FresnelSine(x)	A+B*log(i)^4	1.601878E-03	0.94311959	-1.592284E+03
J0(x)	A+B*i	4.810387E-04	0.99764929	-1.883203E+03
J0(x)	A+B/i	1.678762E-03	0.97137029	-1.180779E+03
J0(x)	A+B*sqrt(i)	8.004041E-04	0.99349185	-1.597050E+03
J0(x)	A+B*log(i)^4	3.007313E-04	0.99908125	-2.147191E+03
J1(x)	A+B*i	2.047183E-03	0.968967	-1.126734E+03
J1(x)	A+B/i	7.786972E-04	0.99550998	-1.708625E+03
J1(x)	A+B*sqrt(i)	1.507544E-03	0.98317129	-1.310935E+03
J1(x)	A+B*log(i)^4	7.468312E-05	0.9999587	-3.119935E+03
J2(x)	A+B*i	2.679841E-03	0.93629402	-9.646192E+02
J2(x)	A+B/i	3.360988E-03	0.89979348	-8.282798E+02
J2(x)	A+B*sqrt(i)	1.320073E-04	0.99984542	-2.777034E+03
J2(x)	A+B*log(i)^4	7.834976E-04	0.9945545	-1.704926E+03
J3(x)	A+B*i	3.779512E-03	0.85623962	-7.576292E+02
J3(x)	A+B/i	4.012442E-03	0.83797379	-7.216265E+02
J3(x)	A+B*sqrt(i)	1.523244E-03	0.97664892	-1.304698E+03
J3(x)	A+B*log(i)^4	1.594634E-03	0.97440884	-1.277125E+03
J4(x)	A+B*i	2.466356E-03	0.93297018	-1.014595E+03
J4(x)	A+B/i	3.320463E-04	0.99878506	-2.221739E+03
J4(x)	A+B*sqrt(i)	3.020917E-03	0.89943791	-8.924978E+02
J4(x)	A+B*log(i)^4	2.017245E-03	0.95515916	-1.135602E+03
J5(x)	A+B*i	1.867550E-04	0.99957111	-2.568176E+03
J5(x)	A+B/i	1.022408E-03	0.98714572	-1.544705E+03
J5(x)	A+B*sqrt(i)	3.096010E-03	0.88212974	-8.777166E+02
J5(x)	A+B*log(i)^4	1.252682E-03	0.98070336	-1.422422E+03
ln(x)	A+B*i	5.082938E-04	0.9993241	-4.952170E+03
ln(x)	A+B/i	2.274471E-05	0.99999865	-1.055049E+04
ln(x)	A+B*sqrt(i)	9.233272E-05	0.9999777	-8.025771E+03
ln(x)	A+B*log(i)^4	3.885866E-04	0.99960497	-5.436085E+03
log(x)	A+B*i	1.878376E-04	0.99951062	-6.746028E+03
log(x)	A+B/i	8.591439E-06	0.99999898	-1.230486E+04
log(x)	A+B*sqrt(i)	9.373801E-05	0.99987812	-7.998551E+03
log(x)	A+B*log(i)^4	1.575541E-04	0.9996557	-7.062836E+03
log10Gamma(x)	A+B*i	5.556837E-03	0.9999861	-6.183669E+02
log10Gamma(x)	A+B/i	9.033335E-03	0.99996327	3.349545E+02
log10Gamma(x)	A+B*sqrt(i)	5.249994E-03	0.99998759	-7.298125E+02
log10Gamma(x)	A+B*log(i)^4	7.161076E-03	0.99997692	-1.207423E+02
Si(x)	A+B*i	5.055574E-04	0.99583383	-1.307189E+03
Si(x)	A+B/i	1.204362E-03	0.97635658	-9.808044E+02
Si(x)	A+B*sqrt(i)	1.948387E-03	0.9381206	-7.999292E+02
Si(x)	A+B*log(i)^4	1.780438E-03	0.94832871	-8.338227E+02
sinh(x)	A+B*i	8.981500E-03	0.99987929	-1.574903E+02
sinh(x)	A+B/i	2.239623E-05	1	-6.163508E+03

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
sinh(x)	A+B*sqrt(i)	2.973224E-03	0.99998677	-1.265221E+03
sinh(x)	A+B*log(i)^4	3.703570E-03	0.99997947	-1.045132E+03
tan(x)	A+B*i	9.498008E-07	1	-2.019180E+03
tan(x)	A+B/i	9.737687E-07	1	-2.014146E+03
tan(x)	A+B*sqrt(i)	1.643552E-06	1	-1.908410E+03
tan(x)	A+B*log(i)^4	1.873671E-06	1	-1.881940E+03
tanh(x)	A+B*i	7.375270E-06	0.99999978	-4.513638E+03
tanh(x)	A+B/i	7.128660E-07	1	-5.920268E+03
tanh(x)	A+B*sqrt(i)	6.806113E-06	0.99999981	-4.561986E+03
tanh(x)	A+B*log(i)^4	3.296665E-05	0.99999564	-3.612225E+03
tinv(0.95,x)	A+B*i	2.292200E-03	0.67759499	-2.355751E+03
tinv(0.95,x)	A+B/i	1.978363E-03	0.7598354	-2.644640E+03
tinv(0.95,x)	A+B*sqrt(i)	1.897803E-03	0.77899651	-2.726207E+03
tinv(0.95,x)	A+B*log(i)^4	1.429837E-03	0.87455027	-3.281721E+03
tinv(0.975,x)	A+B*i	2.611488E-03	0.86215138	-2.099890E+03
tinv(0.975,x)	A+B/i	3.264208E-03	0.78463154	-1.662174E+03
tinv(0.975,x)	A+B*sqrt(i)	3.034159E-03	0.81391864	-1.805563E+03
tinv(0.975,x)	A+B*log(i)^4	4.142991E-03	0.65306	-1.194431E+03
trigamma(x)	A+B*i	2.388676E-03	0.64306243	-2.288290E+03
trigamma(x)	A+B/i	2.561551E-03	0.58952771	-2.149800E+03
trigamma(x)	A+B*sqrt(i)	2.450037E-03	0.62448862	-2.238018E+03
trigamma(x)	A+B*log(i)^4	1.976817E-03	0.75553831	-2.663386E+03

Alternating Sine/Cosine Series of Order 7

The next table shows a summary of results for the Sine series of the order 7:

$Y = a_0 + a_1 * \sin(S_1 * gx(1,A_1,B_1) + Os_1) + a_1 * \cos(S_2 * gx(2,A_2,B_2) + Os_2)$

$+ \ldots + a_7 * \sin(S_7 * gx(7,A_7,B_7) + Os_7)$

The output text files for this series are located in the following folder:

Fourier-Shammas Series Approximations 7 Sine Cosine

The files are named using the following general format:

fxName_n_7_run1.txt

Where fxName is the function name and n is the gx series number which varies between 1 and 4. Each output files specifies the function being approximated and the gx series being used. For example, the file for the inverse hyperboplic cosine (acosh) using the first gx series is named acosh_1_7_run1.txt.

Function	*gx(i,A,B)*	*MSSE*	*RsqrAdj*	*AICc*
10^x	A+B*i	8.167708E-07	1	-2.045299E+03
10^x	A+B/i	7.995725E-09	1	-Inf
10^x	A+B*sqrt(i)	3.236746E-08	1	-2.697375E+03
10^x	A+B*log(i)^4	3.345933E-07	1	-2.225573E+03
acosh(x)	A+B*i	4.985566E-03	0.96942805	-8.258858E+02
acosh(x)	A+B/i	5.479484E-03	0.9630705	-6.386584E+02
acosh(x)	A+B*sqrt(i)	3.393323E-03	0.98583735	-1.588435E+03
acosh(x)	A+B*log(i)^4	3.744698E-03	0.98275244	-1.393146E+03
arccos(x)	A+B*i	6.314912E-04	0.99970603	-7.019022E+02
arccos(x)	A+B/i	5.833070E-04	0.99974918	-7.179350E+02
arccos(x)	A+B*sqrt(i)	6.008198E-04	0.99973389	-7.119596E+02
arccos(x)	A+B*log(i)^4	6.437226E-04	0.99969453	-6.980270E+02
arcsin(x)	A+B*i	5.920946E-04	0.99974157	-7.149145E+02
arcsin(x)	A+B/i	6.288091E-04	0.99970852	-7.027620E+02
arcsin(x)	A+B*sqrt(i)	6.293035E-04	0.99970806	-7.026032E+02
arcsin(x)	A+B*log(i)^4	5.849467E-04	0.99974777	-7.173680E+02
arctan(x)	A+B*i	1.965458E-09	1	-3.263282E+03
arctan(x)	A+B/i	3.860130E-10	1	-Inf
arctan(x)	A+B*sqrt(i)	2.958316E-10	1	-3.487894E+03
arctan(x)	A+B*log(i)^4	5.720682E-09	1	-3.047470E+03
asinh(x)	A+B*i	2.354958E-03	0.99394148	-2.326043E+03
asinh(x)	A+B/i	6.880586E-03	0.9482809	-1.795375E+02
asinh(x)	A+B*sqrt(i)	5.470839E-03	0.967303	-6.385397E+02
asinh(x)	A+B*log(i)^4	3.564158E-03	0.98612241	-1.496405E+03
atanh(x)	A+B*i	1.191710E-03	0.9957188	-3.686954E+03
atanh(x)	A+B/i	1.204501E-03	0.99562641	-3.665602E+03
atanh(x)	A+B*sqrt(i)	1.204262E-03	0.99562814	-3.665998E+03
atanh(x)	A+B*log(i)^4	1.585799E-03	0.99241911	-3.115556E+03
Ci(x)	A+B*i	1.636637E-03	0.97335309	-8.912194E+02
Ci(x)	A+B/i	1.129916E-03	0.9872991	-1.036455E+03
Ci(x)	A+B*sqrt(i)	2.700400E-03	0.92745656	-6.949229E+02
Ci(x)	A+B*log(i)^4	2.900169E-03	0.91632633	-6.669461E+02
cosh(x)	A+B*i	6.329143E-03	0.99993906	-5.041269E+02
cosh(x)	A+B/i	3.076377E-06	1	-8.148553E+03
cosh(x)	A+B*sqrt(i)	3.542468E-03	0.99998091	-1.085629E+03
cosh(x)	A+B*log(i)^4	1.186406E-03	0.99999786	-2.181711E+03
digamma(x)	A+B/i	3.423804E-03	0.98414233	-1.564485E+03
digamma(x)	A+B*i	3.628419E-03	0.98219031	-1.450601E+03
digamma(x)	A+B*sqrt(i)	2.377105E-03	0.99235604	-2.280358E+03
digamma(x)	A+B*log(i)^4	1.556007E-03	0.99672475	-3.111775E+03
erf(x)	A+B*i	2.005404E-06	0.99999999	-3.775752E+03
erf(x)	A+B/i	3.512675E-09	1	-6.454249E+03
erf(x)	A+B*sqrt(i)	2.025187E-09	1	-6.686177E+03
erf(x)	A+B*log(i)^4	2.718046E-07	1	-4.619126E+03
exp(x)	A+B*i	2.049171E-05	0.99999997	-2.670659E+03
exp(x)	A+B/i	4.959934E-09	1	-Inf
exp(x)	A+B*sqrt(i)	3.685444E-06	1	-3.360342E+03
exp(x)	A+B*log(i)^4	3.159840E-07	1	-4.347837E+03
FresnelCosine(x)	A+B*i	1.782510E-03	0.93935353	-1.640939E+03

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
FresnelCosine(x)	A+B/i	1.437978E-03	0.96053193	-1.834675E+03
FresnelCosine(x)	A+B*sqrt(i)	1.646768E-03	0.94823854	-1.712384E+03
FresnelCosine(x)	A+B*log(i)^4	1.644699E-03	0.94836857	-1.713519E+03
FresnelSine(x)	A+B*i	9.632233E-04	0.97938128	-1.996136E+03
FresnelSine(x)	A+B/i	1.161663E-03	0.9700106	-1.845904E+03
FresnelSine(x)	A+B*sqrt(i)	3.379535E-04	0.99746182	-2.836132E+03
FresnelSine(x)	A+B*log(i)^4	6.995993E-04	0.98912307	-2.252598E+03
J0(x)	A+B*i	2.290959E-03	0.94648676	-1.001927E+03
J0(x)	A+B/i	7.655448E-04	0.99402459	-1.617956E+03
J0(x)	A+B*sqrt(i)	7.010555E-04	0.99498892	-1.667412E+03
J0(x)	A+B*log(i)^4	6.264231E-04	0.99599906	-1.730672E+03
J1(x)	A+B*i	2.356631E-03	0.9587358	-1.037880E+03
J1(x)	A+B/i	2.003472E-03	0.97017661	-1.135615E+03
J1(x)	A+B*sqrt(i)	3.132604E-03	0.92708763	-8.665319E+02
J1(x)	A+B*log(i)^4	2.463859E-03	0.95489529	-1.011094E+03
J2(x)	A+B*i	1.406803E-03	0.98238396	-1.348460E+03
J2(x)	A+B/i	3.615180E-03	0.88366737	-7.802791E+02
J2(x)	A+B*sqrt(i)	2.605237E-03	0.93958619	-9.775051E+02
J2(x)	A+B*log(i)^4	3.353100E-04	0.99899923	-2.211740E+03
J3(x)	A+B*i	1.924965E-03	0.96258092	-1.159680E+03
J3(x)	A+B/i	9.268821E-04	0.99132446	-1.599644E+03
J3(x)	A+B*sqrt(i)	1.241917E-03	0.98442485	-1.423508E+03
J3(x)	A+B*log(i)^4	3.933460E-03	0.84375826	-7.294836E+02
J4(x)	A+B*i	3.150268E-03	0.89026851	-8.631469E+02
J4(x)	A+B/i	2.891340E-03	0.90756536	-9.147788E+02
J4(x)	A+B*sqrt(i)	2.331402E-03	0.93990052	-1.044359E+03
J4(x)	A+B*log(i)^4	4.761687E-03	0.74929787	-6.144519E+02
J5(x)	A+B*i	1.376192E-03	0.97663114	-1.361703E+03
J5(x)	A+B/i	4.247426E-05	0.99997774	-3.455566E+03
J5(x)	A+B*sqrt(i)	3.250609E-03	0.86962064	-8.442712E+02
J5(x)	A+B*log(i)^4	9.868604E-04	0.98798314	-1.561897E+03
ln(x)	A+B*i	4.327914E-04	0.99950944	-5.237902E+03
ln(x)	A+B/i	1.577127E-05	0.99999935	-1.120625E+04
ln(x)	A+B*sqrt(i)	3.602899E-04	0.99966003	-5.568293E+03
ln(x)	A+B*log(i)^4	1.187321E-04	0.99996308	-7.568584E+03
log(x)	A+B*i	2.077279E-04	0.99940082	-6.560618E+03
log(x)	A+B/i	3.671833E-06	0.99999981	-1.383266E+04
log(x)	A+B*sqrt(i)	6.011793E-05	0.99994981	-8.794956E+03
log(x)	A+B*log(i)^4	1.753628E-04	0.99957298	-6.865827E+03
log10Gamma(x)	A+B*i	5.408946E-03	0.99998682	-6.672582E+02
log10Gamma(x)	A+B/i	8.745970E-03	0.99996554	2.755588E+02
log10Gamma(x)	A+B*sqrt(i)	5.058324E-03	0.99998847	-7.987498E+02
log10Gamma(x)	A+B*log(i)^4	6.114307E-03	0.99998316	-4.267617E+02
Si(x)	A+B*i	2.632331E-03	0.88642497	-6.826205E+02
Si(x)	A+B/i	1.023719E-03	0.98282235	-1.037725E+03
Si(x)	A+B*sqrt(i)	2.554838E-03	0.89301359	-6.938558E+02
Si(x)	A+B*log(i)^4	1.608156E-03	0.95761049	-8.679063E+02
sinh(x)	A+B*i	4.438859E-03	0.99997046	-8.596038E+02
sinh(x)	A+B/i	6.205499E-06	1	-7.445466E+03

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
sinh(x)	A+B*sqrt(i)	1.487716E-03	0.99999668	-1.954945E+03
sinh(x)	A+B*log(i)^4	3.840267E-03	0.99997789	-1.004749E+03
tan(x)	A+B*i	2.728026E-06	1	-1.801692E+03
tan(x)	A+B/i	3.082821E-07	1	-2.242117E+03
tan(x)	A+B*sqrt(i)	2.201620E-07	1	-2.310120E+03
tan(x)	A+B*log(i)^4	1.714681E-06	1	-1.895491E+03
tanh(x)	A+B*i	4.271436E-06	0.99999993	-4.838329E+03
tanh(x)	A+B/i	3.699216E-07	1	-6.311070E+03
tanh(x)	A+B*sqrt(i)	9.381767E-07	1	-5.750821E+03
tanh(x)	A+B*log(i)^4	8.760732E-06	0.99999969	-4.405895E+03
tinv(0.95,x)	A+B*i	2.055185E-03	0.74055521	-2.565863E+03
tinv(0.95,x)	A+B/i	1.532674E-03	0.85570781	-3.141419E+03
tinv(0.95,x)	A+B*sqrt(i)	1.593225E-03	0.84408167	-3.065399E+03
tinv(0.95,x)	A+B*log(i)^4	1.019704E-03	0.93613088	-3.940938E+03
tinv(0.975,x)	A+B*i	2.549175E-03	0.86851639	-2.143241E+03
tinv(0.975,x)	A+B/i	3.315833E-03	0.77753706	-1.627354E+03
tinv(0.975,x)	A+B*sqrt(i)	3.501185E-03	0.75197093	-1.520636E+03
tinv(0.975,x)	A+B*log(i)^4	3.548858E-03	0.74517058	-1.494101E+03
trigamma(x)	A+B*i	2.759045E-03	0.52330889	-1.998561E+03
trigamma(x)	A+B/i	2.608802E-03	0.57381155	-2.109540E+03
trigamma(x)	A+B*sqrt(i)	2.444024E-03	0.62594949	-2.238856E+03
trigamma(x)	A+B*log(i)^4	1.971289E-03	0.75665639	-2.664903E+03

Alternating Cosine/Sine Series of Order 7

The next table shows a summary of results for the Sine series of the order 7:

$Y = a_0 + a_1 * \cos(S_1 * gx(1,A_1,B_1) + Os_1) + a_2 * \sin(S_2 * gx(2,A_2,B_2) + Os_2)$

$+ \ldots + a_7 * \cos(S_7 * gx(7,A_7,B_7) + Os_7)$

The output text files for this series are located in the following folder:

Fourier-Shammas Series Approximations 7 Cosine Sine

The files are named using the following general format:

fxName_n_7_cossin_run1.txt

Where fxName is the function name and n is the gx series number which varies between 1 and 4. Each output files specifies the function being approximated and the gx series being used. For example, the file for the inverse hyperboplic cosine (acosh) using the first gx series is named acosh_1_7_cossin _run1.txt.

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
10^x	A+B*i	1.733633E-06	1	-1.893271E+03
10^x	A+B/i	1.205508E-08	1	-Inf
10^x	A+B*sqrt(i)	6.840077E-08	1	-2.546254E+03
10^x	A+B*log(i)^4	1.232088E-07	1	-2.427378E+03
acosh(x)	A+B*i	4.876763E-03	0.97074788	-8.696193E+02
acosh(x)	A+B/i	5.318000E-03	0.9652151	-6.979474E+02
acosh(x)	A+B*sqrt(i)	3.280868E-03	0.9867605	-1.655232E+03
acosh(x)	A+B*log(i)^4	2.232378E-03	0.99387044	-2.418383E+03
arccos(x)	A+B*i	6.319463E-04	0.99970561	-7.017567E+02
arccos(x)	A+B/i	5.687922E-04	0.99976151	-7.230251E+02
arccos(x)	A+B*sqrt(i)	5.905998E-04	0.99974287	-7.154251E+02
arccos(x)	A+B*log(i)^4	5.781717E-04	0.99975358	-7.197212E+02
arcsin(x)	A+B*i	7.087058E-04	0.99962975	-6.786002E+02
arcsin(x)	A+B/i	5.809727E-04	0.99975118	-7.187450E+02
arcsin(x)	A+B*sqrt(i)	5.983976E-04	0.99973604	-7.127756E+02
arcsin(x)	A+B*log(i)^4	5.348588E-04	0.99978912	-7.354506E+02
arctan(x)	A+B*i	4.864884E-10	1	-Inf
arctan(x)	A+B/i	6.840164E-10	1	-3.469463E+03
arctan(x)	A+B*sqrt(i)	6.320781E-10	1	-3.491451E+03
arctan(x)	A+B*log(i)^4	5.876026E-09	1	-3.042046E+03
asinh(x)	A+B*i	6.180782E-03	0.95826629	-3.942699E+02
asinh(x)	A+B/i	5.071782E-03	0.97189903	-7.901706E+02
asinh(x)	A+B*sqrt(i)	4.766636E-03	0.97517872	-9.143974E+02
asinh(x)	A+B*log(i)^4	4.413177E-03	0.97872337	-1.068643E+03
atanh(x)	A+B*i	1.203366E-03	0.99563465	-3.667487E+03
atanh(x)	A+B/i	1.189761E-03	0.9957328	-3.690229E+03
atanh(x)	A+B*sqrt(i)	1.221645E-03	0.99550102	-3.637336E+03
atanh(x)	A+B*log(i)^4	1.288397E-03	0.99499593	-3.530936E+03
Ci(x)	A+B*i	2.980873E-03	0.91160472	-6.561869E+02
Ci(x)	A+B/i	1.143326E-03	0.98699584	-1.031831E+03
Ci(x)	A+B*sqrt(i)	1.745991E-03	0.96967323	-8.658652E+02
Ci(x)	A+B*log(i)^4	2.829560E-03	0.92035107	-6.766081E+02
cosh(x)	A+B*i	5.827848E-03	0.99994833	-5.868090E+02
cosh(x)	A+B/i	4.263750E-06	1	-7.821504E+03
cosh(x)	A+B*sqrt(i)	3.518707E-03	0.99998116	-1.092372E+03
cosh(x)	A+B*log(i)^4	2.841251E-03	0.99998772	-1.306649E+03
digamma(x)	A+B/i	4.337751E-03	0.97454631	-1.100268E+03
digamma(x)	A+B*i	2.191051E-03	0.99350579	-2.440265E+03
digamma(x)	A+B*sqrt(i)	3.349368E-03	0.98482435	-1.607611E+03
digamma(x)	A+B*log(i)^4	3.365231E-03	0.98468027	-1.598341E+03
erf(x)	A+B*i	2.238303E-06	0.99999999	-3.729386E+03
erf(x)	A+B/i	4.997563E-10	1	-Inf
erf(x)	A+B*sqrt(i)	8.211449E-09	1	-6.095936E+03
erf(x)	A+B*log(i)^4	9.298540E-06	0.99999978	-3.128399E+03
exp(x)	A+B*i	2.066736E-05	0.99999997	-2.667228E+03
exp(x)	A+B/i	4.223523E-09	1	-Inf
exp(x)	A+B*sqrt(i)	9.225216E-07	1	-3.917127E+03
exp(x)	A+B*log(i)^4	1.479230E-06	1	-3.727316E+03
FresnelCosine(x)	A+B*i	1.054144E-03	0.97878997	-2.114754E+03

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
FresnelCosine(x)	A+B/i	1.021731E-03	0.98007425	-2.142924E+03
FresnelCosine(x)	A+B*sqrt(i)	1.207615E-03	0.97216454	-1.992155E+03
FresnelCosine(x)	A+B*log(i)^4	9.916956E-04	0.98122853	-2.169837E+03
FresnelSine(x)	A+B*i	4.427747E-04	0.99564314	-2.619470E+03
FresnelSine(x)	A+B/i	8.500687E-04	0.9839411	-2.096361E+03
FresnelSine(x)	A+B*sqrt(i)	3.747859E-04	0.99687842	-2.753168E+03
FresnelSine(x)	A+B*log(i)^4	1.586773E-04	0.99944045	-3.442473E+03
J0(x)	A+B*i	2.080495E-04	0.99955867	-2.350136E+03
J0(x)	A+B/i	4.651214E-04	0.99779423	-1.897995E+03
J0(x)	A+B*sqrt(i)	6.630802E-04	0.9955171	-1.698711E+03
J0(x)	A+B*log(i)^4	3.899445E-04	0.99844964	-1.997072E+03
J1(x)	A+B*i	6.189895E-05	0.99997153	-3.228850E+03
J1(x)	A+B/i	1.497366E-03	0.98334111	-1.310903E+03
J1(x)	A+B*sqrt(i)	1.060310E-03	0.99164674	-1.518680E+03
J1(x)	A+B*log(i)^4	2.488015E-03	0.9540065	-1.005220E+03
J2(x)	A+B*i	4.385556E-04	0.99828805	-2.050144E+03
J2(x)	A+B/i	8.557031E-05	0.99993482	-3.033901E+03
J2(x)	A+B*sqrt(i)	4.324684E-03	0.83352446	-6.724023E+02
J2(x)	A+B*log(i)^4	2.288589E-03	0.95337945	-1.055517E+03
J3(x)	A+B*i	4.800121E-04	0.99767324	-1.995769E+03
J3(x)	A+B/i	5.546876E-05	0.99996893	-3.294879E+03
J3(x)	A+B*sqrt(i)	1.448107E-03	0.97882377	-1.331040E+03
J3(x)	A+B*log(i)^4	1.814358E-03	0.96675752	-1.195304E+03
J4(x)	A+B*i	2.460385E-03	0.93306666	-1.011943E+03
J4(x)	A+B/i	1.269909E-03	0.98216876	-1.410089E+03
J4(x)	A+B*sqrt(i)	1.761891E-03	0.96567629	-1.212969E+03
J4(x)	A+B*log(i)^4	3.813821E-04	0.99839174	-2.134234E+03
J5(x)	A+B*i	2.166832E-03	0.94206647	-1.088428E+03
J5(x)	A+B/i	3.877698E-03	0.8144644	-7.380789E+02
J5(x)	A+B*sqrt(i)	2.633727E-03	0.91441042	-9.709576E+02
J5(x)	A+B*log(i)^4	2.291916E-03	0.9351848	-1.054643E+03
ln(x)	A+B*i	5.196244E-04	0.99929284	-4.908406E+03
ln(x)	A+B/i	9.003681E-06	0.99999979	-1.221637E+04
ln(x)	A+B*sqrt(i)	3.610590E-04	0.99965858	-5.564451E+03
ln(x)	A+B*log(i)^4	3.748340E-04	0.99963203	-5.496980E+03
log(x)	A+B*i	1.972695E-04	0.99945963	-6.653707E+03
log(x)	A+B/i	3.580076E-06	0.99999982	-1.387826E+04
log(x)	A+B*sqrt(i)	8.896684E-05	0.99989009	-8.088652E+03
log(x)	A+B*log(i)^4	3.596933E-05	0.99998203	-9.720538E+03
log10Gamma(x)	A+B*i	4.842358E-03	0.99998943	-8.843585E+02
log10Gamma(x)	A+B/i	8.485616E-03	0.99996756	2.162661E+02
log10Gamma(x)	A+B*sqrt(i)	9.129301E-04	0.99999962	-4.157951E+03
log10Gamma(x)	A+B*log(i)^4	4.362127E-03	0.99999143	-1.089273E+03
Si(x)	A+B*i	2.583068E-03	0.89063623	-6.897239E+02
Si(x)	A+B/i	7.705362E-04	0.99026832	-1.144551E+03
Si(x)	A+B*sqrt(i)	2.097857E-03	0.92786373	-7.679550E+02
Si(x)	A+B*log(i)^4	2.404509E-03	0.9052335	-7.166575E+02
sinh(x)	A+B*i	1.137632E-02	0.99980594	8.341468E+01
sinh(x)	A+B/i	6.401684E-06	1	-7.414279E+03

Function	gx(i,A,B)	MSSE	RsqrAdj	AICc
sinh(x)	A+B*sqrt(i)	9.227521E-04	0.99999872	-2.433537E+03
sinh(x)	A+B*log(i)^4	4.629776E-03	0.99996786	-8.174086E+02
tan(x)	A+B*i	5.471855E-07	1	-2.126215E+03
tan(x)	A+B/i	2.878838E-07	1	-2.255945E+03
tan(x)	A+B*sqrt(i)	4.397086E-07	1	-2.170387E+03
tan(x)	A+B*log(i)^4	1.834250E-06	1	-1.881875E+03
tanh(x)	A+B*i	9.918657E-06	0.9999996	-4.331164E+03
tanh(x)	A+B/i	2.224273E-07	1	-6.617302E+03
tanh(x)	A+B*sqrt(i)	9.243806E-07	1	-5.759739E+03
tanh(x)	A+B*log(i)^4	8.509582E-07	1	-5.809561E+03
tinv(0.95,x)	A+B*i	1.626748E-03	0.83745122	-3.024545E+03
tinv(0.95,x)	A+B/i	2.539966E-03	0.60372303	-2.150341E+03
tinv(0.95,x)	A+B*sqrt(i)	2.174515E-03	0.70955237	-2.455128E+03
tinv(0.95,x)	A+B*log(i)^4	1.567449E-03	0.84908596	-3.097401E+03
tinv(0.975,x)	A+B*i	3.443237E-03	0.76011326	-1.553380E+03
tinv(0.975,x)	A+B/i	2.603573E-03	0.86284488	-2.101813E+03
tinv(0.975,x)	A+B*sqrt(i)	2.792141E-03	0.84225803	-1.964622E+03
tinv(0.975,x)	A+B*log(i)^4	3.211285E-03	0.79134441	-1.690212E+03
trigamma(x)	A+B*i	2.382986E-03	0.6443994	-2.288984E+03
trigamma(x)	A+B/i	2.904367E-03	0.47177074	-1.896823E+03
trigamma(x)	A+B*sqrt(i)	2.193185E-03	0.69878968	-2.453489E+03
trigamma(x)	A+B*log(i)^4	2.389936E-03	0.64232213	-2.283211E+03

Selected Results

The next section displays the output results for selected approximations for the best Fourier-Shammas series model. Most of the output files are located in the folders creaed by the ZIP file that you download with this study. I am limiting the selection of output files to reduce the page count of this report.

Results for the Arc Tangent

Using Power A+B* i

```
Fitting arctan(x) in range (0.000000, 1.000000)
Fourier Shammas Series factor is A+B*i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
```

	Estimate	SE	tStat	pValue
(Intercept)	-77.4983957157382	0.000152079768578284	-509590.436914988	0
x1	78.7493163920393	0.000153488753863432	513062.45187258	0
x2	-0.0529485581413543	1.51542345677453e-06	-34939.7773306555	0
x3	-0.125816946384779	1.74031888164054e-06	-72295.3406482471	0
x4	0.0218689152509385	2.81255311507535e-07	77754.674689415	0
x5	0.000285618935911643	2.78698359979526e-08	10248.3177845978	2.88589334493451e-283
x6	2.73932962999732e-09	4.36017728721606e-09	0.628261065904125	0.531372804570908
x7	-1.87608029331471e-09	4.36056436213761e-09	-0.430237954885966	0.668017913754476

```
Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 3.09e-08
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 8.12e+14, p-value = 0
```

	SumSq	DF	MeanSq	F	pValue
Total	5.42875862162711	100	0.0542875862162711		
Model	5.42875862162703	7	0.775536945946718	812054586930813	0
Residual	8.88178419700125e-14	93	9.55030558817339e-16		

```
Model is ----------------------------------
y = -7.749840e+01 +
    7.874932e+01 * cos(5.802763e-02 * x - 0.174033) +
    -5.294856e-02 * cos(2.269672e+00 * x - 1.509172) +
    -1.258169e-01 * cos(2.404814e+00 * x + 0.995005) +
    2.186892e-02 * cos(3.781743e+00 * x - 1.079027) +
    2.856189e-04 * cos(7.266786e+00 * x - 1.300617) +
    2.739330e-09 * cos(9.611503e+01 * x - 1.988917) +
    -1.876080e-09 * cos(3.914861e+02 * x + 1.185871) +

List of factors: [0.058028, 2.269672, 2.404814, 3.781743, 7.266786, 96.115032,
391.486073]
List of offsets: [-0.174033, -1.509172, 0.995005, -1.079027, -1.300617, -1.988917,
1.185871]
Fitting arctan(x) in range (0.000000, 1.000000)
MSS of errors squared = 2.938997e-09
R-Squared =   1.00000000
R-Squared Adjusted =   1.00000000
```

```
Particle swarm AICc =  -3.182009e+03
AIC =  -3.199574e+03
AICc =  -3.182009e+03
```

Comments on the Above Output

The above output (typical of all output text files) shows the following information:

- The name of the fitting function and the range of x values used to fit that function.
- The expressin of the Shammas sequence used.
- The linear regression coefficients for the trigonometric terms. These coefficients are accompanied by their standard errors, student-t values, and p-values.
- The ANOVA table for the regression that includes the F statistic and its p-value.
- The model used in fitting, showing the various terms. This is the model you employ to calculate the function values for x values of your choosing. Notice that the term inside the trigonometric functions shows the values of C_i * gx(i,A_i,B_i).
- The list of factors and offset values.
- The mean square of the sum of errors squared.
- The regression coefficient of determination and its adjusted value. The latter is used to select the best models.
- The AIC and AICc statistics.

Using Power A+B/i

```
Fitting arctan(x) in range (0.000000, 1.000000)
Fourier Shammas Series factor is A+B/i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
                         Estimate            SE     tStat     pValue
                   ____________________      __     _____     ______

    (Intercept)        3.44786139999561      0       Inf        0
    x1              -0.0072407454183919      0      -Inf        0
    x2             7.26941033532173e-05      0       Inf        0
    x3               -0.108550539920097      0      -Inf        0
    x4                 3.53364677726021      0       Inf        0
    x5             -0.00386495329494814      0      -Inf        0
    x6                 22.2784146360353      0       Inf        0
    x7                 17.6809836009353      0       Inf        0


Number of observations: 101, Error degrees of freedom: 93
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: Inf, p-value = 0
                     SumSq           DF           MeanSq            F      pValue
                ________________    ___    __________________    ___    ______

    Total       5.42875862162703    100    0.0542875862162703
    Model       5.42875862162703      7     0.775536945946719    Inf       0
    Residual                   0     93                     0

Model is ----------------------------------
y = 3.447861e+00 +
    -7.240745e-03 * cos(4.668818e+00 * x + 1.967357) +
    7.269410e-05 * cos(8.507213e+00 * x - 1.372306) +
    -1.085505e-01 * cos(2.264147e+00 * x + 1.204349) +
    3.533647e+00 * cos(5.927327e-01 * x - 1.697053) +
    -3.864953e-03 * cos(3.692650e+00 * x + 0.710857) +
    2.227841e+01 * cos(2.694951e-01 * x - 1.970184) +
    1.768098e+01 * cos(4.110465e-01 * x + 1.242603) +

List of factors: [4.668818, 8.507213, 2.264147, 0.592733, 3.692650, 0.269495,
0.411046]
List of offsets: [1.967357, -1.372306, 1.204349, -1.697053, 0.710857, -1.970184,
1.242603]
Fitting arctan(x) in range (0.000000, 1.000000)
MSS of errors squared = 3.252243e-10
R-Squared =    1.00000000
R-Squared Adjusted =    1.00000000
```

```
Particle swarm AICc =  -1.000000e+99
AIC =  -Inf
AICc =  -Inf
```

Using Power A+B*sqrt(i)

```
Fitting arctan(x) in range (0.000000, 1.000000)
Fourier Shammas Series factor is A+B*sqrt(i)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
```

	Estimate	SE	tStat	pValue
(Intercept)	0.154269836418009	3.66362543774854e-05	4210.85176526164	2.42369409913552e-247
x1	-0.342049503311843	5.58496978832163e-05	-6124.46470215614	1.79308992667247e-262
x2	-0.356583786910724	0.000182743363436148	-1951.28173305903	2.82320330080563e-216
x3	1.81231712530661	0.000321110657640525	5643.90213212871	3.58128440284591e-259
x4	1.3571222725687	0.000452446447049014	2999.52023365472	1.2160871405538e-233
x5	0.140837617281105	3.25450623277958e-05	4327.46497341379	1.91059139929347e-248
x6	0.0164914152487537	6.6330811543786e-06	2486.23752143714	4.62619891458404e-226
x7	0.000384590295915387	1.54335678774442e-07	2491.90789174199	3.74295274039294e-226

```
Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 1.38e-08
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 4.06e+15, p-value = 0
```

	SumSq	DF	MeanSq	F	pValue
Total	5.42875862162703	100	0.0542875862162703		
Model	5.42875862162701	7	0.775536945946716	4.06027293465406e+15	0
Residual	1.77635683940025e-14	93	1.91006111763468e-16		

```
Model is ---------------------------------
y = 1.542698e-01 +
    -3.420495e-01 * cos(1.896662e+00 * x - 0.946040) +
    -3.565838e-01 * cos(2.615621e+00 * x + 0.047433) +
    1.812317e+00 * cos(1.520508e+00 * x - 1.973653) +
    1.357122e+00 * cos(2.087334e+00 * x + 0.549190) +
    1.408376e-01 * cos(2.898250e+00 * x - 1.987333) +
    1.649142e-02 * cos(4.352553e+00 * x - 0.791914) +
    3.845903e-04 * cos(6.745402e+00 * x - 0.514223) +

List of factors: [1.896662, 2.615621, 1.520508, 2.087334, 2.898250, 4.352553,
6.745402]
List of offsets: [-0.946040, 0.047433, -1.973653, 0.549190, -1.987333, -0.791914, -
0.514223]
Fitting arctan(x) in range (0.000000, 1.000000)
MSS of errors squared = 1.252821e-09
R-Squared =    1.00000000
R-Squared Adjusted =   1.00000000
```



```
Particle swarm AICc =  -3.353725e+03
AIC =  -3.371290e+03
AICc =  -3.353725e+03
```

Using Power A+B*log10(i)^4

```
Fitting arctan(x) in range (0.000000, 1.000000)
Fourier Shammas Series factor is A+B*log(i)^4

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
```

	Estimate	SE	tStat	pValue
(Intercept)	-23.2562081293887	0.00100827977295861	-23065.2332349657	4.96483326435232e-316
x1	23.9465675379464	0.00102323063407828	23402.903256036	1.28498865872362e-316
x2	-0.0300087926552712	3.72078078391044e-06	-8065.18695888683	1.36715213046907e-273
x3	0.357792860231518	1.68979432061123e-05	21173.7520873013	1.41836007196087e-312
x4	-0.0503455970073738	3.64760893165676e-06	-13802.3559955773	2.72476220837681e-295
x5	1.11308439859315e-09	1.61067744725611e-08	0.0691065986234153	0.945053141663664
x6	-2.31560160747668e-09	1.61689870065988e-08	-0.143212534374086	0.886432173712071
x7	0.0607881339784335	6.10528821872056e-06	9956.63624725196	4.23149563675436e-282

```
Number of observations: 101, Error degrees of freedom: 93
Root Mean Squared Error: 1.14e-07
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 5.92e+13, p-value = 0
```

	SumSq	DF	MeanSq	F	pValue
Total	5.42875862162709	100	0.0542875862162709		
Model	5.42875862162588	7	0.775536945946554	59187652108647.3	0
Residual	1.21858079182857e-12	93	1.31030192669739e-14		

```
Model is ----------------------------------
y = -2.325621e+01 +
    2.394657e+01 * cos(1.137377e-01 * x - 0.210570) +
    -3.000879e-02 * cos(4.318501e+00 * x + 0.504511) +
    3.577929e-01 * cos(1.059072e+00 * x - 2.000000) +
    -5.034560e-02 * cos(4.656682e+00 * x + 0.507062) +
    1.113084e-09 * cos(1.371190e+02 * x + 0.235206) +
    -2.315602e-09 * cos(4.490059e+02 * x - 1.953266) +
    6.078813e-02 * cos(6.235578e+02 * x - 0.317420) +

List of factors: [0.113738, 4.318501, 1.059072, 4.656682, 137.119005, 449.005853,
623.557838]
List of offsets: [-0.210570, 0.504511, -2.000000, 0.507062, 0.235206, -1.953266, -
0.317420]
Fitting arctan(x) in range (0.000000, 1.000000)
MSS of errors squared = 1.092659e-08
R-Squared =   1.00000000
```

```
R-Squared Adjusted =   1.00000000
Particle swarm AICc =  -2.916759e+03
AIC =  -2.934325e+03
AICc =  -2.916759e+03
```

RESULTS FOR THE ERROR FUNCTION

Using Power A+B* i

```
Fitting erf(x) in range (0.000000, 2.100000)
Fourier Shammas Series factor is A+B*i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
```

	Estimate	SE	tStat	pValue
(Intercept)	2.90724848694267	3.67002237212995e-05	79216.0971284599	0
x1	-3.19835454663475	4.60800007020234e-05	-69408.7347636327	0
x2	-0.843455483319476	8.08339616069946e-06	-104344.197234853	0
x3	0.0437954161246887	2.6041324670508e-05	1681.76606523735	0
x4	-0.0172733062486685	2.3466805020218e-05	-736.074051571423	0
x5	-6.7232188296064e-07	1.68274579244114e-06	-0.399538591022301	0.689916436580853
x6	1.25683183332312e-05	1.73928581764337e-06	7.22613742131273	9.85909996220573e-12
x7	2.6107426713682e-06	1.68263782578695e-06	1.55157730995806	0.122321005255727

```
Number of observations: 211, Error degrees of freedom: 203
Root Mean Squared Error: 1.73e-05
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 8.71e+09, p-value = 0
```

	SumSq	DF	MeanSq	F	pValue
Total	18.2280044423161	210	0.086800021153886		
Model	18.2280043816505	7	2.60400062595007	8713544174.42407	0
Residual	6.06655703450087e-08	203	2.98845174113344e-10		

```
Model is ---------------------------------
y = 2.907248e+00 +
    -3.198355e+00 * cos(4.007440e-01 * x + 0.031665) +
    -8.434555e-01 * cos(1.256657e+00 * x + 1.919856) +
    4.379542e-02 * cos(3.440187e+00 * x - 1.710789) +
    -1.727331e-02 * cos(3.770055e+00 * x - 1.999933) +
    -6.723219e-07 * cos(2.247492e+02 * x - 0.556331) +
    1.256832e-05 * cos(1.398491e+01 * x - 1.995606) +
    2.610743e-06 * cos(4.568996e+01 * x + 2.000000) +

List of factors: [0.400744, 1.256657, 3.440187, 3.770055, 224.749202, 13.984913,
45.689956]
```

```
List of offsets: [0.031665, 1.919856, -1.710789, -1.999933, -0.556331, -1.995606,
2.000000]
Fitting erf(x) in range (0.000000, 2.100000)
MSS of errors squared = 1.167316e-06
R-Squared =   1.00000000
R-Squared Adjusted =   1.00000000
Particle swarm AICc =  -4.004112e+03
AIC =  -4.020825e+03
AICc =  -4.004112e+03
```

Using Power A+B/i

```
Fitting erf(x) in range (0.000000, 2.100000)
Fourier Shammas Series factor is A+B/i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
```

	Estimate	SE	tStat	pValue
(Intercept)	1.01668337350373	5.03296155975247e-06	202004.994759731	0
x1	-0.223228677340984	4.07917756793915e-06	-54723.9421729223	0
x2	0.000621486788734503	1.91280646392866e-07	3249.08348259158	0
x3	-1.51650118377876	1.22765200293228e-05	-123528.587918772	0
x4	0.623038840695041	1.1512463275148e-06	541186.387139232	0
x5	-0.000922142480303495	3.63077831571623e-08	-25397.9284913071	0
x6	0.0551555688956853	5.84523906787855e-07	94359.8170326046	0
x7	0.008061238091598	3.09514476813815e-07	26044.7852862376	0

```
Number of observations: 211, Error degrees of freedom: 203
Root Mean Squared Error: 4.43e-08
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.33e+15, p-value = 0
```

	SumSq	DF	MeanSq	F	pValue
Total	18.2280044423161	210	0.086800021153886		
Model	18.2280044423157	7	2.60400063461652	1.32849184509458e+15	0
Residual	3.97903932025656e-13	203	1.960117891752e-15		

```
Model is ----------------------------------
y = 1.016683e+00 +
    -2.232287e-01 * cos(1.178073e+00 * x + 0.271740) +
    6.214868e-04 * cos(4.323665e+00 * x + 0.541183) +
    -1.516501e+00 * cos(3.653893e-01 * x + 0.665255) +
    6.230388e-01 * cos(1.143186e+00 * x - 0.868095) +
    -9.221425e-04 * cos(4.512981e+00 * x + 1.768828) +
    5.515557e-02 * cos(2.599066e+00 * x - 1.784169) +
```

```
    8.061238e-03 * cos(3.655807e+00 * x - 1.597784) +

List of factors: [1.178073, 4.323665, 0.365389, 1.143186, 4.512981, 2.599066, 3.655807]
List of offsets: [0.271740, 0.541183, 0.665255, -0.868095, 1.768828, -1.784169, -1.597784]
Fitting erf(x) in range (0.000000, 2.100000)
MSS of errors squared = 2.982498e-09
R-Squared =   1.00000000
R-Squared Adjusted =   1.00000000
Particle swarm AICc =  -6.523324e+03
AIC =  -6.540036e+03
AICc =  -6.523324e+03
```

Using Power A+B*sqrt(i)

```
Fitting erf(x) in range (0.000000, 2.100000)
Fourier Shammas Series factor is A+B*sqrt(i)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
```

	Estimate	SE	tStat	pValue
(Intercept)	0.272229716048142	1.37188392850141e-07	1984349.48024732	0
x1	0.0729447603439714	1.21586241818224e-07	599942.55314698	0
x2	0.984755846338809	2.83323554074325e-07	3475728.83432232	0
x3	0.234391487864624	1.90268401218504e-07	1231899.18222653	0
x4	0.0232115061046311	1.89237877258176e-07	122657.823269513	0
x5	-0.0147770814211736	9.15614322392877e-08	-161389.802013527	0
x6	-0.00139521906412026	8.18222015996416e-09	-170518.396821819	0
x7	0.000105569522811397	1.68319271308626e-08	6271.98074175518	0

```
Number of observations: 211, Error degrees of freedom: 203
Root Mean Squared Error: 1.18e-08
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.86e+16, p-value = 0
```

	SumSq	DF	MeanSq	F	pValue
Total	18.228004442316	210	0.0868000211538858		
Model	18.228004442316	7	2.60400063461657	1.85988858313245e+16	0
Residual	2.8421709430404e-14	203	1.40008420839429e-16		

```
Model is ----------------------------------
y = 2.722297e-01 +
    7.294476e-02 * cos(2.146382e+00 * x - 1.999697) +
    9.847558e-01 * cos(7.385953e-01 * x - 1.995217) +
    2.343915e-01 * cos(1.407839e+00 * x - 0.907070) +
    2.321151e-02 * cos(2.218105e+00 * x - 0.171731) +
    -1.477708e-02 * cos(3.393709e+00 * x + 1.291051) +
```

```
    -1.395219e-03 * cos(4.478933e+00 * x + 1.813417) +
    1.055695e-04 * cos(5.010661e+00 * x - 0.371688) +

List of factors: [2.146382, 0.738595, 1.407839, 2.218105, 3.393709, 4.478933,
5.010661]
List of offsets: [-1.999697, -1.995217, -0.907070, -0.171731, 1.291051, 1.813417, -
0.371688]
Fitting erf(x) in range (0.000000, 2.100000)
MSS of errors squared = 9.365843e-10
R-Squared =   1.00000000
R-Squared Adjusted =   1.00000000
Particle swarm AICc =  -7.000241e+03
AIC =  -7.016954e+03
AICc =  -7.000241e+03
```

Using Power A+B*log10(i)^4

```
Fitting erf(x) in range (0.000000, 2.100000)
Fourier Shammas Series factor is A+B*log(i)^4

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
```

	Estimate	SE	tStat	pValue
(Intercept)	0.280971647996033	9.91500962181725e-07	283380.106235879	0
x1	0.910881992010581	9.38594491760995e-06	97047.4470078745	0
x2	0.0114448851223339	7.27475333401952e-06	1573.23342756011	0
x3	-0.195147375118356	2.95447959264438e-06	-66051.3532075849	0
x4	0.00641490655789046	9.9638082890534e-07	6438.20753249349	0
x5	5.96832277748813e-08	3.93545949885084e-07	0.151655042549184	0.879609625937866
x6	-5.44060526594415e-08	3.93291125867749e-07	-0.138335317226905	0.890112541201963
x7	4.15281300609867e-07	3.94594391701555e-07	1.05242575501164	0.293855376939901

```
Number of observations: 211, Error degrees of freedom: 203
Root Mean Squared Error: 4.04e-06
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 1.6e+11, p-value = 0
```

	SumSq	DF	MeanSq	F	pValue
Total	18.2280044423161	210	0.0868000211538862		
Model	18.2280044390037	7	2.60400063414339	159586815519.845	0
Residual	3.312379703857e-09	203	1.63171413983104e-11		

```
Model is ---------------------------------
y = 2.809716e-01 +
    9.108820e-01 * cos(9.026541e-01 * x - 1.972477) +
    1.144489e-02 * cos(1.450688e+00 * x + 1.117892) +
    -1.951474e-01 * cos(1.989321e+00 * x + 1.943422) +
    6.414907e-03 * cos(3.823935e+00 * x - 1.708967) +
```



```
    5.968323e-08 * cos(2.324797e+02 * x + 1.907620) +
    -5.440605e-08 * cos(5.092083e+02 * x - 0.555069) +
    4.152813e-07 * cos(4.907164e+01 * x + 1.999295) +

List of factors: [0.902654, 1.450688, 1.989321, 3.823935, 232.479715, 509.208277,
49.071639]
List of offsets: [-1.972477, 1.117892, 1.943422, -1.708967, 1.907620, -0.555069,
1.999295]
Fitting erf(x) in range (0.000000, 2.100000)
MSS of errors squared = 2.727623e-07
R-Squared =   1.00000000
R-Squared Adjusted =   1.00000000
Particle swarm AICc =  -4.617642e+03
AIC =  -4.634355e+03
AICc =  -4.617642e+03
```

RESULTS FOR THE EXPONENTIAL FUNCTION

Using Power A+B* i

```
Fitting exp(x) in range (0.000000, 2.000000)
Fourier Shammas Series factor is A+B*i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
```

	Estimate	SE	tStat	pValue
(Intercept)	78.9009569635739	0.022793896205444	3461.49496568864	0
x1	-107.518430504293	0.0314843183671398	-3414.98358803631	0
x2	11.5900582276611	0.00512930639555154	2259.57611690203	0
x3	0.504605451163145	0.000695502032006969	725.526925790619	0
x4	0.0119763634904029	6.65188987432189e-05	180.044524438611	6.01240857925874e-217
x5	-0.000759461586675105	1.76068162328832e-05	-43.1345211212407	4.77053440132935e-101
x6	-8.52250929085225e-06	1.01247920986014e-05	-0.841746596656488	0.400971592839485
x7	2.56662419580418e-06	1.01403993598019e-05	0.253108788395324	0.800453542820344

```
Number of observations: 201, Error degrees of freedom: 193
Root Mean Squared Error: 0.000101
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 9.09e+09, p-value = 0
```

	SumSq	DF	MeanSq	F	pValue
Total	650.160904122337	200	3.25080452061169		
Model	650.160902149316	7	92.8801288784738	9085491301.19031	0
Residual	1.97302097149077e-06	193	1.02229065880351e-08		

```
Model is ----------------------------------
y = 7.890096e+01 +
```



```
    -1.075184e+02 * cos(1.162995e-01 * x + 0.744507) +
    1.159006e+01 * cos(7.235885e-01 * x + 1.452368) +
    5.046055e-01 * cos(1.896148e+00 * x - 2.000000) +
    1.197636e-02 * cos(3.825493e+00 * x + 0.236809) +
    -7.594616e-04 * cos(6.283185e+00 * x - 0.642565) +
    -8.522509e-06 * cos(3.258060e+01 * x + 0.289778) +
    2.566624e-06 * cos(3.571268e+02 * x + 1.284814) +

List of factors: [0.116300, 0.723589, 1.896148, 3.825493, 6.283185, 32.580596,
357.126795]
List of offsets: [0.744507, 1.452368, -2.000000, 0.236809, -0.642565, 0.289778,
1.284814]
Fitting exp(x) in range (0.000000, 2.000000)
MSS of errors squared = 6.988275e-06
R-Squared =   1.00000000
R-Squared Adjusted =   1.00000000
Particle swarm AICc =  -3.103126e+03
AIC =  -3.119876e+03
AICc =  -3.103126e+03
```

Using Power A+B/i

```
Fitting exp(x) in range (0.000000, 2.000000)
Fourier Shammas Series factor is A+B/i

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
```

	Estimate	SE	tStat	pValue
(Intercept)	3728.61386048558	0	Inf	0
x1	-5152.94633350584	0	-Inf	0
x2	295.618575152064	0	Inf	0
x3	-4836.00263784752	0	-Inf	0
x4	860.592798213585	0	Inf	0
x5	0.0355379301593068	0	Inf	0
x6	7576.53713195384	0	Inf	0
x7	-9513.27949486371	0	-Inf	0

```
Number of observations: 201, Error degrees of freedom: 193
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: Inf, p-value = 0
```

	SumSq	DF	MeanSq	F	pValue
Total	650.160904122279	200	3.2508045206114		
Model	650.160904122279	7	92.8801291603256	Inf	0
Residual	0	193	0		

```
Model is ---------------------------------
y = 3.728614e+03 +
    -5.152946e+03 * cos(1.105529e-01 * x + 0.899204) +
    2.956186e+02 * cos(6.012871e-01 * x + 1.689626) +
    -4.836003e+03 * cos(2.798885e-01 * x - 0.200883) +
    8.605928e+02 * cos(4.941991e-01 * x - 2.000000) +
    3.553793e-02 * cos(1.985559e+00 * x - 1.281113) +
    7.576537e+03 * cos(3.426192e-01 * x - 1.384832) +
    -9.513279e+03 * cos(3.278779e-01 * x - 1.914950) +

List of factors: [0.110553, 0.601287, 0.279889, 0.494199, 1.985559, 0.342619,
0.327878]
List of offsets: [0.899204, 1.689626, -0.200883, -2.000000, -1.281113, -1.384832, -
1.914950]
Fitting exp(x) in range (0.000000, 2.000000)
MSS of errors squared = 8.297127e-09
R-Squared =   1.00000000
R-Squared Adjusted =   1.00000000
Particle swarm AICc =  -1.000000e+99
AIC =  -Inf
AICc =  -Inf
```

Using Power A+B*sqrt(i)

```
Fitting exp(x) in range (0.000000, 2.000000)
Fourier Shammas Series factor is A+B*sqrt(i)

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
```

	Estimate	SE	tStat	pValue
(Intercept)	32.4908247654358	0.0073876918945854	4397.9669467874	0
x1	145.186752430577	0.0361756746731611	4013.38064161362	0
x2	95.8380994099605	0.0265606640255505	3608.27196630955	0
x3	2.28108691474686	0.00172711246043586	1320.75181379398	0
x4	0.376870134249709	0.000513303632098466	734.205079962136	0
x5	0.0678245900405468	0.000148357284351612	457.170609026517	7.60821763840627e-295
x6	-0.00312600295029077	1.69677816594128e-05	-184.231681727035	7.29710766981515e-219
x7	2.24930047435485e-07	4.04111470368899e-07	0.556603966796969	0.578442856025946

```
Number of observations: 201, Error degrees of freedom: 193
Root Mean Squared Error: 4.03e-06
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 5.71e+12, p-value = 0
```

	SumSq	DF	MeanSq	F	pValue
Total	650.160904122337	200	3.25080452061169		
Model	650.160904119201	7	92.8801291598858	5714612040046.92	0


```
    Residual    3.13684722641483e-09    193    1.62530944373825e-11

Model is ----------------------------------
y = 3.249082e+01 +
    1.451868e+02 * cos(3.081638e-01 * x - 1.931449) +
    9.583810e+01 * cos(4.655299e-01 * x + 1.357102) +
    2.281087e+00 * cos(1.505975e+00 * x - 1.989451) +
    3.768701e-01 * cos(2.180258e+00 * x + 0.262553) +
    6.782459e-02 * cos(2.825833e+00 * x + 1.788472) +
    -3.126003e-03 * cos(4.348684e+00 * x - 0.068136) +
    2.249300e-07 * cos(5.507066e+01 * x + 1.999942) +

List of factors: [0.308164, 0.465530, 1.505975, 2.180258, 2.825833, 4.348684,
55.070659]
List of offsets: [-1.931449, 1.357102, -1.989451, 0.262553, 1.788472, -0.068136,
1.999942]
Fitting exp(x) in range (0.000000, 2.000000)
MSS of errors squared = 2.787044e-07
R-Squared =   1.00000000
R-Squared Adjusted =   1.00000000
Particle swarm AICc =  -4.398304e+03
AIC =  -4.415054e+03
AICc =  -4.398304e+03
```

Using Power A+B*log10(i)^4

```
Fitting exp(x) in range (0.000000, 2.000000)
Fourier Shammas Series factor is A+B*log(i)^4

Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6 + x7

Estimated Coefficients:
```

	Estimate	SE	tStat	pValue
(Intercept)	90.7577066847246	0.0275729052614785	3291.55400288994	0
x1	247.577657202732	0.0759745833908334	3258.69055351218	0
x2	48.9087166619499	0.0185035303208154	2643.2100152764	0
x3	1.63941317715401	0.001460832009695	1122.24620372078	0
x4	-0.00255217282481911	2.18023592737152e-05	-117.059479333321	3.38263250096352e-181
x5	6.50722189218777e-06	3.73797311918474e-06	1.74084234549204	0.0833048561830746
x6	4.29400993889072e-06	3.72854234209018e-06	1.15165915924225	0.250885805113976
x7	8.76618498569205e-07	3.25467705059408e-06	0.269341161946988	0.787954892069954

```
Number of observations: 201, Error degrees of freedom: 193
Root Mean Squared Error: 3.26e-05
R-squared: 1,  Adjusted R-Squared: 1
F-statistic vs. constant model: 8.74e+10, p-value = 0
```

	SumSq	DF	MeanSq	F	pValue
Total	650.160904122338	200	3.25080452061169		


```
    Model            650.16090391728      7        92.8801291310401    87418570722.0928      0
    Residual    2.05057858693181e-07    193    1.06247595177814e-09

Model is ---------------------------------
y = 9.075771e+01 +
    2.475777e+02 * cos(1.000000e-01 * x - 1.961620) +
    4.890872e+01 * cos(4.924164e-01 * x + 1.463483) +
    1.639413e+00 * cos(1.371679e+00 * x - 1.999586) +
    -2.552173e-03 * cos(4.537290e+00 * x + 1.638796) +
    6.507222e-06 * cos(1.716614e+01 * x + 1.989504) +
    4.294010e-06 * cos(1.940678e+01 * x + 0.038162) +
    8.766185e-07 * cos(8.421201e+02 * x + 0.802461) +

List of factors: [0.100000, 0.492416, 1.371679, 4.537290, 17.166137, 19.406781,
842.120051]
List of offsets: [-1.961620, 1.463483, -1.999586, 1.638796, 1.989504, 0.038162,
0.802461]
Fitting exp(x) in range (0.000000, 2.000000)
MSS of errors squared = 2.252917e-06
R-Squared =   1.00000000
R-Squared Adjusted =   1.00000000
Particle swarm AICc =  -3.558193e+03
AIC =  -3.574943e+03
AICc =  -3.558193e+03
```

Conclusions

The Fourier-Shammas series produce medium quality results. Some functions lend themselves to be well approximated by the Fourier-Shammas series. Other functions are difficult to approximate, regardless of the Fourier-Shammas series used.

Next

The next study looks at adding a linear term and a quadratic term to the Fourier-Shammas series. The hope for this addition is to make these two new terms represent a rough baseline approximation that is refined by the trigonometric terms. The study is titled *The Hybrid Quadratic Fourier-Shammas series*.

Document History

Date	***Version***	***Comments***
11/7/2020	1.00.00	Initial release.

Date	***Version***	***Comments***

