Shammas Polynomials

By

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This article discusses a new brand of polynomial that is aimed at regression analysis. While traditional polynomials play a very important role in calculus, numerical analysis, and statistical curve fitting, using them in certain types of curve fitting can be frustrating. I am talking about using polynomials to fit data whose X variable (that is, the independent variable) runs into high numbers. In these cases raising high values of X to high powers creates huge numbers. Solving the regression coefficients for such polynomials may well call error and instability in calculating the polynomial coefficients. I suggest a remedy for this problem—The Shammas Polynomial. I have to somewhat apologize for the polynomial name, since the other generic names I thought of turned out to be already taken.

The goal of the Shammas Polynomials is to use lower-powers with the independent variable and obtain a better fit than normal polynomials with the same number of terms.

What is a Shammas Polynomial?

A typical polynomial of order N is defined as:

$$Y(X) = A_0 + \sum A_i X^i \qquad \text{for } i = 1 \text{ to } N$$

For a moment, let's take a step back and redefine polynomials using the following and more general form:

$$Y(X) = A_0 + \sum A_i X^{P(i)} \qquad \text{for } i = 1 \text{ to } N$$

Where P(i) is a general function for the power of each term in the polynomial. In the case of traditional polynomials, P(i) is simply equal to i.

What if we define P(i) in terms of i and some other constants? Here is an example:

$$P(i) = i / \alpha + \beta$$

Let's call α *the state of the polynomial*, and call β *the shift factor*. To tame the polynomials terms, you can have α set values higher than 1. Likewise, β can be 0.1 and higher. Values of α that exceed 1 reduce the value of the powers of $X^{P(i)}$. The higher α is the more damped the

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powers are. The use of the shift factor β allows, if needed, a limited compensation to increase the calculator powers.

You can assign different expressions for P(i). Each expression generates a variant of the Shammas Polynomial. The following table shows the proposed set of Shammas Polynomials.

Type of Shammas Polynomial	***P(i)***	***Comment***
Linear Shammas Polynomial	$i / \alpha + \beta$	When $\alpha=1$ and $\beta=0$ The Shammas Polynomial is equivalent to a regular polynomial.
Logarithmic Shammas Polynomial	$\ln(i+1) * \alpha + \beta$	
Square root Shammas Polynomial	$\sqrt{i} * \alpha + \beta$	
Reciprocal	$\alpha / i + \beta$	

You can define your own flavor of the Shammas Polynomial. You need to remember one simple rule. The expression for P(i) cannot generate the same value for any two different values of i. Such duplicate values create redundant terms in the models used for curve fitting.

Fitting Data with a Shammas Polynomial

To use a Shammas Polynomial in curve fitting I suggest the following method:

1. Select the type of Shammas Polynomial to use.
2. Select the number of terms (which is equivalent to the order of a regular polynomial).
3. Select the values for α and β.
4. For each observation, calculate the variable for each term. The value for each variable is based on the values of the independent variable X, the value of the power I, and the values of α and β. For example, if you have a 4-term Linear Shammas Polynomial, the variables for the regression are calculated using:

$$X\text{^}(1/ \alpha + \beta)$$

$$X\text{^}(2/ \alpha + \beta)$$

$X^{(3/\alpha + \beta)}$

$X^{(4/\alpha + \beta)}$

5. Perform a multiple regression to calculate the regression coefficients and also the regression statistics (such as R^2, F statistic, error sum of squares, and so on). It is a good idea to calculate the entries for the regression ANOVA table.
6. Repeat steps 3 to 5 for different values of α, until you find the best value for α.
7. Repeat steps 3 to 5 for different values of β, until you find the best value for β.
8. Repeat steps 1 to 7, if need be, to fit your data with other types of Shammas Polynomials. You can then select the very best model and its coefficients.

I recommend that you start with $\alpha = 1$ and $\beta = 0$. Alter the values for α until you get the best fit. Then alter the values of β until you get an even better fit.

It is also a good idea to perform a polynomial regression to compare the results of fitting the data with a regular polynomial and with one or more types of Shammas Polynomials.

Sample Results

Shammas Polynomials are meant to produce improved curve fitting compared to regular polynomials that have the same number of terms. Of course not every case will put the Shammas Polynomials ahead of regular polynomials.

The cases I am interested in deal with long range time series, such as stock prices and market indices that are taken over a long period of time. This section looks at how Shammas Polynomials perform with the DOW, CAC, DAX, and FTSE indices taken over a range of several years.

The DOW Jones Index

The following table shows the result of fitting the DOW Jones Index for the period between October 1995 and August 2008. The table compares the regression fitting using a 4-order polynomial against the Linear, Logarithmic, Square Root, and Reciprocal Shammas Polynomials.

Shammas Model	Regular Polynomials	Linear	Logarithmic	Square Root	Reciprocal
Adjusted R^2	0.850606274	0.915507486	0.851800446	0.916048831	0.849788444
Residual SS	1963027605	1110228262	1947336226	1103115016	1973773856
F Stat	4578.752091	8712.635906	4622.117542	8773.995839	4549.451035
α		1	2.4	2	2
β		1	1	0.3	0.2

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Shammas Model	Regular Polynomials	Linear	Logarithmic	Square Root	Reciprocal
A0	3473.45326	5120.85312	6702.908709	5186.324407	7344.807524
A1	15.42880386	0.020777917	-375.811575	0.004586171	0.001378591
A2	-0.012893107	-2.36355E-05	85.27016985	-2.67224E-05	-24.32871434
A3	4.32508E-06	9.32045E-09	-13.26792501	2.26198E-07	596.738466
A4	-4.68235E-10	-1.21795E-12	1.048469326	-1.34395E-09	-1140.874048

The Square Root Shammas Polynomial shows the best results, and is closely followed by the Linear Shammas Model. The other two Shammas Polynomials performed about the same as the regular 4-order polynomial.

The Linear Shammas Polynomial fits the DJI index with the following polynomial:

$$DJI = A_0 + A_1 t^2 + A_2 t^3 + A_3 t^4 + A_4 t^5$$

The above polynomial skips the term of time raised to power 1. Performing a 5-order polynomial curve fit confirms that the term of time raised to 1 is statistically insignificant.

The Square Root Shammas Polynomial fits the following model:

$$DJI = A_0 + A_1 t^{2.3} + A_2 t^{3.13} + A_3 t^{3.76} + A_4 t^{4.3}$$

The powers of the Linear and Square Root Shammas Polynomials are somewhat close to each other and are able to fit the data with similar goodness of fit.

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The CAC Index

Here is another example that uses the French CAC index data from January 1990 to August 2008. The following table compares the regression fitting using a 4-order polynomial against the Linear, Logarithmic, Square Root, and Reciprocal Shammas Polynomials.

Shammas Model	Regular Polynomials	Linear	Logarithmic	Square Root	Reciprocal
Adjusted R^2	0.70924439	0.718937076	0.645753745	0.720806724	0.645450616
Residual SS	2898180137	2801565835	3531039210	2782929646	3534060722
F Stat	2836.703159	2974.584483	2120.115493	3002.28222	2117.309814
α		1	1	2	1
β		0.5	0.2	0	0.2
A0	2668.289144	2139.773236	-439.4167468	2022.088205	-1051.996037
A1	-4.813418407	-0.096367857	4394.147687	-0.004401385	-2.100878531
A2	0.005543048	0.000121317	-5115.195614	2.68621E-05	1060.836605
A3	-1.81679E-06	-4.10031E-08	2723.202135	-2.10301E-07	-8206.282973
A4	1.89512E-10	4.30067E-12	-565.5951761	1.13403E-09	9140.411051

The Square Root Shammas Polynomial shows the best results, and is closely followed by the Linear Shammas Model. The other two Shammas Polynomials performed not as well as the regular 4-order polynomial.

The Square Root Shammas Polynomial fits the following model:

$$CAC = A_0 + A_1\,t^2 + A_2\,t^{2.83} + A_3\,t^{3.46} + A_4\,t^4$$

The Linear Shammas Polynomial fits the DJI index with the following polynomial:

$$CAC = A_0 + A_1\,t^{1.5} + A_2\,t^{2.5} + A_3\,t^{3.5} + A_4\,t^{4.5}$$

The powers of the Linear and Square Root Shammas Polynomials are somewhat close to each other and are able to fit the data with similar goodness of fit.

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The DAX Index

A third example uses the German DAX index data from October 1995 to August 2008. The following table compares the regression fitting using a 4-order polynomial against the Linear, Logarithmic, Square Root, and Reciprocal Shammas Polynomials.

Shammas Model	Regular Polynomials	Linear	Logarithmic	Square Root	Reciprocal
Adjusted R^2	0.670293782	0.690664248	0.694065346	0.711687097	0.682368394
Residual SS	2573816826	2414796933	2388246622	2250684280	2479557641
F Stat	1648.74906	1810.630375	1839.758555	2001.681848	1742.19821
α		1.9	3	1.9	3
β		0	-0.2	0	-0.2
A0	500.9412185	5045.750141	3978.654248	5953.837282	5589.118881
A1	13.66240913	-749.2136901	-288.2143597	-892.3167915	6.51819E-06
A2	-0.011832878	53.13958177	30.55482752	60.19979533	-4.749848283
A3	3.60308E-06	-1.188852656	-2.769392949	-1.325560379	391.4127767
A4	-3.14521E-10	0.008467432	0.142931859	0.009373356	-1390.732181

The Square Root Shammas Polynomial shows the best results. The other Shammas Polynomials did better than the 4-order polynomial.

The Square Root Shammas Polynomial fits the following model:

$$DAX = A_0 + A_1\, t^{1.9} + A_2\, t^{2.69} + A_3\, t^{3.29} + A_4\, t^{3.8}$$

The example presented show that the Square Root Shammas Polynomial performs well with the three cases studied. Generalizing this conclusion may not serve best fitting other data.

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The FTSE Index

The fourth example uses the British FTSE index data from October 1995 to August 2008. The following table compares the regression fitting using a 4-order polynomial against the Linear, Logarithmic, Square Root, and Reciprocal Shammas Polynomials.

Shammas Model	Regular Polynomials	Linear	Logarithmic	Square Root	Reciprocal
Adjusted R^2	0.652536202	0.899474907	0.639355625	0.904651481	0.631488047
Residual SS	969783354.2	280270333.6	1006570798	266120979.8	1028529478
F Stat	1523.586611	7253.163526	1438.30946	7693.266209	1390.314319
α		1	2	2	2
β		1.2	0.1	0.4	0.2
A0	2168.709723	3531.330906	5764.235357	3449.107705	5337.953954
A1	11.4505974	0.003365689	-652.1151752	0.001567875	0.001085756
A2	-0.011403113	-4.13839E-06	227.5555516	-9.61996E-06	-18.79870601
A3	4.12897E-06	1.68542E-09	-50.45793724	8.3239E-08	455.5978755
A4	-4.84747E-10	-2.241E-13	5.285896627	-5.00924E-10	-876.1792613

The Square Root and Linear Shammas Polynomials yield good fits with the data. These fits are significantly better than fitting the data with the 4-order polynomial. By contrast, the polynomial fit give better results than the Logorithmic and Reciprocal Shammas Polynomials.

The Square Root Shammas Polynomial fits the following model:

$$FTSE = A_0 + A_1\, t^{2.4} + A_2\, t^{3.23} + A_3\, t^{3.86} + A_4\, t^{4.4}$$

The Linear Shammas Polynomial fits the DJI index with the following polynomial:

$$FTSE = A_0 + A_1\, t^{2.2} + A_2\, t^{3.2} + A_3\, t^{4.2} + A_4\, t^{5.2}$$

Conclusion

This article has shown that the Shammas Polynomials provide better curve fit than comparable polynomial. Since this study is limited, more analysis is needed to generalize the conclusion drawn in this article. This article serves to open the door for further investigating Shammas Polynomials.

One of the surprises of using the Shammas Polynomials with the four cases studied, is that the polynomials seem to skip the linear terms and opt for terms with powers that exceed the power 4 present in the 4-order polynomial used for comparison. These results seem a bit counter intuitive regarding the original intention for defining the Shammas Polynomials.

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