# Roots of Two Nonlinear Equations for the HP-67

To find the roots for the following two simultaneous nonlinear equations:

f(x,y) = 0
g(x,y) = 0

let Fx(x,y) = df(x,y)/dx = (f(x+hx,y) - f(x,y)) / hx
let Fy(x,y) = df(x,y)/dy = (f(x,y+hy) - f(x,y)) / hy
let Gx(x,y) = dg(x,y)/dx = (g(x+hx,y) - g(x,y)) / hx
let Gy(x,y) = dg(x,y)/dx = (g(x,y+hy) - g(x,y)) / hy

Where,
hx = 0.001 * (1 + |x|)
hy = 0.001 * (1 + |y|)

To refine the guesses for x and y use the following equations:
J = Fy(x,y) * Gx(x,y)- Fx(x,y) * Gy(x,y)
x = x - (Fy(x,y) * g(x,y) - f(x,y) * Gy(x,y)) / J
x = y - (Gx(x,y) * f(x,y) - g(x,y) * Fx(x,y)) / J

## Algorithm

Input: x, y, TolerX, TolerY, and MaxIter

```
Iter = 0
Do
  Iter = Iter + 1
  f = f(x,Y)
  g = g(x,Y)
  h = 0.001 * (1+|y|)
  fx = (f(x+h,y) - f)/h
```

```
  gx = (g(x+h,y) - g)/h
  h = 0.001 * (1+|y|)
  fy = (f(x,y+h) - f)/h
  gy = (g(x,y+h) - g)/h
  J = fy * gx - fx * gy
  DiffX = (fy * g - f * gy) / J
  DiffY = (gx * f - g * fx) / J
  x = x - DiffX
  y = y - DiffY
Until (|DiffX| < TolerX and |DiffY| < TolerY) or
      (Iter > MaxIter)
Return Iter, x, y
```

# HP-67 Implementation

## Instructions

Note: Numeric output is designated by red italic text.

| *Step* | *Input* | *Command* | *Output* |
|---|---|---|---|
| 1 | Move program pointer to label 9 | [GTO][9] | |
| 2 | Switch to program mode. | [PRGM] | |
| 3 | Edit the commands between LBL 9 and RTN to implement your f(x,y) function. | | |
| 4 | When done switch out of program mode. | [RUN] | |
| 5 | Move program pointer to label 8 | [GTO][8] | |
| 6 | Switch to program mode. | [PRGM] | |
| 7 | Edit the commands between LBL 8 and RTN to implement your g(x,y) function. | | |
| 8 | When done switch out of program mode. | [RUN] | |
| 9 | Enter the initial guesses for variables x and y. First enter the guess for variable x. | x[ENTER] | |
| 10 | Enter initial guess for y. | y[A] | |
| 11 | Enter tolerance limit for x. | TolerX[ENTER] | |

| Step | Input | Command | Output |
|---|---|---|---|
| 12 | Enter tolerance limit for y. | TolerY[f][A] | |
| 13 | Enter maximum number of iterations and start the iterations. | MaxIter[B] | |
| 14 | To view the number of iterations. | | iterations |
| 15 | To view the root for variable x. | [R/S] | x |
| 16 | To view the root for variable y. | [R/S] | y |
| 17 | To investigate other possible roots, go to step 9. | | |
| 18 | To solve for a different functions f(x,y) and/or g(x,y) go to step 1. | | |

## Example

Note: Since the example uses the existing code for functions f(x,y) and g(x,y) found I labels 9 and 8, we will start with the step that simply runs the programs. The code calculates the values for the following functions:

f(xy) = x^2 + y^3 – 31 = 0

g(x,y) = x * y – 6 = 0

The example uses the following input:

- Initial guess for x is 5.
- Initial guess for y is 5.
- Tolerance for the root of variable x is 1E-8.
- Tolerance for the root of variable y is 1E-8.
- The maximum number of iterations is 55.

| Step | Comment | Command | Output |
|---|---|---|---|
| 1 | Enter 5 for the initial guess for x. | 5 [ENTER] | |
| 2 | Enter 5 for the initial guess for y. | 5 [A] | |
| 3 | Enter 1E-8 for the tolerance limit for x. | 1[EEX]8[CHS][ENTER] | |
| 4 | Enter 1E-8 for the tolerance limit for y. | 1[EEX]8[CHS][f][A] | |
| 5 | Enter 55 for the maximum number of iterations and start the iterations. | 55[B] | |
| 6 | The program displays the | | 7.00000 |

| Step | Comment | Command | Output |
|---|---|---|---|
| | number of iterations. | | |
| 7 | To view the root for variable x. | [R/S] | 2.00000 |
| 8 | To view the root for variable y. | [R/S] | 3.00000 |

## Memory Map

**R0 = x**
**R1 = y**
**R2 = hx, hy, J**
**R3 = f(x,y)**
**R4 = g(x,y)**
**R5 = Fx(x,y)**
**R6 = Fy(x,y)**
**R7 = Gx(x,y)**
**R8 = Gy(x,y)**
**R9 = Iter**
**RA = Toler x**
**RB = Toler y**
**RC = Delta x**
**RD = Delta y**
**RE = MaxIter**

## Listing

| Program Step | Comment |
|---|---|
| ♦LBL A | Enter values for initial guesses for x and y. |
| STO 1 | |
| X<>Y | |
| STO 0 | |
| RTN | |
| ♦LBL a | Enter values for initial guesses for the tolerances of x and y. |
| STO A | |
| X<>Y | |
| STO E | |
| RTN | |

| *Program Step* | *Comment* |
|---|---|
| ♦LBL B | Enter the maximum number of iterations and proceed with the iterations. |
| STO D | |
| 0 | |
| STO 9 | I = 0 |
| ♦LBL 0 | Start the main loop |
| RCL 1 | |
| RCL 0 | |
| PAUSE | Pause to display x |
| GSB 9 | Calculate f(x,y) |
| STO 3 | Store f(x,y) |
| RCL 1 | |
| RCL 0 | |
| GSB 8 | Calculate g(x,y) |
| STO 4 | Store g(x,y) |
| RCL 0 | |
| ABS | |
| 1 | |
| STO+ 9 | I = I + 1 |
| + | |
| .001 | |
| * | |
| STO 2 | h = 0.001 * (1 + ABS(x)) |
| RCL 1 | |
| RCL 0 | |
| RCL 2 | |
| + | |
| GSB 9 | Calculate f(x+h,y) |
| RCL 3 | |
| - | |
| RCL 2 | |
| / | |
| STO 5 | Store Fx(x,y) |
| RCL 1 | |
| RCL 0 | |
| RCL 2 | |
| + | |

| *Program Step* | *Comment* |
|---|---|
| GSB 8 | Calculate g(x+h,y) |
| RCL 4 | |
| - | |
| RCL 2 | |
| / | |
| STO 7 | Store Gx(x,y) |
| RCL 1 | |
| PAUSE | Pause to display y |
| ABS | |
| 1 | |
| + | |
| .001 | |
| * | |
| STO 2 | h = 0.001 * (1 + ABS(y)) |
| RCL 1 | |
| + | |
| RCL 0 | |
| GSB 9 | Calculate f(x,y+h) |
| RCL 3 | |
| - | |
| RCL 2 | |
| / | |
| STO 6 | Store Fy(x,y) |
| RCL 1 | |
| RCL 2 | |
| + | |
| RCL 0 | |
| GSB 8 | Calculate g(x,y+h) |
| RCL 4 | |
| - | |
| RCL 2 | |
| / | |
| STO 8 | Store Gy(x,y) |
| RCL05 | |
| * | |
| RCL 6 | |
| RCL 7 | |

| *Program Step* | *Comment* |
|---|---|
| * | |
| X<>Y | |
| - | |
| STO 2 | J = fy * gx - fx * gy |
| 1/X | Put 1/J in the stack |
| RCL 6 | |
| RCL 4 | |
| * | |
| RCL 3 | |
| RCL 8 | |
| * | |
| - | |
| * | Mutliply (fy * g - f * gy) by 1/J |
| STO B | DiffX = (fy * g - f * gy) / J |
| STO- 0 | X = X - DiffX |
| RCL 7 | |
| RCL 3 | |
| * | |
| RCL 4 | |
| RCL 5 | |
| * | |
| - | |
| RCL 2 | |
| / | |
| STO C | DiffY = (gx * f - g * fx) / J |
| ST0- 1 | Y = Y - DiffY |
| RCL D | |
| RCL 9 | |
| X>Y? | Is Iter > MaxIter? |
| GTO 1 | Exit loop |
| RCL E | |
| RCL B | |
| ABS | |
| X>Y? | Is ABS(DiffX) > TolerX |
| GTO 0 | Resume the next iteration |
| RCL A | |
| RCL C | |

| *Program Step* | *Comment* |
|---|---|
| ABS | |
| X>Y? | Is ABS(DiffY) > TolerY |
| GTO 0 | Resume the next iteration |
| ♦LBL 1 | Display results |
| RCL 9 | |
| R/S | Display the number of iterations |
| RCL 0 | |
| R/S | Display root for variable x |
| RCL 1 | |
| R/S | Display root for variable y |
| RTN | |
| | |
| ♦LBL 9 | Function f(x,y) |
| X^2 | |
| X<>Y | |
| 3 | |
| Y^X | |
| + | |
| 31 | |
| - | |
| RTN | |
| ♦LBL 8 | Function g(x,y) |
| * | |
| 6 | |
| - | |
| RTN | |